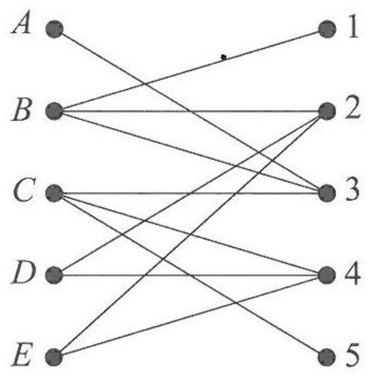
AQA MD01 Decision 1 Mathematics 18 June 2014

Question Paper and Worked Solutions

Please note, this document represents my own solutions to the questions, is entirely unofficial and is not related to the mark scheme (which I have not seen). Therefore, while it should help you see how to do the questions, it won't include every valid method or give you a break down of the mark allocation. If you spot any errors, or think you have found a better solution, please [email me](#) so I can update it.

1 Five people, A, B, C, D and E, are to be allocated to five tasks, 1, 2, 3, 4 and 5. The following bipartite graph shows the tasks that each person is able to undertake.



(a) Represent this information in an adjacency matrix.

[2 marks]

(b) Initially, A is allocated to task 3, B to task 2 and C to task 4.

(i) Demonstrate, by using an alternating-path algorithm from this initial matching, how each person can be allocated to a different task.

[3 marks]

(ii) Find a different allocation of people to tasks.

[1 mark]

1.

a)

	A	**B**	**C**	**D**	**E**
1	0	1	0	0	0
2	0	1	0	1	1
3	1	1	1	0	0
4	0	0	1	1	1
5	0	0	1	0	0

b)

i.

The alternating path algorithm:

1 From your initial matching, find a vertex on the left-hand subset not in the initial match and connect this vertex to a vertex on the right-hand set.

2. If the right-hand vertex is not in the initial matching then add this to the initial matching and repeat step 1. If the right hand vertex is in the initial matching go to step 3.

3. Add the new edge to the matching and remove from the initial matching the edge linking this vertex to the left-hand side. Repeat step 1 using the vertex that has just been removed from the matching.

4. Continue until there is a complete matching or no further improvement can be made.

1. Link D-2, 2. Break 2-B, 3. Link B-1, 4. Link E-4, 5. Break 4-C, 6. Link C-5, 7. Link A-3

Final (complete) matching: A-3, B-1, C-5, D-2, E-4.

ii.

Alternative matching: (noticing that D and E can both be assigned to either 2 or 4)

A-3, B-1, C-5, D-4, E-2

Given the constraints (eg A must go to 3), these two are the only complete matchings.

2 A document which is currently written in English is to be translated into six other European Union languages. The cost of translating a document varies, as it is harder to find translators for some languages.

The costs, in euros, are shown in the table below.

(a) (i) On the **table below**, showing the order in which you select the edges, use Prim's algorithm, starting from E, to find a minimum spanning tree for the graph connecting D, E, F, G, H, I and S.

[5 marks]

(ii) Find the length of your minimum spanning tree.

[1 mark]

(iii) Draw your minimum spanning tree.

[2 marks]

(b) It is given that the graph has a unique minimum spanning tree.

State the final two edges that would be added to complete the minimum spanning tree in the case where:

(i) Prim's algorithm starting from H is used;

(ii) Kruskal's algorithm is used.

[3 marks]

Answer space for question 2

	Danish (D)	English (E)	French (F)	German (G)	Hungarian (H)	Italian (I)	Spanish (S)
Danish (D)	–	120	140	80	170	140	140
English (E)	120	–	70	80	130	130	110
French (F)	140	70	–	90	190	85	90
German (G)	80	80	90	–	110	100	100
Hungarian (H)	170	130	190	110	–	140	150
Italian (I)	140	130	85	100	140	–	60
Spanish (S)	140	110	90	100	150	60	–

2. a) i.
1. Label the column corresponding to the start vertex with a 1. Delete the row corresponding to that vertex.
2. Ring the smallest available value in any labelled column.
3. Label the column corresponding to the ringed vertex with a 2, etc, and delete the row corresponding to it.
4. Repeat steps 2 and 3 until all rows have been deleted.
5. Write down the order in which edges were selected and the length of the minimum spanning tree.

