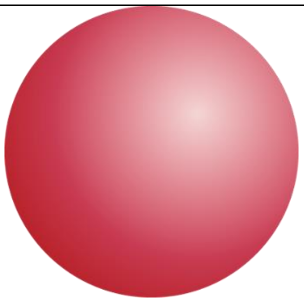
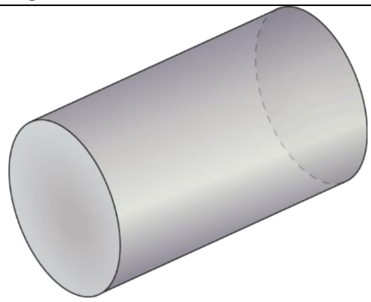
Slicing Shapes



Volume: $\frac{4}{3}\pi r^3$

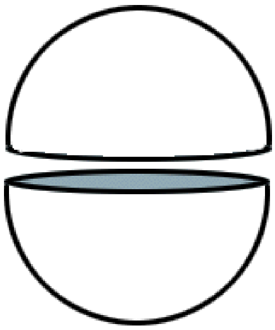
Surface Area: $4\pi r^2$



Volume: $\pi r^2 h$

Surface Area: $2\pi r^2 + 2\pi r h$
(each end is a circle of radius r, and the curved face is a rectangle of length h and width $2\pi r$)

A **sphere** of radius $3m$ is to be cut in half to increase the overall surface area.

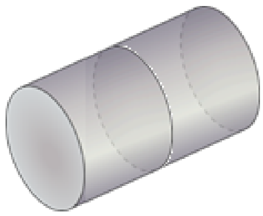


Calculate the original surface area of the whole sphere:

Calculate the additional surface area this circular cut provides:

By what proportion has the surface area increased as a result of the cut?

A **cylinder** of radius $3m$ and height $4m$ is to be cut in half parallel to the ends.

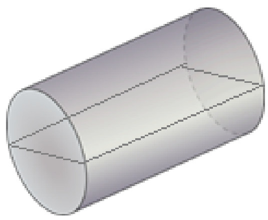


Calculate the original surface area of the whole cylinder:

Calculate the additional surface area this circular cut provides:

By what proportion has the surface area increased as a result of the cut?

The **cylinder** (of radius $3m$ and height $4m$) is instead cut in through the diameter.



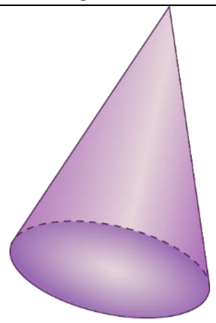
Write down the original surface area of the whole cylinder:

Calculate the additional surface area this rectangular cut provides:

By what proportion has the surface area increased as a result of the cut?

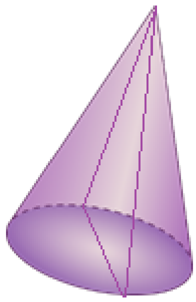
By what proportion would the surface area be increased if you cut a cylinder in both these directions?
What about cutting a sphere twice?

Slicing Shapes Extension



Volume: $\frac{1}{3}\pi r^2 h$
Surface Area: $\pi r^2 + \pi r l$
(the base is a circle of radius r, and the curved face is a sector of radius l and arc length $2\pi r$).

A **cone** of radius $3m$ and height $12m$ is to be cut vertically through the diameter.



Find the slant height and hence the original surface area of the whole cone:

Calculate the additional surface area this triangular cut provides.

By what proportion has the overall surface area increased as a result of the cut?

A **cone** of radius $3m$ and height $12m$ is to be cut horizontally, parallel to the base, a distance of $12\sqrt[3]{2}\ m$ from the top.

