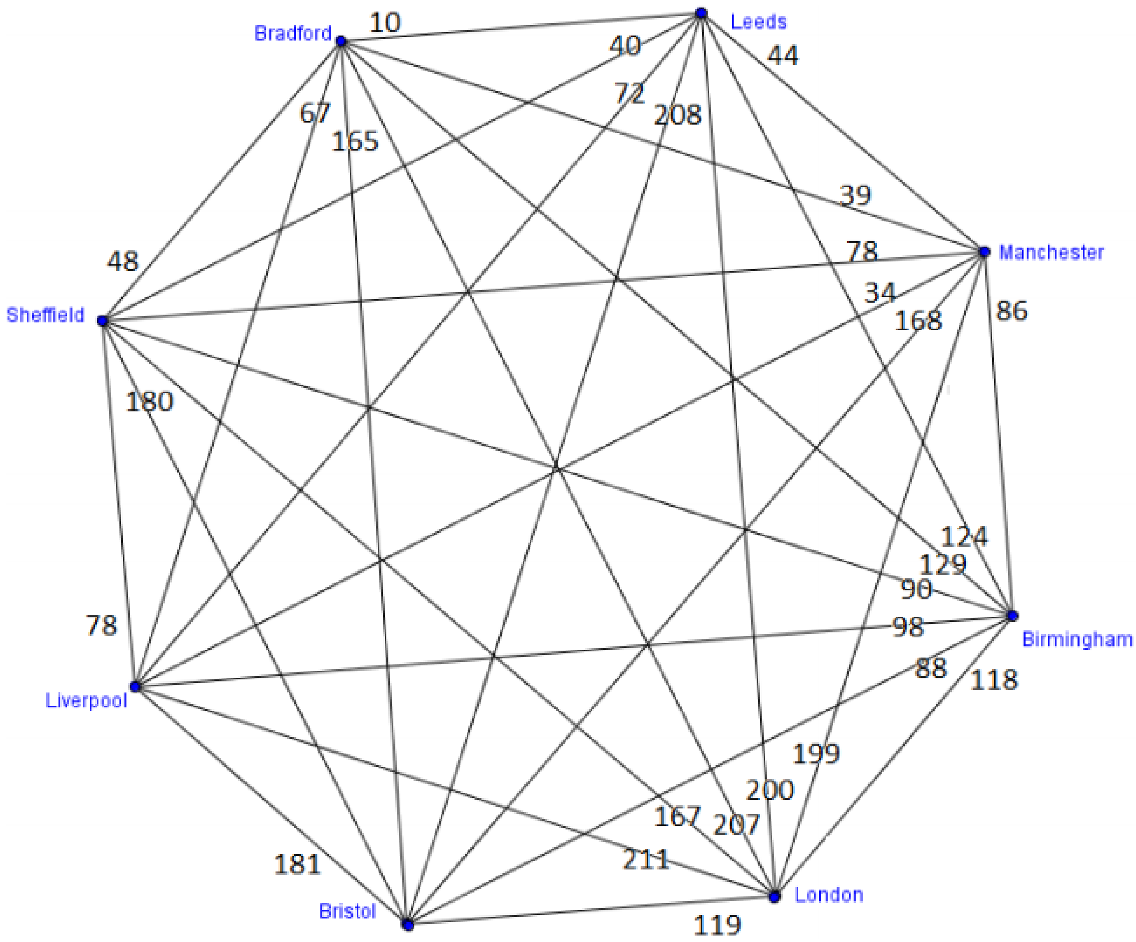
Spanning Trees

The following network shows the distance, in miles, between the 8 largest cities in England.



A new form of internet connection cable is to be routed along existing motorways to link these cities to one another. We do not require the most efficient route between any pair of cities, only that all pairs of cities are **connected** (ie, there is some route between them).

Use Kruskal's algorithm (below) to find a minimum spanning tree for the network.

