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General Certificate of Education (A-level)
January 2011

Mathematics MPC2

(Specification 6360)

Pure Core 2

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC2

Q	Solution	Marks	Total	Comments
1(a)	Arc = $r\theta$	M1		arc = $r\theta$ seen or used. PI by correct θ
	$4 = 5\theta \Rightarrow \theta = \frac{4}{5} = 0.8$	A1	2	(θ =) $\frac{4}{5}$ OE
(b)	Area of sector = $\frac{1}{2}r^2\theta$	M1		Area $=\frac{1}{2}r^2\theta$ seen or used within **(b)**. PI
	$= \frac{1}{2}\times 5^2 \times 0.8 = 10$ (cm^2)	A1F	2	Ft on 12.5×c's exact value for θ in part (a) provided $5 \le$ c's area ≤ 20
	Total		**4**	
2(a)(i)	(p =) 3	B1	1	
(ii)	(q =) –3	B1F	1	If not correct, ft on $-p$
(iii)	(r =) $\frac{1}{2}$	B1	1	OE
(b)	$2^{\frac{1}{2}} \times 2^x = 2^{-3} \Rightarrow 2^{\frac{1}{2}+x} = 2^{-3}$	M1		Using a law of indices or logs correctly to combine at least two of the powers of 2 PI
	$\Rightarrow x = -3\frac{1}{2}$	A1F	2	If not correct, ft on $x = q - r$ provided method shown
	Total		**5**	
3(a)	$10^2 = 8^2 + 5^2 - 2\times8\times5\cos\theta$	M1		Use of the cosine rule PI by next line
	$\cos\theta = \frac{8^2+5^2-10^2}{2\times8\times5} (= -\frac{11}{80} = -0.1375)$	m1		Rearrangement
	θ = 97.90(32...) = 97.9° (to nearest 0.1°)	A1	3	CSO (Must see either exact value for $\cos\theta$ or at least 4sf value for either $\cos\theta$ or θ before the printed answer 97.9°) AG
(b)(i)	Area = $\frac{1}{2}\times8\times5\sin\theta$	M1		OE
	= 19.810… = 19.8 (cm^2) to 3sf	A1	2	Condone > 3sf
(ii)	Area of triangle = $0.5\times BC\times AD$	M1		Or valid method to find sin B or sin C or B or C
	AD = [Ans.**(b)(i)**]÷[0.5×BC]	m1		Or AD = 5 sin B; or AD = 8 sin C OE
	$AD = \frac{19.810..}{5} = 3.962.. = 3.96$ (cm) to 3sf	A1	3	Condone > 3sf
	Total		**8**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$h = 0.5$ $f(x) = \sqrt{27x^3 + 4}$ $I \approx h/2\{\ldots\}$ $\{\ldots\} = f(0) + f(1.5) + 2[f(0.5) + f(1)]$	B1 M1		PI OE summing of areas of the 'trapezia'..
	$\{\ldots\} = \sqrt{4} + \sqrt{95.125} + 2(\sqrt{7.375} + \sqrt{31})$ $= 2 + 9.7532\ldots + 2(2.7156\ldots + 5.5677\ldots)$	A1		OE Accept 2dp rounded or truncated as evidence for surds
	$(I \approx)$ $0.25 \times 28.32012\ldots = 7.08$ (to 3sf)	A1	4	Must be 7.08
(b)	$g(x) = \sqrt{27\left(\frac{1}{3}x\right)^3 + 4} = \sqrt{x^3 + 4}$	M1 A1	 2	Any form which simplifies to $\sqrt{kx^3 + 4}$, $k \neq 27$, $k \neq 0$ or which simplifies to $x^3 + 4$ ACF
	Total		**6**	
5(a)	$(1-x)^3 = 1 - 3x + 3x^2 - x^3$	M1 A1	 2	3 terms correct or 1 (±)3 (±)3 (±)1 seen All correct
(b)	$(1+y)^4 = 1 + 4y + 6y^2 + 4y^3 + y^4$	M1 A1		4 terms correct, accept unsimplified All 5 terms correct and simplified at some stage
