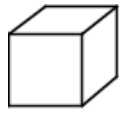# Volume

*Volume is a measure of **real space** (eg cupboard space, box capacity, room in a bottle).*

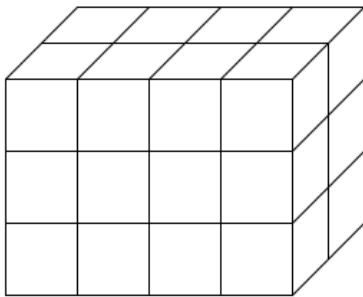| Length is 1-dimensional	Area is 2-dimensional	**Volume is 3-dimensional**
*Often measured in metres (m) or **centimetres** (cm):*	*Often measured in square metres (m^2) or **square centimetres** (cm^2):*	***Often measured in cubic metres (m^3) or** cubic centimetres (cm^3):*
——	☐	

Common units for volume include:

Cubic centimetres (cm^3): A small 6-sided dice has a volume of about $1cm^3$.

Cubic metres (m^3): The space under a dining room table has a volume of around $1m^3$.

Cubic kilometres (km^3): A large lake might hold about $1km^3$.

> Bonus unit: Because a cubic metre is a million times bigger than a cubic centimetre*, to measure the volume of, say, a school bag, which is around $30000cm^3$ but only $0.03m^3$, we use **litres**. A litre is $1000ml$ which is exactly the same as $1000cm^3$. A bag is around $30l$.

*A cubic metre measures $100cm$ by $100cm$ by $100cm$, so its volume in cubic centimetres must be $1,000,000cm^3$. So $1m^3 = 1,000,000cm^3$.

Volume is **how many cubes could fit inside** a shape:
This cuboid contains **2 layers of 3 rows of 4** cubes.
Or, equivalently, **4 layers of 2 columns of 3** cubes, etc.

Therefore the volume (the number of cubes) is:
$$2 \text{ lots of } 3 \text{ lots of } 4 = 2 \times 3 \times 4 = \mathbf{24cm^3}$$
$$(\mathbf{24 \; cubic \; centimetres})$$
(we include cm^3 to show the *size* of cube we're using)

> ***Don't*** *just memorise 'multiply all the lengths' because a) that doesn't tell the whole story and b) for most problems you are given, using that simplistic idea will get you the wrong answer. Use the info above to learn **why** we multiply.*
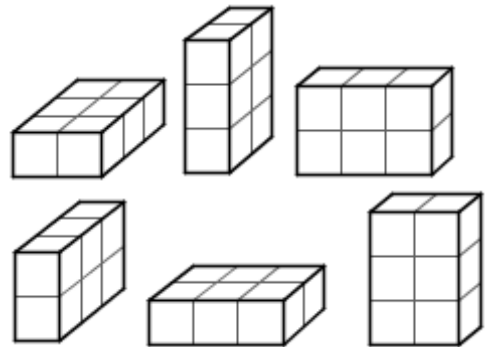

Does it matter which order you multiply in?
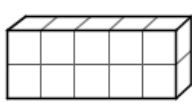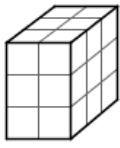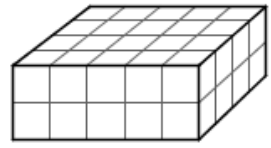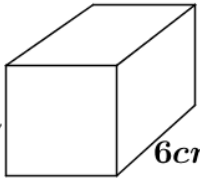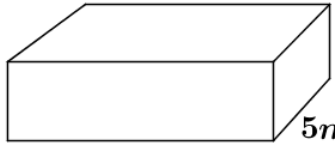*And which measurement counts as the length?*

Look at all the cuboids shown opposite. What do you notice?

How many cubes are there in each one?

The words 'length' and 'width' are not important. **You can multiply the three dimensions *in any order* and you'll get the same answer.**
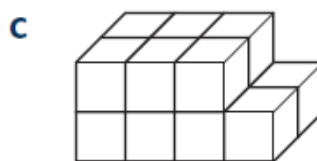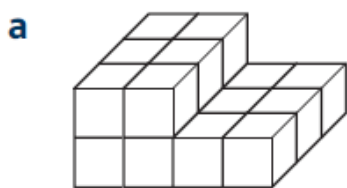

**Check you understand:** *Find the volume of each of the cuboids shown below.*

Volume = _____ cm^3

Volume = _____ cm^3

Volume = _____ cm^3

4cm 4cm 6cm
Volume = _____ cm^3

3m 10m 5m
Volume = _____ m^3

10mm 40mm 20mm
Volume = _____ mm^3

Volume Problems

1. The shapes below are **prisms** (the same all the way back) made from **centimetre cubes.**
   Find the volume of each shape. Give units with your answers.

