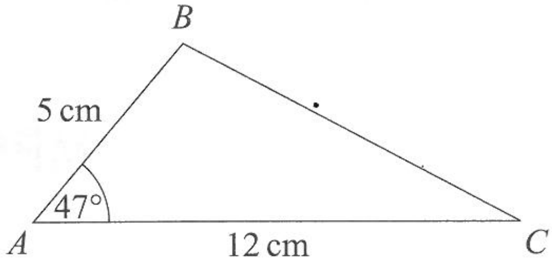
AQA MPC2 Core 2 Mathematics 22 May 2014

Question Paper and Worked Solutions

Please note, this document represents my own solutions to the questions, is entirely unofficial and is not related to the mark scheme (which I have not seen). Therefore, while it should help you see how to do the questions, it won't include every valid method or give you a break down of the mark allocation. If you spot any errors, or think you have found a better solution, please email me so I can update it.

1 The diagram shows a triangle ABC.



The size of angle BAC is 47° and the lengths of AB and AC are 5 cm and 12 cm respectively.

(a) Calculate the area of the triangle ABC, giving your answer to the nearest cm^2.

[2 marks]

(b) Calculate the length of BC, giving your answer, in cm, to one decimal place.

[3 marks]

1.

a)

$$Area = \frac{1}{2}ab\sin C = \frac{1}{2}(5)(12)\sin 47 = \mathbf{22cm^2\ to\ the\ nearest\ cm^2}$$

b)

$$a^2 = b^2 + c^2 - 2bc\cos A = 5^2 + 12^2 - 2(5)(12)\cos 47 = 87.16\ldots \quad \Rightarrow \quad a = \mathbf{9.3cm\ to\ 1\ d.p.}$$

2 (a) Find $\int\left(1+3x^{\frac{1}{2}}+x^{\frac{3}{2}}\right)\mathrm{d}x$.

[3 marks]

(b) (i) The expression $(1+y)^3$ can be written in the form $1+3y+ny^2+y^3$.
Write down the value of the constant n.

[1 mark]

(ii) Hence, or otherwise, expand $(1+\sqrt{x})^3$.

[1 mark]

(c) Hence find the exact value of $\int_0^1(1+\sqrt{x})^3\,\mathrm{d}x$.

[3 marks]

2.

a)

$$\int 1+3x^{\frac{1}{2}}+x^{\frac{3}{2}}\,dx = x+\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+C = \boldsymbol{x+2x^{\frac{3}{2}}+\frac{2x^{\frac{5}{2}}}{5}+C}$$

b)

i.

$$(1+y)^3 = 1+3y+3y^2+y^3 \quad \Rightarrow \quad \boldsymbol{n=3}$$

ii.

$$Let\ y=\sqrt{x} \quad \Rightarrow \quad \left(1+\sqrt{x}\right)^3 = \boldsymbol{1+3x^{\frac{1}{2}}+3x+x^{\frac{3}{2}}}$$

c)

$$\int_0^1\left(1+\sqrt{x}\right)^3 dx = \int_0^1 1+3x^{\frac{1}{2}}+3x+x^{\frac{3}{2}}\,dx = \left[x+2x^{\frac{3}{2}}+\frac{3x^2}{2}+\frac{2x^{\frac{5}{2}}}{5}\right]_0^1 = \left(1+2+\frac{3}{2}+\frac{2}{5}\right)-(0) = \frac{49}{10} = \mathbf{4.9}$$

3 The first term of a geometric series is 54 and the common ratio of the series is $\frac{8}{9}$.

(a) Find the sum to infinity of the series.

[2 marks]

(b) Find the second term of the series.

[1 mark]

(c) Show that the 12th term of the series can be written in the form $\frac{2^p}{3^q}$, where p and q are integers.

[3 marks]

3.

a)

$$S_\infty = \frac{a}{1-r} = \frac{54}{1-\frac{8}{9}} = \frac{54}{\frac{1}{9}} = 54\times 9 = \mathbf{486}$$

b)

$$S_n = \frac{a(1-r^n)}{1-r} \quad \Rightarrow \quad S_2 = \frac{54\left(1-\left(\frac{8}{9}\right)^2\right)}{1-\frac{8}{9}} = \frac{54\left(\frac{17}{81}\right)}{\frac{1}{9}} = \mathbf{102}$$

c)

$$U_n = ar^{n-1} \quad \Rightarrow \quad U_{12} = 54\times\left(\frac{8}{9}\right)^{11} = (2\times 3^3)\times\left(\frac{2^3}{3^2}\right)^{11} = 2\times 3^3\times\frac{2^{33}}{3^{22}} = \boldsymbol{\frac{2^{34}}{3^{19}}}$$

4 A curve has equation $y = \frac{1}{x^2} + 4x$.

(a) Find $\frac{dy}{dx}$.