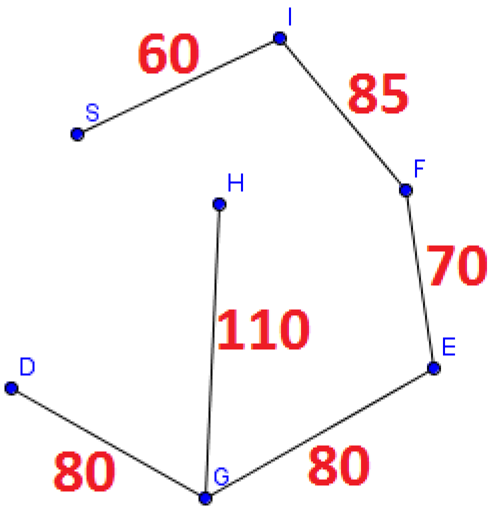
Answer space for question 2

	4	1	2	3	7	5	6
	Danish (D)	English (E)	French (F)	German (G)	Hungarian (H)	Italian (I)	Spanish (S)
Danish (D)	–	120	140	80	170	140	140
English (E)	120	–	70	80	130	130	110
French (F)	140	70		90	190	85	90
German (G)	80	80	90	–	110	100	100
Hungarian (H)	170	130	190	110		140	150
Italian (I)	140	130	85	100	140		60
Spanish (S)	140	110	90	100	150	60	

Minimum spanning tree: **E-F (70), E-G (80), F-I (85), G-D (80), G-H (110), I-S (60)**.

ii.
Total weighting (ie, cost): $70 + 80 + 85 + 80 + 110 + 60 = 485$, so **485 Euros.**

iii.



Note: without losing information, the graph can be arranged with points in any position – this arrangement is one which ensures there are no crossing edges, but it is not essential. Also note that, while the weighting is not required by this question, it will be helpful in answering the following part.

b)
Applying Prim's algorithm starting from H would link the following vertices, in this order: H-G (110), G-D (80), G-E (80), E-F (70), F-I (85), I-S (60). Therefore the final two edges added would be **F-I and I-S**.
Applying Kruskal's algorithm (since it does not require edges to be added only if they connect) would yield: I-S, F-E, G-E, G-D, I-F, G-H, with the final two edges added being **I-F and G-H**.

Note that in both of these, the order of G-D and G-E could be reversed. The key idea is that both would yield the diagram given above (since there is a unique spanning tree), so the only thing to determine is the order in which the chosen edges are chosen.

3 The network below shows 11 towns, A, B, ..., K. The number on each edge shows the time, in minutes, to travel between a pair of towns.

(a) (i) Use Dijkstra's algorithm on the diagram below to find the minimum time to travel from A to K.

[6 marks]

(ii) State the corresponding route.

[1 mark]

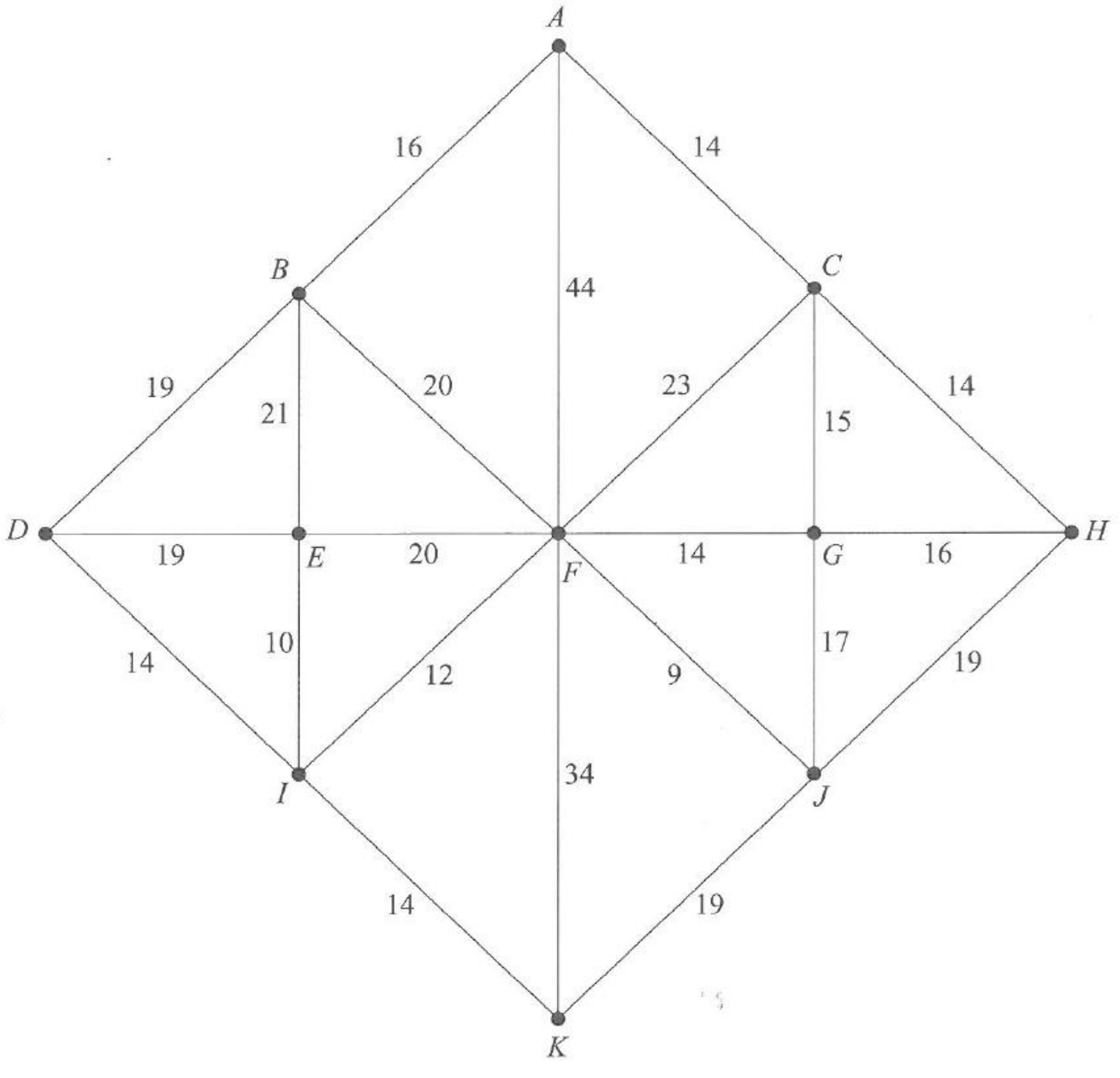
(b) On a particular day, Jenny travels from A to K but visits her friend at D on her way. Find Jenny's minimum travelling time.

