°C

100
90
80
70
60
50
40
30
20
10
0
10
20
30

Adding & Subtracting Negative Numbers

Negative numbers were once described as 'imaginary'.

They are harder to visualise than 1, 2 and 3, or even $\frac{1}{2}$ or $\frac{3}{4}$.

But they are really useful for measuring things that involve **direction**:
- **Height** (if sea level is 0, below sea level is negative)
- **Temperature** (water freezes at 0°, negative is even colder)
- **Money** (if I *borrow* it's like having *less than* £0, so negative)

Complete the statements to work out the rules of negatives:

*When you **add** a **positive**,* the number ***goes up**.*
*When you **add zero**,* the number ***stays the same**.*
*When you **add** a **negative**,* the number _____.

*When you **subtract** a **positive**,* the number _____.
*When you **subtract zero**,* the number _____.
*When you **subtract** a **negative**,* the number _____.

Think of adding and subtracting like **moving up or down**.
You *can't* tell if your answer will be positive or negative immediately, but you *can* tell if you should move **up** or **down**.

Try the following questions:

1) $5 + 7 =$ **2)** $5 - 7 =$ **3)** $7 - 5 =$

4) $5 + -7 =$ **5)** $5 - -7 =$ **6)** $7 - -5 =$

7) Add up all these numbers: $4, -3, 12, -10, -1, 0, -9$
Remember that numbers can be added in any order you like.

Test Yourself: Adding & Subtracting Negative Numbers

We use + to mean 'increases by' or 'rises by', and – to mean 'decreases by' or 'falls by'.

Exercise 1

Calculate the final temperature.

1) 5°C increases by 9°C
2) 5°C falls by 3°C
3) 12°C falls by 15°C
4) –2°C increases by 4°C
5) –5°C falls by 8°C
6) 9°C – 4°C
7) –8°C – 12°C
8) –4°C + 2°C
9) 8°C – 12°C
10) –6°C – 5°C
11) –17°C + 3°C
12) –1°C + 15°C
13) 0°C – 6°C
14) 12°C – 12°C
15) –6°C + 6°C
16) –17°C – 6°C
17) –43°C + 26°C
18) –17°C + 26°C
19) –7°C – 19°C
20) –31°C + 27°C

Change in temperature is how much the temperature has risen or fallen.
The change from 5° to 8° is 3°, but the change from 8° to 5° is −3° (since it's gone down).

Exercise 2

What is the change in temperature between each of the following?

1) 3°C and 7°C
2) 17°C and 23°C
3) –5°C and 4°C
4) –7°C and 2°C
5) –6°C and –3°C
6) –7°C and 0°C
7) 5°C and 2°C
8) 7°C and –2°C
9) 5°C and –3°C
10) –2°C and –7°C
11) –8°C and –4°C
12) 0°C and –12°C
13) –17°C and –12°C
14) 8°C and –16°C
15) –9°C and –15°C
16) –12°C and 22°C
17) –12°C and 34°C
18) –16°C and –8°C
19) –16°C and 0°C
20) 12°C and –20°C

$10 - ? = -6$ *means "How much do I need to go down from 10 to get to −6?"*

Exercise 3

In each of the following, write down the number represented by the '?'

1) $5 - ? = 1$
2) $3 - ? = 3$
3) $4 - ? = -2$
4) $7 - ? = -9$
5) $-2 + ? = 3$
6) $-5 + ? = 1$
7) $-4 - ? = -7$
8) $-3 + ? = 4$
9) $? + 3 = 5$
10) $? - 4 = 3$
11) $5 - ? = -2$
12) $5 - ? = -3$
13) $? + 2 = -7$
14) $4 + ? = -9$
15) $? - 2 = -2$
16) $7 + ? = 0$
17) $8 + ? = 1$
18) $10 - ? = -6$
19) $4 + ? = -6$
20) $? - 14 = -4$

Multiplying Negatives

Does multiplication always make your number bigger?

Try these questions (you can double-check answers on your calculator):

Start with:	Multiply by:	Answer:	Higher or lower than 12?
12	2		
12	1		
12	0.5		
12	0		
12	−0.5		
12	−1		
12	−2		

Remember it can be helpful to think in terms of **higher** and **lower** rather than **bigger** and **smaller**, since −£4000 certainly seems 'bigger' than £12, but it's also a lot lower.

Now we are dealing with **negative** numbers, they have **direction** as well as **size**. When you multiply by a negative, you change both the **size** and the **direction**.

