AQA MM2B Mechanics 2 Mathematics 23 June 2014

Question Paper and Worked Solutions

Please note, this document represents my own solutions to the questions, is entirely unofficial and is not related to the mark scheme (which I have not seen). Therefore, while it should help you see how to do the questions, it won't include every valid method or give you a break down of the mark allocation. If you spot any errors, or think you have found a better solution, please [email me](#) so I can update it.

1 An eagle has caught a salmon of mass $3\ \mathrm{kg}$ to take to its nest. When the eagle is flying with speed $8\ \mathrm{m\,s^{-1}}$, it drops the salmon. The salmon falls a vertical distance of 13 metres back into the sea.

The salmon is to be modelled as a particle. The salmon's weight is the only force that acts on it as it falls to the sea.

(a) Calculate the kinetic energy of the salmon when it is dropped by the eagle.
[2 marks]

(b) Calculate the potential energy lost by the salmon as it falls to the sea.
[2 marks]

(c) (i) Find the kinetic energy of the salmon when it reaches the sea.
[2 marks]

(ii) Hence find the speed of the salmon when it reaches the sea.
[2 marks]

1.

a)

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(3)(8)^2 = \boldsymbol{96\ J}$$

b)

$$GPE = mgh = 3g(13) = \boldsymbol{382.2\ J}$$

c)

i.

$$KE_I + GPE_I = KE_F + GPE_F \quad \Rightarrow \quad 96 + 382.2 = KE + 0 \quad \Rightarrow \quad KE = \boldsymbol{478.2\ J}$$

ii.

$$\frac{1}{2}mv^2 = 478.2 \quad \Rightarrow \quad \frac{1}{2}(3)v^2 = 478.2 \quad \Rightarrow \quad v = \boldsymbol{17.9\ ms^{-1}\ to\ 3\ s.f.}$$

2 A particle has mass $6\,\mathrm{kg}$. A single force $(24\mathrm{e}^{-2t}\mathbf{i} - 12t^3\mathbf{j})$ newtons acts on the particle at time t seconds. No other forces act on the particle.

(a) Find the acceleration of the particle at time t.

[2 marks]

(b) At time $t = 0$, the velocity of the particle is $(-7\mathbf{i} - 4\mathbf{j})\,\mathrm{m\,s^{-1}}$

Find the velocity of the particle at time t.

[4 marks]

(c) Find the speed of the particle when $t = 0.5$.

[4 marks]

2.

a)

$$F = ma \quad \Rightarrow \quad \begin{bmatrix} 24e^{-2t} \\ -12t^3 \end{bmatrix} = 6a \quad \Rightarrow \quad a = \frac{1}{6}\begin{bmatrix} 24e^{-2t} \\ -12t^3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{4e^{-2t}} \\ \boldsymbol{-2t^3} \end{bmatrix}$$

b)

$$v = \int a \, dt = \begin{bmatrix} -2e^{-2t} \\ -\frac{1}{2}t^4 \end{bmatrix} + C$$

$$v = \begin{bmatrix} -7 \\ -4 \end{bmatrix} \ at\ t = 0 \quad \Rightarrow \quad \begin{bmatrix} -7 \\ -4 \end{bmatrix} = \begin{bmatrix} -2e^0 \\ -\frac{1}{2}0^4 \end{bmatrix} + C = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + C \quad \Rightarrow \quad C = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \quad \Rightarrow \quad \boldsymbol{v} = \begin{bmatrix} \boldsymbol{-2e^{-2t} - 5} \\ \boldsymbol{-\frac{1}{2}t^4 - 4} \end{bmatrix}$$

c)