[3 marks]

(b) The point $P(-1, -3)$ lies on the curve. Find an equation of the normal to the curve at the point P.

[3 marks]

(c) Find an equation of the tangent to the curve that is parallel to the line $y = -12x$.

[5 marks]

4.

a)

$$y = \frac{1}{x^2} + 4x = x^{-2} + 4x \quad \Rightarrow \quad \boldsymbol{\frac{dy}{dx} = -2x^{-3} + 4 = -\frac{2}{x^3} + 4}$$

b)

$$x = -1 \quad \Rightarrow \quad \frac{dy}{dx} = \left(-\frac{2}{(-1)^3} + 4\right) = 6 = gradient\ of\ tangent \quad \Rightarrow \quad gradient\ of\ normal = -\frac{1}{6}$$

$$y - y_1 = m(x - x_1) \quad \Rightarrow \quad y - -3 = -\frac{1}{6}(x - -1) \quad \Rightarrow \quad \boldsymbol{y + 3 = -\frac{1}{6}(x + 1) \quad or \quad x + 6y + 19 = 0}$$

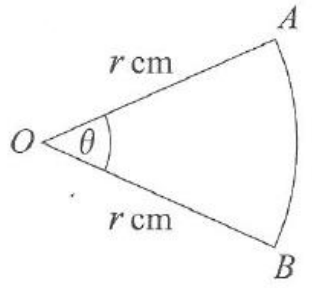
c)

$$\frac{dy}{dx} = -12 \quad \Rightarrow \quad -\frac{2}{x^3} + 4 = -12 \quad \Rightarrow \quad \frac{2}{x^3} = 16 \quad \Rightarrow \quad 2 = 16x^3 \quad \Rightarrow \quad x^3 = \frac{1}{8} \quad \Rightarrow \quad x = \frac{1}{2}$$

$$x = \frac{1}{2} \quad \Rightarrow \quad y = \frac{1}{\left(\frac{1}{2}\right)^2} + 4\left(\frac{1}{2}\right) = 6$$

$$y - y_1 = m(x - x_1) \quad \Rightarrow \quad \boldsymbol{y - 6 = -12\left(x - \frac{1}{2}\right) \quad or \quad y = -12x + 12}$$

5 The diagram shows a sector OAB of a circle with centre O and radius r cm.



The angle AOB is θ radians.

The area of the sector is 12 cm^2.

The perimeter of the sector is four times the length of the arc AB.

Find the value of r.

[6 marks]

5.

$$l = r\theta \quad \Rightarrow \quad r\theta + 2r = 4r\theta \quad \Rightarrow \quad \theta + 2 = 4\theta \quad \Rightarrow \quad 2 = 3\theta \quad \Rightarrow \quad \theta = \frac{2}{3}$$

$$A = \frac{1}{2}r^2\theta \quad \Rightarrow \quad 12 = \frac{1}{2}r^2\left(\frac{2}{3}\right) \quad \Rightarrow \quad r^2 = 36 \quad \Rightarrow \quad \boldsymbol{r = 6cm}$$

6 (a) Sketch, on the axes given below, the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.

[2 marks]

(b) Describe the geometrical transformation that maps the graph of $y = \sin x$ onto the graph of $y = \sin 5x$.

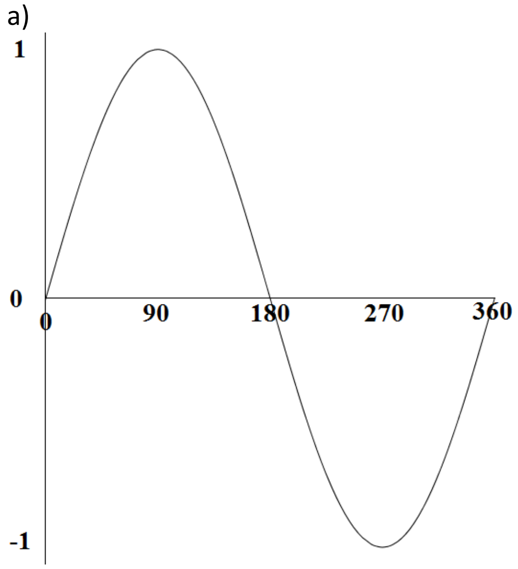
[2 marks]

(c) Describe the single geometrical transformation that maps the graph of $y = \sin 5x$ onto the graph of $y = \sin(5x + 10°)$.