Circle the correct word:

When we multiply by a **positive / negative** number, the direction is **unchanged**.

When we multiply by a **positive / negative** number, the direction is **reversed**.

Multiplying by 4 increases the size, but doesn't change the direction.
Multiplying by −4 increases the size, and also reverses the direction.

Example 1	Example 2	Example 3	Example 4
5×7	5×-7	-5×7	-5×-7
Multiplying by 7 ***doesn't*** *change the direction, so the answer is **positive**.*	*Multiplying by −7* ***does*** *change the direction, so the answer is **negative**.*	*Multiplying by 7* ***doesn't*** *change the direction, so the answer is **negative**.*	*Multiplying by −7* ***does*** *change the direction, so the answer is **positive**.*
35	−35	−35	35

Try these questions:

1. a) $3 \times -6 =$ b) $-5 \times 4 =$ c) $-4 \times -6 =$

2. Multiply all of these numbers: 2, −3, 5, −1, 10, −4, 1, −2
Remember that numbers can be multiplied in any order you like.

Test Yourself: Multiplying Negatives

Recall that multiplying by a negative changes the direction (sign) of your number.

1 Work out

   a $(+2) \times (-4)$ **b** $(-3) \times (-5)$ **c** $(-4) \times (-6)$

   d $(+3) \times (+5)$ **e** $(-2) \times (+5)$ **f** $(-4) \times (+5)$

   g $(-3) \times (+8)$ **h** $(-1) \times (+9)$ **i** $(-4) \times (-4)$

Note that dividing with negatives works the same as multiplying: the number part tells you how the size changes, and the negative sign means a change in direction.

2 Work out

   a $(+6) \div (+3)$ **b** $(-8) \div (+4)$ **c** $(+10) \div (-5)$

   d $(-12) \div (-3)$ **e** $(-8) \div (-4)$ **f** $(-12) \div (-12)$

   g $(-14) \div (+2)$ **h** $(+12) \div (+4)$

To work out the missing numbers, first ignore the signs to work out the size of the number, then look just at the signs to work out the direction (sign) of your number.

3 Find the missing number.

   a $(+10) \div (\ ) = (-2)$ **b** $(-8) \div (\ ) = (+2)$ **c** $(-3) \times (\ ) = (+12)$

   d $(-5) \times (\ ) = (+20)$ **e** $(+5) \times (\ ) = (-25)$ **f** $(\ ) \times (-4) = (+20)$

   g $(\ ) \div (+3) = (+4)$ **h** $(\ ) \div (-4) = (-5)$ **i** $(+16) \div (\ ) = (-2)$

For these mixed questions, remember you can multiply or divide numbers in any order.

4 Work out

   a $\dfrac{(-5) \times (+4)}{(+2)}$ **b** $\dfrac{(-4) \times (-5)}{(+2)}$ **c** $\dfrac{(+6) \times (-4)}{(-3)}$

   d $\dfrac{(-5) \times (-8)}{(-4)}$ **e** $\dfrac{(-6)}{(+2)} + (-2)$ **f** $(-3) - \dfrac{(-6)}{(+2)}$

Adding & Subtracting Negative Numbers SOLUTIONS

*When you **add** a **positive**,*   *the number **goes up**.*
*When you **add** zero,*   *the number **stays the same**.*
*When you **add** a **negative**,*   *the number **goes down**.*
*When you **subtract** a **positive**,*   *the number **goes down**.*
*When you **subtract** zero,*   *the number **stays the same**.*
*When you **subtract** a **negative**,*   *the number **goes up**.*

1) $5 + 7 = \mathbf{12}$ **2)** $5 - 7 = \mathbf{-2}$ **3)** $7 - 5 = \mathbf{2}$

4) $5 + -7 = \mathbf{-2}$ **5)** $5 - -7 = \mathbf{12}$ **6)** $7 - -5 = \mathbf{12}$

7) Add up all these numbers: $4, \ -3, \ 12, \ -10, \ -1, \ 0, \ -9$

$4 + -3 + 12 + -10 + -1 + 0 + -9 = \mathbf{-7}$

Test Yourself: Adding & Subtracting Negative Numbers SOLUTIONS

Exercise 1
1) 14 2) 2   3) –3 4) 2 5) –13 6) 5 7) –20
8) –2 9) –4 10) –11 11) –14 12) 14   13) –6
14) 0 15) 0 16) –23 17) –17 18) 9   19) –26
20) –4

Exercise 2
1) 4   2) 6   3) 9 4) 9 5) 3 6) 7 7) 3 8) 9
9) 8   10) 5   11) 4   12) 12   13) 5   14) 24
15) 6 16) 34 17) 46 18) 8 19) 16 20) 32

Exercise 3
1) 4   2) 0   3) 6   4) 16   5) 5   6) 6   7) 3
8) 7   9) 2   10) 7   11) 7   12) 8   13) –9
14) –13   15) 0   16) –7   17) –7   18) 16
19) –10   20) 10

Multiplying Negatives SOLUTIONS

Does multiplication always make your number bigger?