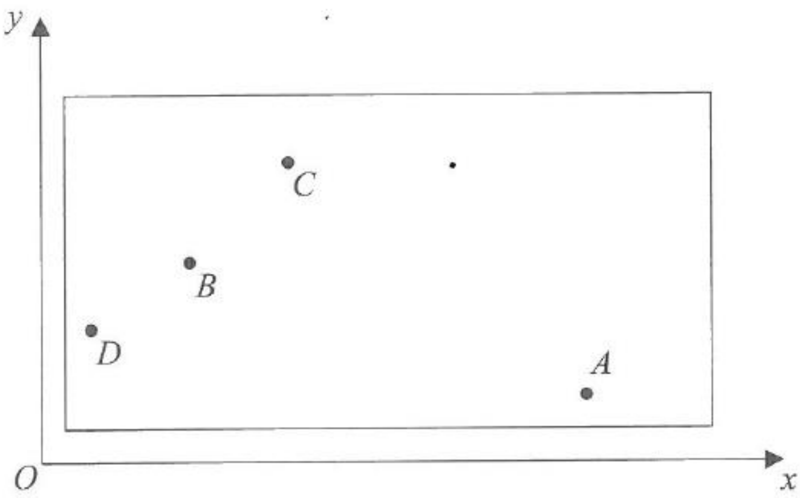
$$t = 0.5 \quad \Rightarrow \quad v = \begin{bmatrix} -2e^{-2(0.5)} - 5 \\ -\frac{1}{2}(0.5)^4 - 4 \end{bmatrix} = \begin{bmatrix} -5.7357\ldots \\ -4.03125 \end{bmatrix}$$

$$\Rightarrow \quad speed = |v| = \sqrt{(-5.7357\ldots)^2 + (-4.03125)^2} = \boldsymbol{7.01\ ms^{-1}\ to\ 3\ s.f.}$$

3 Four tools are attached to a board.

The board is to be modelled as a uniform lamina and the four tools as four particles.

The diagram shows the lamina, the four particles A, B, C and D, and the x and y axes.



The lamina has mass 5 kg and its centre of mass is at the point $(7, 6)$.

Particle A has mass 4 kg and is at the point $(11, 2)$.
Particle B has mass 3 kg and is at the point $(3, 6)$.
Particle C has mass 7 kg and is at the point $(5, 9)$.
Particle D has mass 1 kg and is at the point $(1, 4)$.

Find the coordinates of the centre of mass of the system of board and tools.

[5 marks]

3.

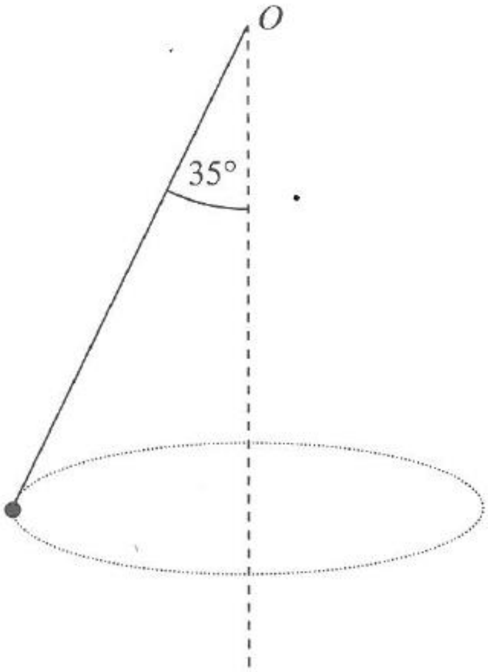
Object	**Mass**	**Centre of mass**
Lamina	5	$(7,6)$
A	4	$(11,2)$
B	3	$(3,6)$
C	7	$(5,9)$
D	1	$(1,4)$
Total	**20**	$(\bar{x}, \bar{y})$

$$5 \times 7 + 4 \times 11 + 3 \times 3 + 7 \times 5 + 1 \times 1 = 20\bar{x} \quad \Rightarrow \quad \bar{x} = \frac{124}{20} = 6.2$$

$$5 \times 6 + 4 \times 2 + 3 \times 6 + 7 \times 9 + 1 \times 4 = 20\bar{y} \quad \Rightarrow \quad \bar{y} = \frac{123}{20} = 6.15$$

$$\boldsymbol{Centre\ of\ mass}: \quad (\mathbf{6.2, 6.15})$$

4 A particle, of mass $0.8\,\text{kg}$, is attached to one end of a light inextensible string. The other end of the string is attached to the fixed point O. The particle is set in motion, so that it moves in a horizontal circle at constant speed, with the string at an angle of $35°$ to the vertical. The centre of this circle is vertically below O, as shown in the diagram.