[1 mark]

(c) On a different day, all roads connected to I are closed due to flooding. Jenny does not visit her friend at D. Find her minimum time to travel from A to K. State the route corresponding to this minimum time.

[2 marks]

Answer space for question 3

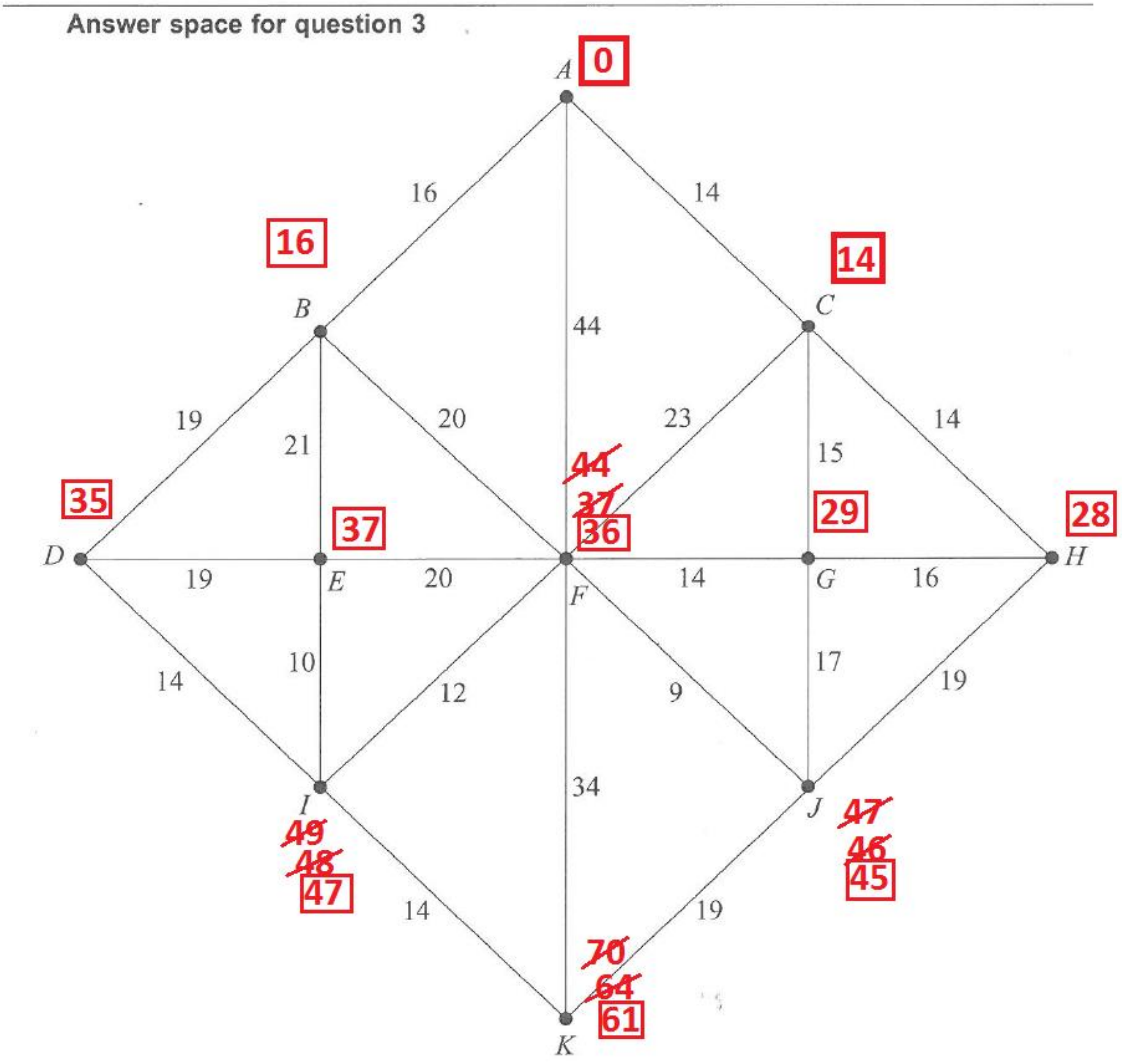


3.

a)

i.

1. Label the start vertex as 0.
2. Box this number (permanent label).
3. Label each vertex that is connected to the start vertex with its distance (temporary label).
4. Box the smallest number.
5. From this vertex, consider the distance to each connected vertex.