[2 marks]

6.

a)



b)

$$y = \sin x \quad \rightarrow \quad y = \sin 5x$$

$$\boldsymbol{Stretch\ in\ the\ x\ direction\ of\ scale\ factor\ \frac{1}{5}}$$

c)

$$y = \sin 5x \quad \rightarrow \quad y = \sin(5x + 10) = \sin\big(5(x + 2)\big)$$

$$\boldsymbol{Translation\ by\ vector\ \begin{bmatrix} -2 \\ 0 \end{bmatrix}}$$

7 (a) Given that $\dfrac{\cos^2 x + 4\sin^2 x}{1 - \sin^2 x} = 7$, show that $\tan^2 x = \dfrac{3}{2}$.

[3 marks]

(b) **Hence** solve the equation $\dfrac{\cos^2 2\theta + 4\sin^2 2\theta}{1 - \sin^2 2\theta} = 7$ in the interval $0° < \theta < 180°$, giving your values of θ to the nearest degree.

[4 marks]

7.

a)

$$\frac{\cos^2 x + 4\sin^2 x}{1 - \sin^2 x} = 7 \implies \frac{\cos^2 x + 4\sin^2 x}{\cos^2 x} = 7 \implies \frac{\cos^2 x}{\cos^2 x} + \frac{4\sin^2 x}{\cos^2 x} = 7 \implies 1 + 4\tan^2 x = 7$$

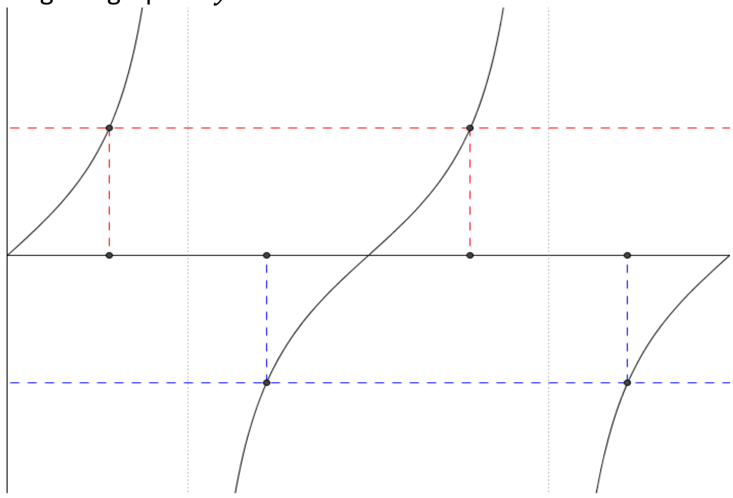
$$\implies 4\tan^2 x = 6 \implies \tan^2 x = \frac{6}{4} = \mathbf{\frac{3}{2}}$$

b)

$$0° < \theta < 180° \implies 0° < 2\theta < 360°$$

$$\frac{\cos^2 2\theta + 4\sin^2 2\theta}{1 - \sin^2 2\theta} = 7 \implies \tan^2 2\theta = \frac{3}{2} \implies \tan 2\theta = \pm\sqrt{\frac{3}{2}} \implies 2\theta = 50.76\ldots° \quad or \quad -50.76\ldots°$$

Using the graph of $y = \tan x$:



$$2\theta = 50.76\ldots°, 230.76\ldots°$$

$$2\theta = -50.76\ldots°, 129.23\ldots°, 309.23\ldots°$$

Within range:

$$2\theta = 50.76\ldots°, 230.76\ldots°, 129.23\ldots°, 309.23\ldots°$$

$$\implies \boldsymbol{\theta = 25°, \quad 65°, \quad 115°, \quad 155°}$$

8 An arithmetic series has first term a and common difference d.

The sum of the first 5 terms of the series is 575.

(a) Show that $a+2d=115$.

[3 marks]

(b) Given also that the 10th term of the series is 87, find the value of d.

[3 marks]

(c) The nth term of the series is u_n. Given that $u_k > 0$ and $u_{k+1} < 0$, find the value of $\sum_{n=1}^{k} u_n$.