The particle moves in a horizontal circle and completes 20 revolutions each minute.

(a) Find the angular speed of the particle in radians per second.

[2 marks]

(b) Find the tension in the string.

[3 marks]

(c) Find the radius of the horizontal circle.

[4 marks]

4.

a)

$$20\ rpm = \frac{20 \times 2\pi}{60}\ rad\ s^{-1} = \frac{2\pi}{3}\ rad\ s^{-1} = \mathbf{2.09\ rad\ s^{-1}\ to\ 3\ s.f.}$$

b)

Resolving vertically:

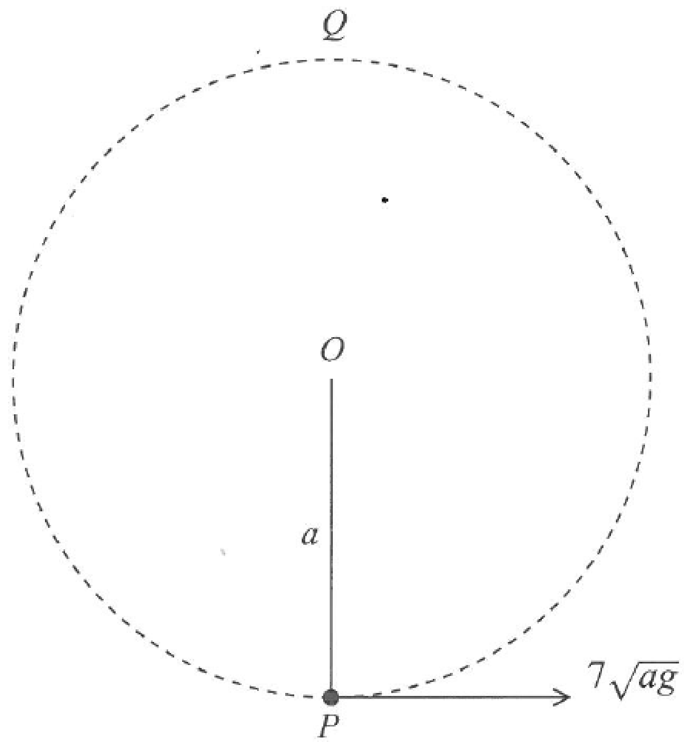
$$T\cos 35 = 0.8g \quad \Rightarrow \quad T = \frac{0.8g}{\cos 35} = \mathbf{9.57\ N\ to\ 3\ s.f.}$$

c)

Resolving radially:

$$T\sin 35 = mr\omega^2 \quad \Rightarrow \quad (9.57\ldots)\sin 35 = 0.8r(2.09\ldots)^2 \quad \Rightarrow \quad r = \frac{(9.57\ldots)\sin 35}{0.8(2.09\ldots)^2} = \mathbf{1.56\ m\ to\ 3\ s.f.}$$

5 A light inextensible string, of length a, has one end attached to a fixed point O. A particle, of mass m, is attached to the other end of the string. The particle is moving in a vertical circle with centre O. The point Q is the highest point of the particle's path. When the particle is at P, vertically below O, the string is taut and the particle is moving with speed $7\sqrt{ag}$, as shown in the diagram.



(a) Find, in terms of g and a, the speed of the particle at the point Q.

[4 marks]

(b) Find, in terms of g and m, the tension in the string when the particle is at Q.