Multiplying by numbers less than 1 makes the number *smaller*.

Try these questions (you can double-check answers on your calculator):

Start with:	Multiply by:	Answer:	Higher or lower than 12?
12	2	24	*Higher*
12	1	12	*The same*
12	0.5	6	*Lower*
12	0	0	*Lower*
12	-0.5	-6	*Lower* *(smaller size & opposite direction)*
12	-1	-12	*Lower* *(same size, but opposite direction)*
12	-2	-24	*Lower* *(larger size, but opposite direction)*

Remember it can be helpful to think in terms of **higher** and **lower** rather than **bigger** and **smaller**, since $-£4000$ certainly seems 'bigger' than £12, but it's also a lot lower. Now we are dealing with **negative** numbers, they have **direction** as well as **size**. When you multiply by a negative, you change both the **size** and the **direction**.

Circle the correct word:

When we multiply by a **positive** / ~~negative~~ number, the direction is **unchanged**.

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Multiplying by 4 increases the size, but doesn't change the direction.
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Example 1	Example 2	Example 3	Example 4
5×7	5×-7	-5×7	-5×-7
*Multiplying by 7 **doesn't** change the direction, so the answer is **positive**.*	*Multiplying by -7 **does** change the direction, so the answer is **negative**.*	*Multiplying by 7 **doesn't** change the direction, so the answer is **negative**.*	*Multiplying by -7 **does** change the direction, so the answer is **positive**.*
35	-35	-35	35

Try these questions:

1. a) $3 \times -6 = $ **-18**   b) $-5 \times 4 = $ **-20**   c) $-4 \times -6 = $ **24**

2. Multiply all of these numbers: $2, -3, 5, -1, 10, -4, 1, -2$

Remember that numbers can be multiplied in any order you like.

$$2 \times -3 \times 5 \times -1 \times 10 \times -4 \times 1 \times -2 = \textbf{2400}$$

Test Yourself: Multiplying Negatives SOLUTIONS

Recall that multiplying by a negative changes the direction (sign) of your number.

1.

a	-8	b	15	c	24
d	15	e	-10	f	-20
c	-24	h	-9	i	16

Note that dividing with negatives works the same as multiplying: the number part tells you how the size changes, and the negative sign means a change in direction.

2.

a	2	b	-2	c	-2
d	4	e	2	f	1
c	-7	h	3		

To work out the missing numbers, first ignore the signs to work out the size of the number, then look just at the signs to work out the direction (sign) of your number.

3 Find the missing number.

a $\ (+10) \div (\ ) = (-2)$
$\quad -5$

b $\ (-8) \div (\ ) = (+2)$
$\quad -4$

c $\ (-3) \times (\ ) = (+12)$
$\quad -4$

d $\ (-5) \times (\ ) = (+20)$
$\quad -4$

e $\ (+5) \times (\ ) = (-25)$
$\quad -5$

f $\ (\ ) \times (-4) = (+20)$
$\quad -5$

g $\ (\ ) \div (+3) = (+4)$
$\quad 12$

h $\ (\ ) \div (-4) = (-5)$
$\quad 20$

i $\ (+16) \div (\ ) = (-2)$
$\quad -8$

For these mixed questions, remember you can multiply or divide numbers in any order.

4 Work out

a $\ \dfrac{(-5) \times (+4)}{(+2)}$
$\quad = -10$

b $\ \dfrac{(-4) \times (-5)}{(+2)}$
$\quad = 10$

c $\ \dfrac{(+6) \times (-4)}{(-3)}$
$\quad = 8$

d $\ \dfrac{(-5) \times (-8)}{(-4)}$
$\quad = -10$

e $\ \dfrac{(-6)}{(+2)} + (-2)$
$\quad = -5$

f $\ (-3) - \dfrac{(-6)}{(+2)}$
$\quad = 0$