6. If a distance is less than the distance already in this vertex, cross out the distance and write in the new distance. If there was no distance at the vertex, write down the new distance.
7. Repeat from step 4 until the destination vertex is boxed.



Label and box 0 at A, Label B, C and F
Box 14 at C, Label F, G and H
Box 16 at B, Label D, E and F
Box 28 at H, Label J (not G)
Box 29 at G, Label J (not F)
Box 35 at D, Label I (not E)
Box 36 at F, Label I, J and K
Box 37 at E, Label I
Box 45 at J, Label K
Box 47 at I, Label K
Box 61 at K

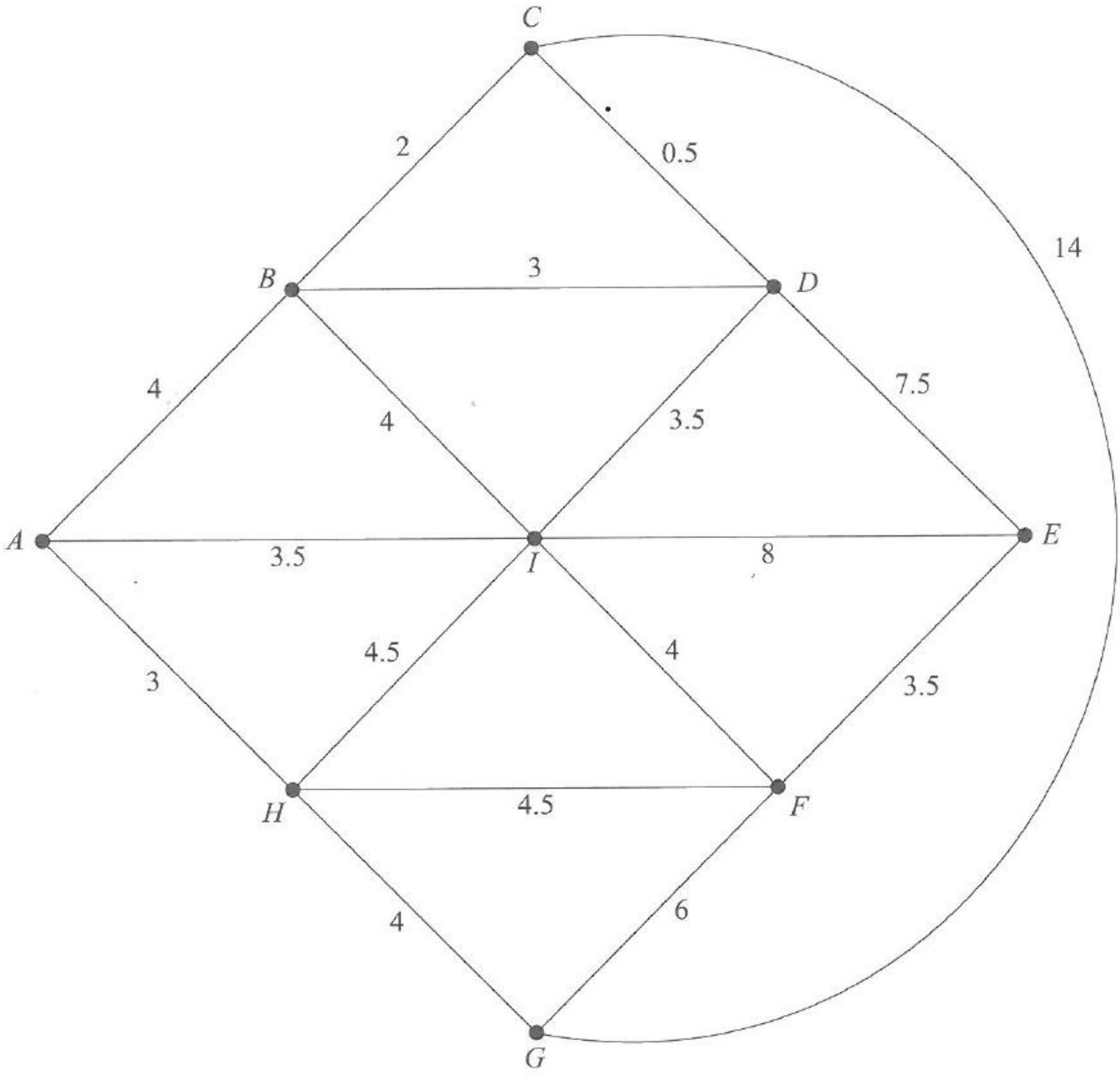
ii.
By tracking backwards:
K-I-E-B-A, so **Route: A-B-E-I-K, Total length: 61**.