Write down the original surface area of the whole cone:

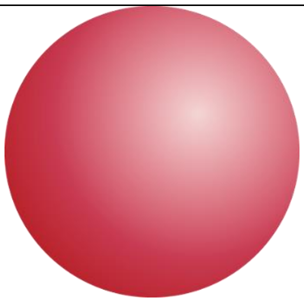
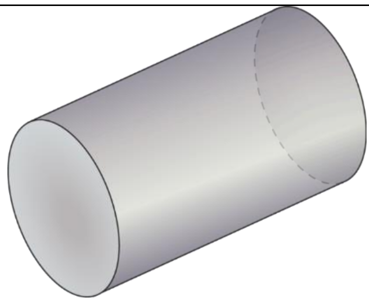
Calculate the additional surface area this circular cut provides:

Calculate the surface area of the frustum (truncated cone):

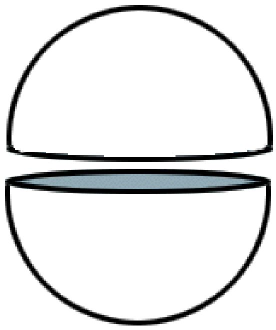
By what proportion has the overall surface area increased as a result of the cut?

By what proportion would the surface area be increased if you cut a cone in both these directions?

Slicing Shapes SOLUTIONS

 **Volume**: $\frac{4}{3}\pi r^3$ **Surface Area**: $4\pi r^2$	 **Volume**: $\pi r^2 h$ **Surface Area**: $2\pi r^2 + 2\pi r h$ (each end is a circle of radius r, and the curved face is a rectangle of length h and width $2\pi r$)

A **sphere** of radius $3m$ is to be cut in half to increase the overall surface area.



Calculate the original surface area of the whole sphere:

$$SA = 4\pi(3)^2 = \mathbf{36\pi\ m^2}$$

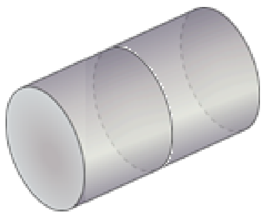
Calculate the additional surface area this circular cut provides:

Increased by two circles of radius 3, so: $2\pi(3)^2 = \mathbf{18\pi\ m^2}$

By what proportion has the surface area increased as a result of the cut?

$$\frac{54\pi}{36\pi} = \mathbf{\frac{3}{2}} \quad \textbf{or} \quad \mathbf{50\%\ increase}$$

A **cylinder** of radius $3m$ and height $4m$ is to be cut in half parallel to the ends.



Calculate the original surface area of the whole cylinder:

$$SA = 2\pi(3)^2 + 2\pi(3)(4) = \mathbf{42\pi\ m^2}$$

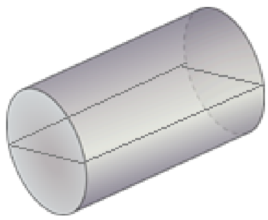
Calculate the additional surface area this circular cut provides:

Increased by two circles of radius 3, so: $2\pi(3)^2 = \mathbf{18\pi\ m^2}$

By what proportion has the surface area increased as a result of the cut?

$$\frac{60\pi}{42\pi} = \mathbf{\frac{10}{7}} \quad \textbf{or} \quad \approx \mathbf{43\%\ increase}$$

The **cylinder** (of radius $3m$ and height $4m$) is instead cut in through the diameter.



Write down the original surface area of the whole cylinder:

$$SA = 2\pi(3)^2 + 2\pi(3)(4) = \mathbf{42\pi\ m^2}$$

Calculate the additional surface area this rectangular cut provides:

Increased by two rectangles of width 6 and length 4, so:

$$2 \times 6 \times 4 = \mathbf{48\ m^2}$$

By what proportion has the surface area increased as a result of the cut?