[3 marks]

5.

a)

Using conservation of energy:

$$KE_I + GPE_I = KE_F + GPE_F \quad \Rightarrow \quad \frac{1}{2}m\left(7\sqrt{ag}\right)^2 + 0 = \frac{1}{2}mv^2 + mg(2a)$$

$$\Rightarrow \quad \frac{49mga}{2} = \frac{1}{2}mv^2 + 2mga \quad \Rightarrow \quad 49ga = v^2 + 4ga \quad \Rightarrow \quad v^2 = 45ga \quad \Rightarrow \quad \boldsymbol{v = \sqrt{45ga}}$$

b)

Resolving radially at Q:

$$mg + T = \frac{mv^2}{r} \quad \Rightarrow \quad mg + T = \frac{m\left(\sqrt{45ga}\right)^2}{a} \quad \Rightarrow \quad T = \frac{45mga}{a} - mg = \boldsymbol{44mg}$$

6 A puck, of mass m kg, is moving in a straight line across smooth horizontal ice. At time t seconds, the puck has speed $v\,\mathrm{m\,s^{-1}}$. As the puck moves, it experiences an air resistance force of magnitude $0.3mv^{\frac{1}{3}}$ newtons, until it comes to rest. No other horizontal forces act on the puck.

When $t = 0$, the speed of the puck is $8\,\mathrm{m\,s^{-1}}$.

Model the puck as a particle.

(a) Show that

$$v = (4 - 0.2t)^{\frac{3}{2}}$$

[6 marks]

(b) Find the value of t when the puck comes to rest.

[2 marks]

(c) Find the distance travelled by the puck as its speed decreases from $8\,\mathrm{m\,s^{-1}}$ to zero.

[5 marks]

6.

a)

$$F = -0.3mv^{\frac{1}{3}} \quad\Rightarrow\quad ma = -0.3mv^{\frac{1}{3}} \quad\Rightarrow\quad a = -0.3v^{\frac{1}{3}} \quad\Rightarrow\quad \frac{dv}{dt} = -0.3v^{\frac{1}{3}}$$

$$\Rightarrow \quad \int v^{-\frac{1}{3}}\,dv = \int -0.3\,dt \quad\Rightarrow\quad \frac{3}{2}v^{\frac{2}{3}} = -0.3t + C$$

$$v = 8\ \ at\ \ t = 0 \quad\Rightarrow\quad \frac{3}{2}\left(8^{\frac{2}{3}}\right) = 0 + C \quad\Rightarrow\quad C = 6 \quad\Rightarrow\quad \frac{3}{2}v^{\frac{2}{3}} = -0.3t + 6 \quad\Rightarrow\quad v^{\frac{2}{3}} = -0.2t + 4$$

$$\Rightarrow \quad v = (-0.2t + 4)^{\frac{3}{2}} = \boldsymbol{(4 - 0.2t)^{\frac{3}{2}}}$$

b)

$$v = 0 \quad\Rightarrow\quad (4 - 0.2t)^{\frac{3}{2}} = 0 \quad\Rightarrow\quad 4 - 0.2t = 0 \quad\Rightarrow\quad t = \frac{4}{0.2} = \mathbf{20}$$

c)