b)
The quickest route to D is 35 (see boxed value at D). The quickest route from D to K is D-I-K for 28. **Total route length: 63**.

c)
By inspection: **A-B-F-J-K (route length 64)**.

4 Paulo sells vegetables from his van. He drives around the streets of a small village. The network shows the streets in the village. The number on each edge shows the time, in minutes, to drive along that street.

Paulo starts from his house located at vertex A and drives along all the streets at least once before returning to his house.



The total of all the times in the diagram is 79.5 minutes.

(a) Find the length of an optimal Chinese postman route around the village, starting and finishing at A. (Shortest routes between vertices may be found by inspection.)

[5 marks]

(b) For an optimal Chinese postman route, state:

(i) the number of times the vertex F would occur;

(ii) the number of times the vertex D would occur.

[2 marks]

(c) Toto is standing for the position of Mayor in the local elections. He intends to travel along all the roads at least once. He can start his journey at any vertex and can finish his journey at any vertex.

(i) Find the length of an optimal route for Toto.

(ii) State the vertices from which Toto could start in order to achieve this optimal route.

[3 marks]

4.

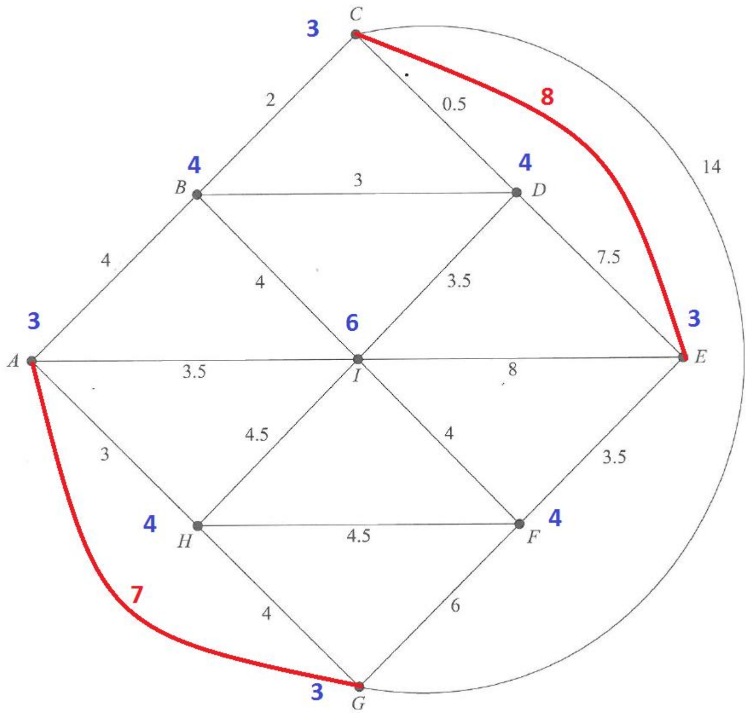
a)

1. List all odd vertices.
2. List all possible pairings of odd vertices.
3. For each pairing find the edges that connect the vertices with the minimum weight.
4. Find the pairings such that the sum of the weights is minimised.
5. On the original graph add the edges that have been found in step 4.
6. The length of an optimal Chinese postman route is the sum of all the edges added to the total found in Step 4.
7. A route corresponding to this minimum weight can then be found by inspection.