Kruskal's algorithm:
To find a minimum spanning tree for a network with n edges.

Step 1: Choose the unused edge with the lowest value.
Step 2: Add this edge to your tree.
Step 3: If there are n-1 edges in your tree, stop. If not, go to step 1.
NOTE: Ensure you do not create a cycle – if choosing an arc would produce a cycle, don't choose it.

State the edges chosen, and the order in which you chose them:

State the total length of cable required:

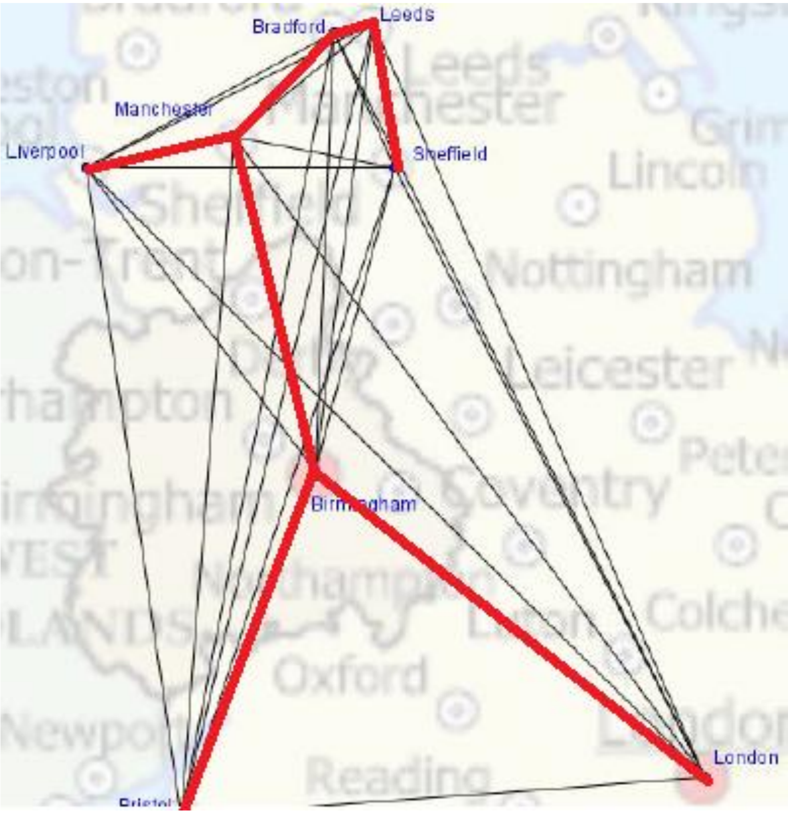
There are officially 51 cities in England.
How many edges would a minimum spanning tree connecting all of these cities have?

Spanning Trees SOLUTIONS

State the edges chosen, and the order in which you chose them:
Bradford-Leeds (10), Manchester-Liverpool (34), Manchester-Bradford (39),
Leeds-Sheffield (40),
[Note: at this point we reject a number of low weighted edges as they would create cycles]
Manchester-Birmingham (86), Birmingham-Bristol (88), Birmingham-London (118)
State the total length of cable required:

$$10 + 34 + 39 + 40 + 86 + 88 + 118 = \boldsymbol{415\ miles}$$

Note: the distance and position preserved graph of the 8 cities allows us to (approximately) visually verify the validity of our spanning tree. Routes that appear, here, to be less efficient than other possibilities may involve more roundabout routes due to actual motorways (lines shown are as-the-crow-flies for simplicity).



There are officially 51 cities in England.
How many edges would a minimum spanning tree connecting all of these cities have?
50 edges.

In the 8 city version, 7 edges were needed.
In general, for a graph with n vertices, $n - 1$ edges are needed for the spanning tree.
The first edge connects two vertices, and each additional edge (since we require no cycles) must connect exactly one more, so a 2 edge tree connects 3 vertices and so on.