	$(1+y)^4 - (1-y)^3 =$ $(4y + 3y) + (6y^2 - 3y^2) + (4y^3 + y^3) + y^4$ $= 7y + 3y^2 + 5y^3 + y^4$ (as required with p=3 and q=5)	A2,1	4	A2 Be convinced as part answer is given (A1 for three terms found correctly or if found correct values for p and q but did not show $7y+y^4$.)
(c)	$\int \left[\left(1+\sqrt{x}\right)^4 - \left(1-\sqrt{x}\right)^3 \right] \mathrm{d}x =$ $\int \left(7\sqrt{x} + 3x + 5x\sqrt{x} + x^2\right) \mathrm{d}x$	 M1		 Use of part (**b**)….$y \to \sqrt{x}$ OE before any integration
	$\int \left(7x^{0.5} + 3x + 5x^{1.5} + x^2\right) \mathrm{d}x$ $= \frac{7x^{1.5}}{1.5} + \frac{3x^2}{2} + \frac{5x^{2.5}}{2.5} + \frac{x^3}{3}$ $(+c)$	 m1		 Correct integration of an x^k term where k is non-integer
	$= \frac{14}{3}x^{1.5} + \frac{3}{2}x^2 + 2x^{2.5} + \frac{1}{3}x^3$ $(+c)$	A2,1F	4	Coeffs simplified; condone absent $(+c)$ Ft on c's p and q ie 2nd term $+\frac{p}{2}x^2$ and 3rd term is $+\frac{2q}{5}x^{2.5}$. (A1F for three of these four ft terms or for four correct ft terms unsimplified)
	Total		**10**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$ar^2 = 36$; $ar^5 = 972$;	M1		For $ar^2 = 36$ or $ar^5 = 972$ or for seeing $36r^3 = 972$
	$r^3 = \dfrac{972}{36}$ $(= 27)$ $\Rightarrow r = 3$	A1	2	CSO AG Full valid completion.
(ii)	$a \times 3^2 = 36$	M1		OE. PI
	$a = 4$	A1	2	Correct answer without working scores the two marks
(b)(i)	$\sum_{n=1}^{20} u_n = S_{20} = \dfrac{a(1-r^{20})}{1-r}$	M1		OE
	$= \dfrac{4(1-3^{20})}{-2} = -2(1-3^{20}) = 2\left(3^{20}-1\right)$	A1	2	CSO AG Be convinced
(ii)	$u_n = a \times 3^{n-1}$	B1		Seen or used
	$4\times 3^{n-1} > 4\times 10^{15} \Rightarrow 3^{n-1} > 10^{15}$			
	$(n-1)\log 3$ $(>)$ $\log 10^{15}$	M1		Or finds values of u_n for appropriate adjacent integer values of n so that u_n's are either side of 4×10^{15}
	$n-1 > \dfrac{15}{\log_{10} 3}$; $n-1 > 31.4...$			
	($n > 32.4...$ and n is an integer so least value of n is) $n = 33$	A1	3	CSO
	Total		**9**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$y = x + 3 + \frac{8}{x^4} = x + 3 + 8x^{-4}$	B1		For $\frac{8}{x^4} = 8x^{-4}$ PI by correct differentiation of 3rd term
	$\frac{dy}{dx} = 1 - 32x^{-5}$ or $1 - \frac{32}{x^5}$	M1 A1	3	kx^{-5} OE For either
(b)	When $x = 1$, $y = 12$	B1		
	When $x = 1$, $\frac{dy}{dx} = 1 - 32 = -31$	M1		Attempt to find value of $\frac{dy}{dx}$ when x=1
	Tangent: $y - 12 = -31(x - 1)$	A1F	3	Only ft on c's answer to (**a**). Any correct (ft on c's (a)) form.
(c)	$1 - 32x^{-5} = 0$	M1		$1 - 32x^{-5} = 0$ or c's $\frac{dy}{dx} = 0$
	$\Rightarrow x^5 = 32$	m1		Attempt to form x^n = const ($\neq$0). PI by next line
	$\Rightarrow x = 2$ (Coordinates of M) (2, 5.5)	A1 A1	4	CSO CSO
(d)(i)	$\int \left(x + 3 + \frac{8}{x^4}\right) dx$			