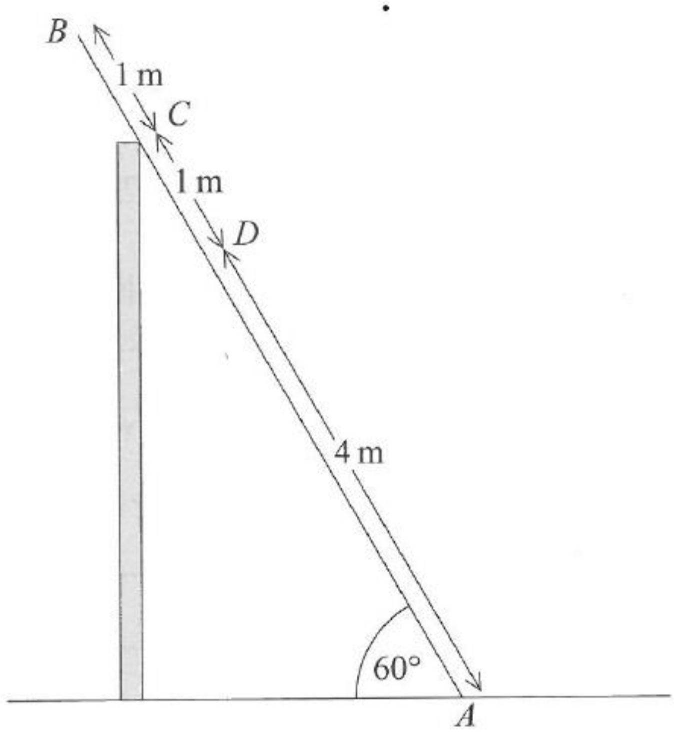
$$x = \int_0^{20} v\,dt = \int_0^{20} (4 - 0.2t)^{\frac{3}{2}}\,dt = \left[\frac{2(4 - 0.2t)^{\frac{5}{2}}}{5(-0.2)}\right]_0^{20} = (0) - (-64) = \boldsymbol{64\ m}$$

Note: Since the speed is always non-negative within this range, distance is equal to displacement.

7 A uniform ladder AB, of length 6 metres and mass 22 kg, rests with its foot, A, on rough horizontal ground. The ladder rests against the top of a smooth vertical wall at the point C, where the length AC is 5 metres. The vertical plane containing the ladder is perpendicular to the wall, and the angle between the ladder and the ground is $60°$. A man, of mass 88 kg, is standing on the ladder.

The man may be modelled as a particle at the point D, where the length of AD is 4 metres.

The ladder is on the point of slipping.



(a) Draw a diagram to show the forces acting on the ladder.

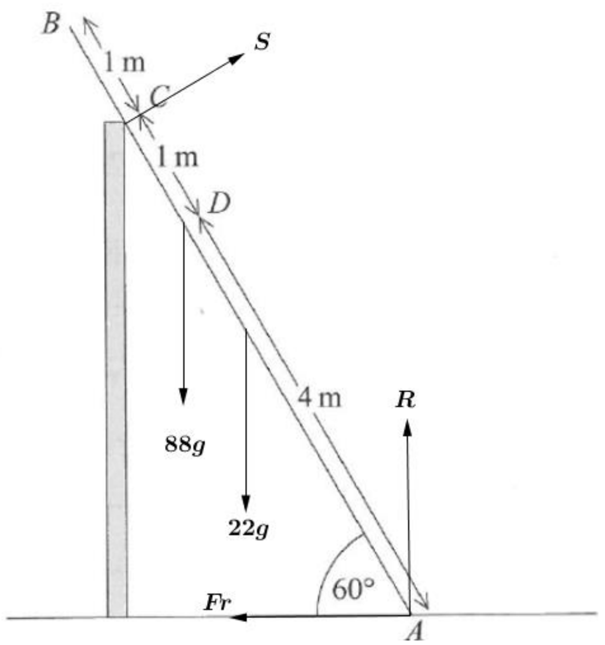
[2 marks]

(b) Find the coefficient of friction between the ladder and the horizontal ground.

[6 marks]

7.

a)



b)

Taking moments about A:

$$5S = 3(22g\cos 60) + 4(88g\cos 60)$$

$$\Rightarrow \quad S = 409.64\ N$$

Resolving forces vertically:

$$R + S\cos 60 = 88g + 22g$$

$$\Rightarrow \quad R = 110g - 409.64\cos 60 = 873.18\ N$$

Resolving forces horizontally:

$$F_r = S\sin 60 = 409.64\sin 60 = 354.75\ldots$$

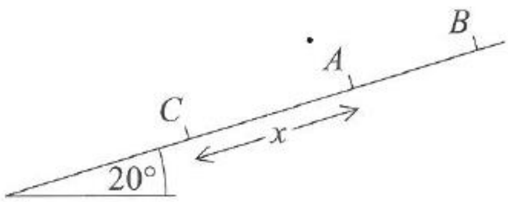
Using friction in limiting equilibrium:

$$F_r = \mu R \quad \Rightarrow \quad \mu = \frac{F_r}{R} = \frac{354.75\ldots}{873.18} = \mathbf{0.406\ to\ 3\ s.f.}$$