a

b

c

d

2. Work out the volumes of the cuboids shown. Give units with your answers.

a

2 cm

6 cm

3 cm

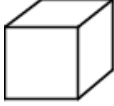b

4 m

10 m

6 m

c

3 mm

12 mm

5 mm
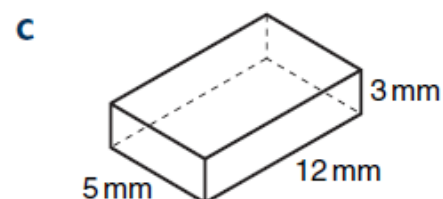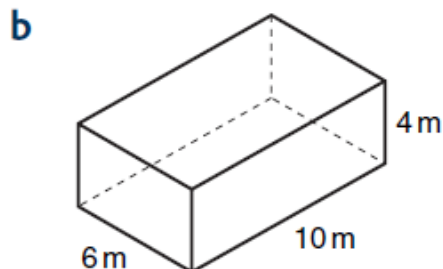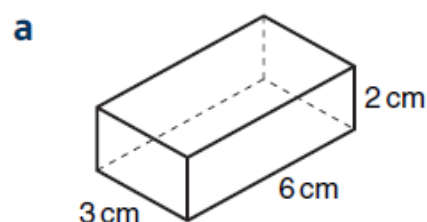
3   Work out the volume of a cuboid which is 40 cm by 30 cm by 20 cm.

4   Work out the volume of a cuboid which is 20 m by 7 m by 5 m.

5   Work out the volume of a cuboid which is 25 mm by 10 mm by 8 mm.

6   A cuboid measures 4 m by 2 m by 50 cm.
   a   Explain why its volume is *not* 400 m^3.
   b   Work out its volume in m^3.

7   Work out the volume, in m^3, of a cuboid which is 5 m by 2 m by 40 cm.

8   A cuboid measures 10 cm by 4 cm by 5 mm.
   Work out its volume
   a   in cm^3 **b**   in mm^3.

Extension Section

1   The volume of a cuboid is 400 cm^3. Its length is 10 cm and its width is 8 cm.
   Work out its height.

2   The volume of a cuboid is 600 cm^3. Its length is 20 cm and its height is 5 cm.
   Work out its width.

3   The volume of a cuboid is 180 cm^3. Its width is 6 cm and its height is 2 cm.
   Work out its length.

Volume SOLUTIONS

*Volume is a measure of **real space** (eg cupboard space, box capacity, room in a bottle).*

Length is 1-dimensional	Area is 2-dimensional	Volume is 3-dimensional
*Often measured in metres (m) or **centimetres** (cm):*	*Often measured in square metres (m^2) or **square centimetres** (cm^2):*	***Often measured in cubic metres (m^3) or cubic centimetres (cm^3):***
——	□	

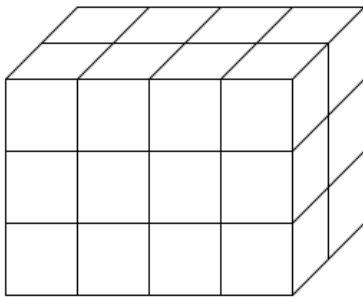Common units for volume include:

Cubic centimetres (cm^3): A small 6-sided dice has a volume of about $1cm^3$.

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$$2 \text{ lots of } 3 \text{ lots of } 4 = 2 \times 3 \times 4 = \textbf{24cm}^\textbf{3}$$
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(we include cm^3 to show the *size* of cube we're using)

> ***Don't*** *just memorise 'multiply all the lengths' because a) that doesn't tell the whole story and b) for most problems you are given, using that simplistic idea will get you the wrong answer. Use the info above to learn **why** we multiply.*

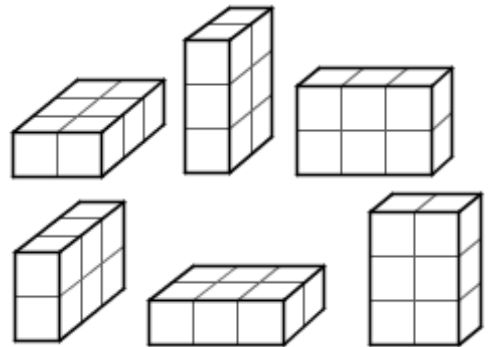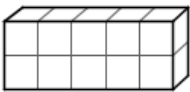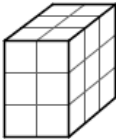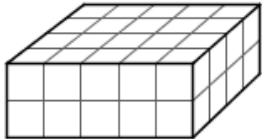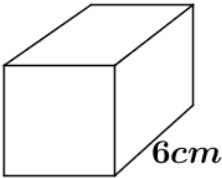|	
Does it matter which order you multiply in? *And which measurement counts as the length?* Look at all the cuboids shown opposite. What do you notice? They are all the same shape, just turned (or viewed differently) How many cubes are there in each one? $1 \times 2 \times 3 = 2 \times 3 \times 1 = \cdots = \textbf{6}$ *The words 'length' and 'width' are not important – even 'height' changes when you knock a cuboid over.* **You can multiply the three dimensions *in any order* and you'll *always* get the same answer.**	