The odd vertices are A, C, E and G, each having order 3.

Pairing	Shortest Route	Total Weighting
A-C, E-G	A-B-C (6), E-G (9.5)	15.5
A-E, C-G	A-I-F-E (11), C-D-I-H-G (12.5)	23.5
A-G, C-E	A-H-G (7), C-D-E (8)	15

Choose the pairing A-G, C-E:



Adding the minimum required repeat routes A-G and C-E:

$$79.5 + 8 + 7 = \mathbf{94.5}$$

b)

i.

Twice. It has order 4 and is not part of any of the repeated roots.

ii.

Three times. It has order 4, but is also part of the necessarily repeated route C-D-E.

c)

Since he can start and finish at different places, only one pair of odd-ordered vertices need to be paired. Choosing the smallest of these (A-C with length 6) yields a route of length:

$79.5 + 6 = \mathbf{85.5}$, starting at **G** and ending at E or starting at **E** and ending at G.

5 The feasible region of a linear programming problem is determined by the following:

$$x \geqslant 1$$
$$y \geqslant 3$$
$$x + y \geqslant 5$$
$$x + y \leqslant 12$$
$$3x + 8y \leqslant 64$$

(a) On the grid below, draw a suitable diagram to represent the inequalities and indicate the feasible region.

[5 marks]

(b) Use your diagram to find, on the feasible region:

(i) the maximum value of $3x + y$;

(ii) the maximum value of $2x + 3y$;

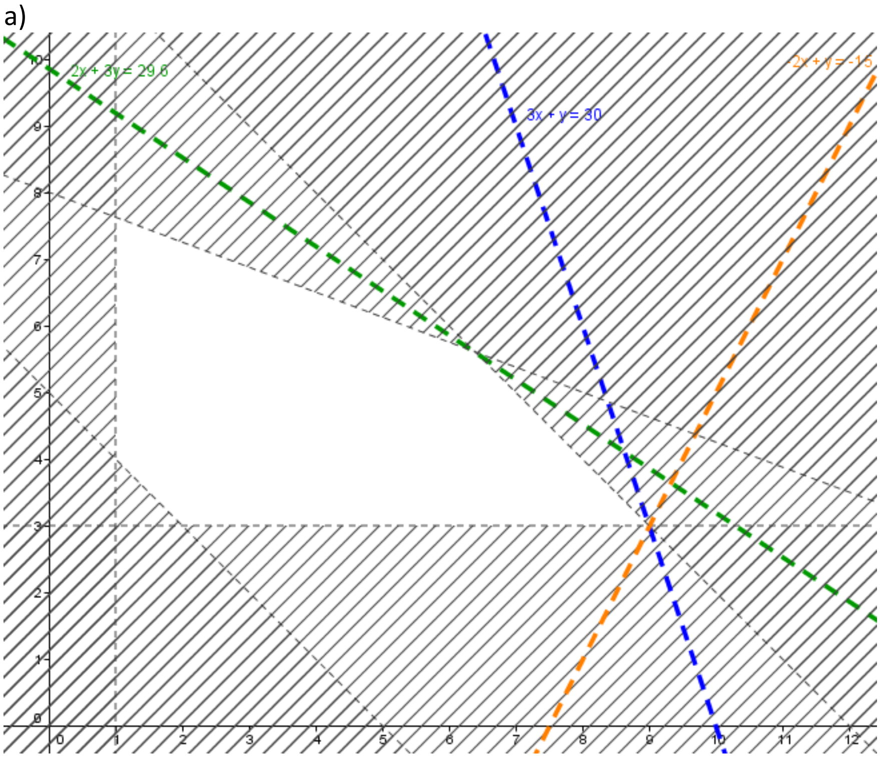
(iii) the minimum value of $-2x + y$.

In each case, state the coordinates of the point corresponding to your answer.

[6 marks]

5.

a)



b)

i.

$3x + y = k$ has a gradient of -3, making it a steeper downward slope than any of the other lines. Therefore the point at which $3x + y$ is maximum must be at the far right of the diagram, shown in blue.

This point is where $x + y = 12$ crosses the line $y = 3$, therefore it is the point $(9,3)$ and the maximum value $3x + y$ takes is 30

ii.

Similarly, $2x + 3y = p$ would have maximum p when $3x + 8y = 64$ crosses $x + y = 12$. Solving simultaneously gives the point as $(6.4, 5.6)$ and the maximum value would be 29.6. (Green)

iii.