[5 marks]

8.

a)

$$S_n = \frac{n}{2}(2a+(n-1)d) \quad \Rightarrow \quad 575 = \frac{5}{2}(2a+4d) \quad \Rightarrow \quad 230 = 2a+4d \quad \Rightarrow \quad \boldsymbol{a+2d=115}$$

b)

$$U_n = a+(n-1)d \quad \Rightarrow \quad 87 = a+9d$$

Solving simultaneously:

$$(a+9d)-(a+2d) = 87-115 \quad \Rightarrow \quad 7d=-28 \quad \Rightarrow \quad \boldsymbol{d=-4}$$

c)

$$d=-4 \quad \Rightarrow \quad a+2(-4)=115 \quad \Rightarrow \quad a=123$$

Solving for $U_n = 0$:

$$U_n = a+(n-1)d \quad \Rightarrow \quad 123+(n-1)(-4)=0 \quad \Rightarrow \quad 123-4n+4=0 \quad \Rightarrow \quad n=\frac{127}{4}=31.75$$

$$\Rightarrow \quad k=31 \quad and \quad k+1=32$$

$$\sum_{n=1}^{k} U_n = S_k = S_{31} = \frac{31}{2}\big(2(123)+(31-1)(-4)\big) = \frac{31}{2}(246-120) = \frac{31}{2}(126) = \mathbf{1953}$$

9 A curve has equation $y = 3 \times 12^x$.

(a) The point $(k, 6)$ lies on the curve $y = 3 \times 12^x$. Use logarithms to find the value of k, giving your answer to three significant figures.

[3 marks]

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^{1.5} 3 \times 12^x \, dx$, giving your answer to two significant figures.

[4 marks]

(c) The curve $y = 3 \times 12^x$ is translated by the vector $\begin{bmatrix} 1 \\ p \end{bmatrix}$ to give the curve $y = f(x)$. Given that the curve $y = f(x)$ passes through the origin $(0, 0)$, find the value of the constant p.

[3 marks]

(d) The curve with equation $y = 2^{2-x}$ intersects the curve $y = 3 \times 12^x$ at the point T. Show that the x-coordinate of T can be written in the form $\dfrac{2 - \log_2 3}{q + \log_2 3}$, where q is an integer. State the value of q.

[5 marks]

9.

a)

$$x = k, y = 6 \quad \Rightarrow \quad 6 = 3 \times 12^k \quad \Rightarrow \quad 2 = 12^k \quad \Rightarrow \quad \ln 2 = \ln 12^k \quad \Rightarrow \quad \ln 2 = k \ln 12$$

$$\Rightarrow \quad k = \frac{\ln 2}{\ln 12} = \mathbf{0.279\ to\ 3\ s.f.}$$

b)

$$\int_0^{1.5} 3 \times 12^x \, dx \approx \frac{1}{2}h\{(y_0 + y_3) + 2(y_1 + y_2)\} \qquad h = \frac{1.5 - 0}{3} = 0.5$$

$$\begin{array}{ll} x_0 = 0 & y_0 = 3 \times 12^0 = 3 \\ x_1 = 0.5 & y_1 = 3 \times 12^{0.5} \approx 10.39 \\ x_2 = 1 & y_2 = 3 \times 12^1 = 36 \\ x_3 = 1.5 & y_3 = 3 \times 12^{1.5} = 124.71 \end{array}$$

$$\int_0^{1.5} 3 \times 12^x \, dx \approx \frac{1}{2}h\{(3 + 124.70 \ldots) + 2(10.39 \ldots + 36)\} = \mathbf{55\ to\ 2\ s.f.}$$

c)

$$y = 3 \times 12^x \quad \longrightarrow \quad y = 3 \times 12^{x-1} + p$$

$$x = 0, y = 0 \quad \Rightarrow \quad 0 = 3 \times 12^{0-1} + p \quad \Rightarrow \quad p = -\frac{3}{12} = -\mathbf{\frac{1}{4}}$$

d)

$$2^{2-x} = 3 \times 12^x \quad \Rightarrow \quad 2 - x = \log_2(3 \times 12^x) \quad \Rightarrow \quad 2 - x = \log_2 3 + x \log_2 12$$

$$\Rightarrow \quad 2 - x = \log_2 3 + x(\log_2 3 + \log_2 4) \quad \Rightarrow \quad 2 - x = \log_2 3 + x(\log_2 3 + 2)$$

$$\Rightarrow \quad 2 - x = \log_2 3 + x \log_2 3 + 2x \quad \Rightarrow \quad 2 - \log_2 3 = 3x + x \log_2 3 \quad \Rightarrow \quad 2 - \log_2 3 = x(3 + \log_2 3)$$

$$\Rightarrow \quad \boldsymbol{x = \frac{2 - \log_2 3}{3 + \log_2 3}}$$