Check you understand: *Find the volume of each of the cuboids shown below.*

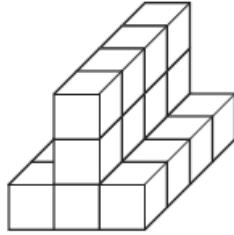| Volume = ____10____ cm^3	Volume = ____18____ cm^3	Volume = ____40____ cm^3
4cm, 6cm, 4cm Volume = ____96____ cm^3	3m, 5m, 10m Volume = ____150____ m^3	10mm, 40mm, 20mm Volume = ____8000____ mm^3

Volume Problems SOLUTIONS

1. The shapes below are **prisms** (the same all the way through) made from **centimetre cubes.**
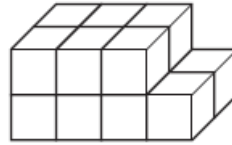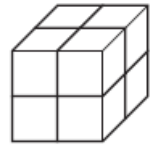   Find the volume of each shape. Give units with your answers.
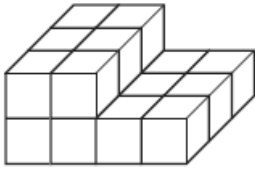
a   **b**   **c**   **d**

$18cm^3$ $16cm^3$ $14cm^3$ $8cm^3$

2. Work out the volumes of the cuboids shown. Give units with your answers.

a   **b**   **c**

2 cm, 6 cm, 3 cm

4 m, 10 m, 6 m

3 mm, 12 mm, 5 mm
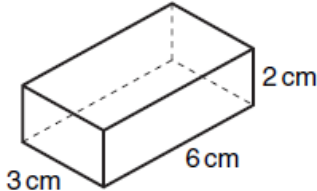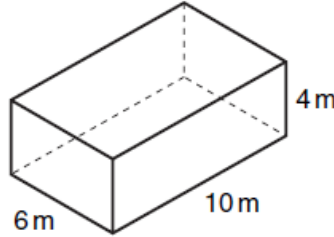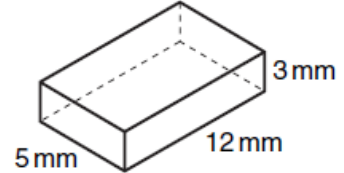
$36cm^3$ $240m^3$ $180mm^3$

3   Work out the volume of a cuboid which is 40 cm by 30 cm by 20 cm.

$$24000cm^3$$

4   Work out the volume of a cuboid which is 20 m by 7 m by 5 m.

$$700m^3$$

5   Work out the volume of a cuboid which is 25 mm by 10 mm by 8 mm.

$$2000mm^3$$

6   A cuboid measures 4 m by 2 m by 50 cm.

   a   Explain why its volume is *not* 400 m³.

   b   Work out its volume in m³.

a) $4 \times 2 \times 50 = 400$ but $50cm \neq 50m$. All lengths need to be in metres to give cubic metres for volume.
b) $4 \times 2 \times 0.5 = 4m^3$

7   Work out the volume, in m³, of a cuboid which is 5 m by 2 m by 40 cm.

$$5 \times 2 \times 0.4 = 4m^3$$

8   A cuboid measures 10 cm by 4 cm by 5 mm.
   Work out its volume

   a   in cm³   **b**   in mm³.

a) $10 \times 4 \times 0.5 = 20cm^3$
b) $100 \times 40 \times 5 = 20000mm^3$

Extension Section

1   The volume of a cuboid is 400 cm³. Its length is 10 cm and its width is 8 cm.
   Work out its height.

$$400 \div (10 \times 8) = \frac{400}{80} = \frac{40}{8} = \frac{20}{4} = 5cm$$

2   The volume of a cuboid is 600 cm³. Its length is 20 cm and its height is 5 cm.
   Work out its width.

$$600 \div (20 \times 5) = 600 \div 100 = 6cm$$

3   The volume of a cuboid is 180 cm³. Its width is 6 cm and its height is 2 cm.
   Work out its length.

$$180 \div (6 \times 2) = \frac{180}{12} = \frac{90}{6} = \frac{30}{2} = 15cm$$