$-2x + y$ is minimum where $y = 3$ crosses $x + y = 12$ so the point is $(9,3)$ and the minimum is $-2x + y = -15$. (Orange)

6 (a) Sarah is solving a travelling-salesman problem.

(i) She finds the following upper bounds: 32, 33, 32, 32, 30, 32, 32.

Write down the best upper bound.

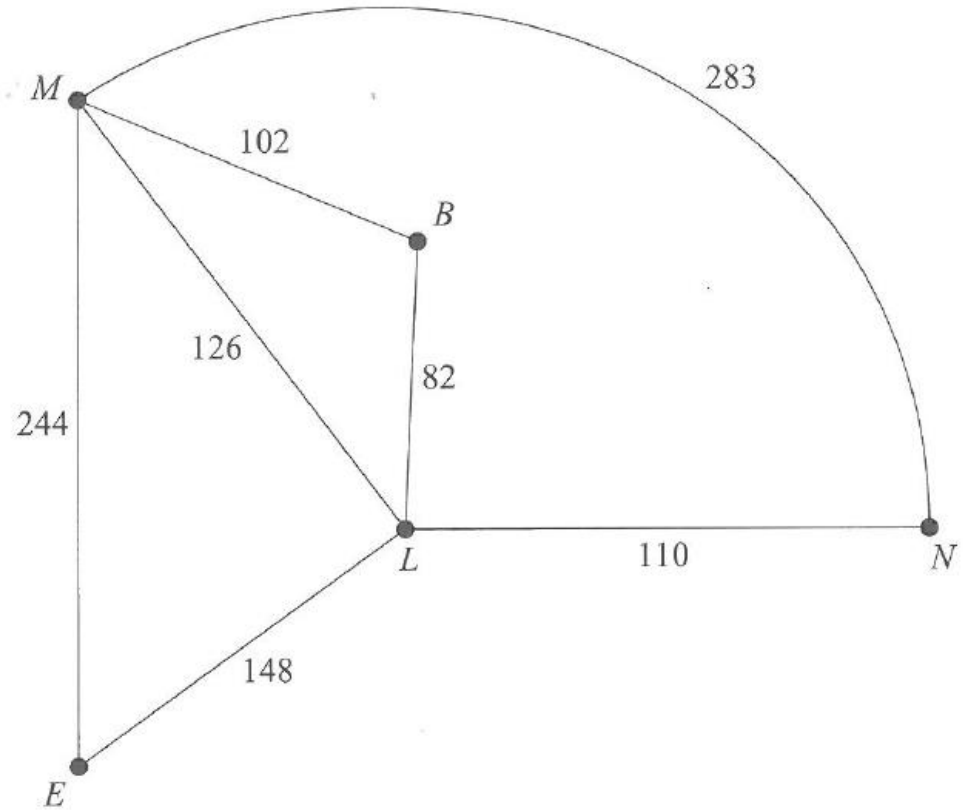
(ii) She finds the following lower bounds: 17, 18, 17, 20, 18, 17, 20.

Write down the best lower bound.

[2 marks]

(b) Rob is travelling by train to a number of cities. He is to start at M and visit each other city at least once before returning to M.

The diagram shows the travelling times, in minutes, between cities. Where no time is shown, there is no direct journey available.



The table below shows the minimum travelling times between all pairs of cities.

	B	E	L	M	N
B	–	230	82	102	192
E	230	–	148	244	258
L	82	148	–	126	110
M	102	244	126	–	236
N	192	258	110	236	–

(i) Explain why the minimum travelling time from M to N is not 283.

[1 mark]

(ii) Find an upper bound for the minimum travelling time by using the tour $MNBELM$.

[1 mark]

(iii) Write down the actual route corresponding to the tour $MNBELM$.

[2 marks]

(iv) Use the nearest-neighbour algorithm, starting from M, to find another upper bound for the minimum travelling time of Rob's tour.

[4 marks]

6.
a)
i.
Best upper bound: 30 (because it is a stricter upper bound – more limiting – than the others).

ii.
Best lower bound: 20 (because it is a stricter lower bound – more limiting – than the others).

b)
i.
By inspection, route M-L-N gives a time of $126 + 110 = 236$. Since $236 < 283$, the minimum time must be no greater than 236 and hence cannot be 283.

ii.
Using the table:

$$236 + 192 + 230 + 148 + 126 = 932$$

iii.
Finding the shortest route (also corresponding to the values in the table) for MNBELM:
M-L-N-L-B-L-E-L-M

iv.
The nearest neighbour algorithm:
1. Choose a start vertex.
2. From your current vertex go to the nearest unvisited vertex.
3. Repeat step 2 until all the vertices have been visited.
4. Return to the start vertex.