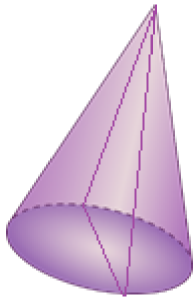
$$\frac{42\pi + 48}{42\pi} = \mathbf{\frac{7\pi + 8}{7\pi}} \quad \textbf{or} \quad \approx \mathbf{36\%\ increase}$$

Note: if you cut the cylinder in **both** directions as described, the S.A. increases by S.F. $\mathbf{\frac{10\pi+8}{7\pi}}$ **or** $\approx \mathbf{79\%\ increase}$

If you cut a sphere in half twice, because you are adding four circles, you double the surface area ($100\%\ increase$)

Slicing Shapes Extension SOLUTIONS

A **cone** of radius $3m$ and height $12m$ is to be cut vertically through the diameter.



Find the slant height and hence the original surface area of the whole cone:

$l = \sqrt{3^2 + 12^2} = 3\sqrt{17}$ so: $SA = \pi(3)^2 + \pi(3)(3\sqrt{17}) = \boldsymbol{9\pi(1+\sqrt{17})\ m^2}$

Calculate the additional surface area this triangular cut provides.

Base is 2×3 and height is 12. Two triangles give an area of $\boldsymbol{72\ m^2}$

By what proportion has the overall surface area increased as a result of the cut?

$$\frac{9\pi(1+\sqrt{17})+72}{9\pi(1+\sqrt{17})} \quad or \quad \approx 50\%\ increase$$

A **cone** of radius $3m$ and height $12m$ is to be cut horizontally, parallel to the base, so as to leave a *similar* cone of half the volume as the top piece.

Write down the original surface area of the whole cone:

$$\boldsymbol{9\pi(1+\sqrt{17})\ m^2}$$

Calculate the additional surface area this circular cut provides:

Given a volume scale factor of 2 from the smaller cone to the larger, the length scale factor must be $2^{\frac{1}{3}}$ (or the cube root of 2), so the radius is $3\sqrt[3]{2}$, and the two circles we are adding have combined area:

$$2 \times \pi\left(3 \times 2^{\frac{1}{3}}\right)^2 = \boldsymbol{9\pi \times 2^{\frac{5}{3}}}$$

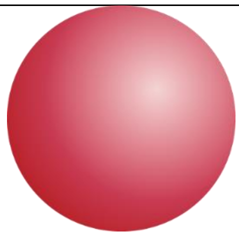
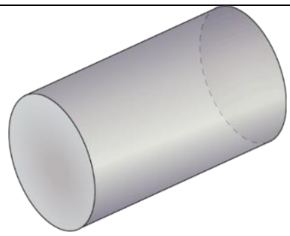
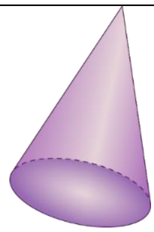
By what proportion has the overall surface area increased as a result of the cut?

$$\frac{\left(9\pi(1+\sqrt{17}) + 9\pi \times 2^{\frac{5}{3}}\right)}{9\pi(1+\sqrt{17})} \quad or \quad \approx 62\%\ increase$$

Note: if you cut the cone in half in **both** directions as described, the surface area increases by scale factor:

$$\frac{\left(9\pi(1+\sqrt{17}) + 9\pi \times 2^{\frac{5}{3}} + 72\right)}{9\pi(1+\sqrt{17})} \quad or \quad \approx 112\%\ increase\ (more\ than\ double)$$

Direct comparison

Shape	 $r = 3m$	 $r = 3m \quad h = 4m$	 $r = 3m \quad h = 12m$
Volume	$36\pi \approx 113m^3$	$36\pi \approx 113m^3$	$36\pi \approx 113m^3$
Surface Area	$36\pi \approx 113m^2$	$42\pi \approx 132m^2$	$9\pi(1+\sqrt{17}) \approx 145m^2$
% increase in S.A after two cuts	100%	79%	112%
New S.A.	$72\pi \approx 226m^2$	$60\pi + 48 \approx 236m^2$	$9\pi(1+\sqrt{17}) + 9\pi \times 2^{\frac{5}{3}} + 72 \approx 307m^2$