8 An elastic string has natural length 1.5 metres and modulus of elasticity 120 newtons. One end of the string is attached to a fixed point, A, on a rough plane inclined at $20°$ to the horizontal. The other end of the elastic string is attached to a particle of mass $4\,\text{kg}$. The coefficient of friction between the particle and the plane is 0.8.

The three points, A, B and C, lie on a line of greatest slope.

The point C is x metres from A, as shown in the diagram. The particle is released from rest at C and moves up the plane.



(a) Show that, as the particle moves up the plane, the frictional force acting on the particle is $29.5\,\text{N}$, correct to three significant figures.

[3 marks]

(b) The particle comes to rest for an instant at B, which is 2 metres from A.

The particle then starts to move back towards A.

(i) Find x.

[8 marks]

(ii) Find the acceleration of the particle as it starts to move back towards A.

[4 marks]

8.

a)

Resolving perpendicular to the slope:

$$R = 4g\cos 20 = 36.83\ldots N$$

Using friction in motion:

$$F_r = \mu R = 0.8 \times 36.83\ldots = 29.46\ldots = \mathbf{29.5\ N\ to\ 3\ s.f.}$$

b)

i.

$$l = 1.5 \quad \lambda = 120 \quad e = x - 1.5 \quad \Rightarrow \quad EPE_I = \frac{\lambda e^2}{2l} = \frac{120(x-1.5)^2}{3} = 40(x-1.5)^2$$

$$l = 1.5 \quad \lambda = 120 \quad e = 2 - 1.5 = 0.5 \quad \Rightarrow \quad EPE_F = \frac{\lambda e^2}{2l} = \frac{120(0.5)^2}{3} = 10$$

$$GPE_I = 0 \qquad GPE_F = mgh = 4g(x+2)\sin 20$$

$$KE_I = 0 \qquad KE_F = 0$$

$$WD_{F_r} = 29.5(x+2)$$

Using conservation of energy and work done against friction:

$$EPE_I + GPE_I + KE_I = EPE_F + GPE_F + KE_F + WD_{F_r}$$

$$40(x-1.5)^2 + 0 + 0 = 10 + 4g(x+2)\sin 20 + 0 + (29.46\ldots)(x+2)$$

$$40(x-1.5)^2 = 10 + 4g\sin 20\, x + 8g\sin 20 + (29.46\ldots)x + 58.9\ldots$$

$$40(x^2 - 3x + 2.25) = 95.75\ldots + (42.87\ldots)x$$

$$40x^2 - 120x + 90 = 95.75\ldots + (42.87\ldots)x$$

$$40x^2 - (162.87\ldots)x - 5.75\ldots = 0$$

$$x = \frac{(162.87\ldots) \pm \sqrt{(-162.87\ldots)^2 - 4(40)(-5.75\ldots)}}{80} = 4.106\ldots \;\; or \; -0.035\ldots$$

$l = 1.5$ means that $x \geq 1.5$ in this case (otherwise it would never have accelerated from C).

$$\Rightarrow \quad \boldsymbol{x = 4.11\ m\ to\ 3\ s.f.}$$

ii.

$$l = 1.5 \quad \lambda = 120 \quad x_B = 0.5 \quad \Rightarrow \quad T = \frac{\lambda x_B}{l} = \frac{120 \times 0.5}{1.5} = 40\ N$$

Resolving forces down the slope and using $F = ma$:

$$4g\sin 20 + 40 - 29.46\ldots = 4a \quad \Rightarrow \quad a = 5.984\ldots = \boldsymbol{5.98\ ms^{-2}\ to\ 3\ s.f.}$$