Note: use the table to carry out this algorithm.

M-B (102)
B-L (82)
L-N (110)
N-E (258)
E-M (244)
Total: $102 + 82 + 110 + 258 + 244 = 796 =$ upper bound.

7 A factory makes batches of three different types of battery: basic, long-life and super.

Each basic batch needs 4 minutes on machine A, 7 minutes on machine B and 14 minutes on machine C.

Each long-life batch needs 10 minutes on machine A, 14 minutes on machine B and 21 minutes on machine C.

Each super batch needs 10 minutes on machine A, 14 minutes on machine B and 28 minutes on machine C.

Machine A is available for 4 hours a day, machine B for 3.5 hours a day and machine C for 7 hours a day.

Each day the factory must make:

more basic batches than the total number of long-life and super batches;

at least as many long-life batches as super batches.

At least 15% of the production must be long-life batches.

Each day, the factory makes x basic, y long-life and z super batches.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, writing your answers with simplified integer coefficients.

[8 marks]

7.
Time requirements, individually for each machine:

$$T_A = 4x + 10y + 10z \quad \Rightarrow \quad 4x + 10y + 10z \leq 240 \quad \Rightarrow \quad \mathbf{2x + 5y + 5z \leq 120}$$

$$T_B = 7x + 14y + 14z \quad \Rightarrow \quad 7x + 14y + 14z \leq 210 \quad \Rightarrow \quad \mathbf{x + 2y + 2z \leq 30}$$

$$T_C = 14x + 21y + 28z \quad \Rightarrow \quad 14x + 21y + 28z \leq 420 \quad \Rightarrow \quad \mathbf{2x + 3y + 4z \leq 60}$$

Battery number requirements:

$$\boldsymbol{x > y + z}$$

$$\boldsymbol{y \geq z}$$

$$y \geq 0.15(x + y + z) \quad \Rightarrow \quad 100y \geq 15x + 15y + 15z \quad \Rightarrow \quad 20y \geq 3x + 3y + 3z \quad \Rightarrow \quad \mathbf{3x - 17y + 3z \leq 0}$$

8 In this question you may use the fact that any simple graph must have an even number of vertices of odd degree.

(a) A simple graph has five vertices and their degrees are

$$x,\ x+1,\ x+1,\ x+2 \text{ and } x+3$$

(i) Show that x must be odd.

[2 marks]

(ii) Find the value of x and draw a graph with vertices having the given degrees.

[3 marks]

(b) A simple graph has 10 vertices.

(i) State the minimum possible degree and maximum possible degree of a vertex.

[2 marks]

(ii) Show that the degrees of the vertices cannot all be different.

[2 marks]

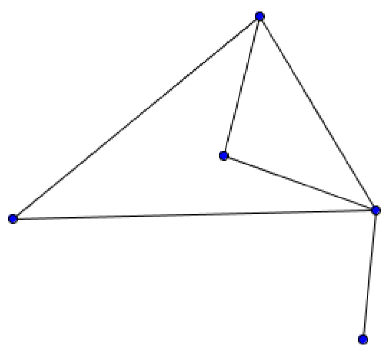
8.
a)
i.
If x is even, the second, third and fifth vertices in the list would be odd. This is three vertices, which is not an even number. Therefore x is odd.

ii.
Since the graph is simple, it contains no duplicate edges and no loops. This means that the maximum order of any vertex is 4, since it can link up with at most every other vertex (not itself). This gives an upper limit on x:

$$x+3 \leq 4 \quad \Rightarrow \quad x \leq 1$$

Since x is odd, and cannot be negative, we have $x = 1$. A simple graph may have a vertex with zero degree (a simple graph does not have to be a connected graph), but in this case x cannot be even.

This gives vertices of degree: $1, 2, 2, 3, 4$. Starting from a pentagon and filling in edges as required, from the vertex with greatest order first, then rearranging points (for clarity only – this is not essential) gives:



b)
i.
Since there can be no duplicate edges or loops, the maximum possible degree is $10 - 1 = 9$. This would involve one vertex linking to each other.

Simple does not imply connected, so there could be a vertex with order 0, although the order cannot be negative. Therefore 0 is the minimum.

ii.
Given that the minimum is 0 and the maximum is 9, for all the vertices to have different degrees we would have to have: $0, 1, 2, \ldots, 8, 9$. There must be an even number of odd vertices in a simple graph, but we have vertices of order $1, 3, 5, 7$ and 9. That is an odd number. Since this was the only way of having all vertices different, it must be impossible.