	$= \frac{x^2}{2} + 3x - \frac{8}{3}x^{-3} + c$	M1 A1 B1	3	Power −3 correctly obtained $-\frac{8}{3}x^{-3}$ $\frac{x^2}{2} + 3x + c$
(ii)	Area = $\left[\frac{x^2}{2} + 3x - \frac{8}{3}x^{-3}\right]_1^2$			
	$= \left(2 + 6 - \frac{1}{3}\right) - \left(\frac{1}{2} + 3 - \frac{8}{3}\right)$	M1		Attempting to calculate F(2) − F(1) where F(x) is c's answer to part (d)(i) provided F is not just the c's integrand (x+3+8/x^4)
	$= \frac{9}{2} + \frac{7}{3} = \frac{41}{6}$	A1	2	OE Accept 6.83 or better provided d(i) used
(e)	$k = -5.5$	B1F	1	Ft on $-y_M$ from part (c).
	Total		**16**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\log_k x^2 - \log_k 5 = 1$	M1		A valid law of logs used correctly
	$\log_k \frac{x^2}{5} = 1$	M1		Another valid law of logs used correctly **or** correct method to reach log f(x) = log 5k
	$\log_k \frac{x^2}{5} = \log_k k$ [or $\log x^2 = \log 5k$]	A1		PI by next line
	$\Rightarrow \frac{x^2}{5} = k$ ie $k = \frac{x^2}{5}$	A1	4	Accept either of these two forms.
(b)	$\log_a y = \frac{3}{2}$; $\log_4 a = b + 2$			
	$\Rightarrow y = a^{\frac{3}{2}}$ $\Rightarrow a = 4^{b+2}$	M1		For either equation
	$y = (4^{b+2})^{\frac{3}{2}}$	m1		Elimination of a from two correct equations not involving logarithms
	$y = 2^{3(b+2)}$; $y = 2^{3b+6}$	A1	3	CSO Either form acceptable
	Total		**7**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
9(a)	$\tan x = -3$ $\Rightarrow x = \tan^{-1}(-3) \qquad (= -71.56...)^\circ$	M1		PI eg by 71(.56..) or −71(.56..) seen
	$x = 108^\circ,\ 288^\circ$	A1,A1	3	Condone more accurate answers. (108.4349…, 288.4349…). [Ignore answers outside interval; If more than 2 answers inside interval −1 from A marks for each extra to a min of 0]
(b)(i)	$7\sin^2\theta + \sin\theta\cos\theta = 6(\cos^2\theta + \sin^2\theta)$	M1		$\cos^2\theta + \sin^2\theta = 1$ used; OE
	$7\sin^2\theta - 6\sin^2\theta + \sin\theta\cos\theta - 6\cos^2\theta = 0$ $\Rightarrow \sin^2\theta + \sin\theta\cos\theta - 6\cos^2\theta = 0$			
	$\Rightarrow \dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\sin\theta}{\cos\theta} - 6 = 0$	M1		$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ used
	$\Rightarrow \tan^2\theta + \tan\theta - 6 = 0$	A1	3	CSO AG
(ii)	$(\tan\theta + 3)(\tan\theta - 2) = 0$	M1		Factorise or other valid method to solve quadratic
	$\tan\theta = -3$ or $\tan\theta = 2$	A1		Need both
	$\theta = 108^\circ,\ 288^\circ; \quad \theta = 63^\circ,\ 243^\circ;$	B2F,1F	4	**Only** ft on (a) for the c's two +'ve $\tan^{-1}(-3)$ vals. [B1 if 3 correct (ft)] Condone more accurate answers. (108.4349…, 288.4349…; 63.4349…, 243.4349…) [Ignore answers outside interval; If more than 2 answers for each inside interval, −1 for each extra from Bs to a min of 0]
	Total		**10**	
	TOTAL		**75**	