How to use factors to solve a quadratic equation:

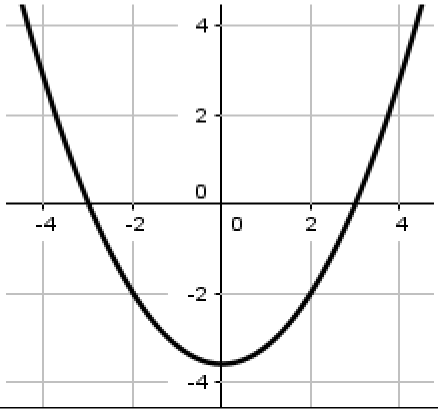
I think of a number	x
I subtract four from the number to create a new number	$x - 4$
I multiply both numbers together	$x(x-4)$
I get an answer of zero.	$x(x-4) = 0$
What could my original number have been?	Find x
If two numbers multiply to make zero, one of them is zero.	$x(x-4) = 0 \quad \Longrightarrow \quad x = 0 \;\; or \;\; x - 4 = 0$
So either my original number was $\mathbf{0}$…	Therefore, either: $\boldsymbol{x} = \mathbf{0}$, or…
… or the second number I created was zero,	$x - 4 = 0$
which means the first one was *four more than zero*: $\mathbf{4}$	$\Longrightarrow \quad \boldsymbol{x} = \mathbf{4}$
Therefore my first number was either $\mathbf{0}$ or $\mathbf{4}$	$\Longrightarrow \quad \boldsymbol{x} = \mathbf{0} \;\; \boldsymbol{or} \;\; \boldsymbol{x} = \mathbf{4}$

How to sketch the graph of a quadratic:

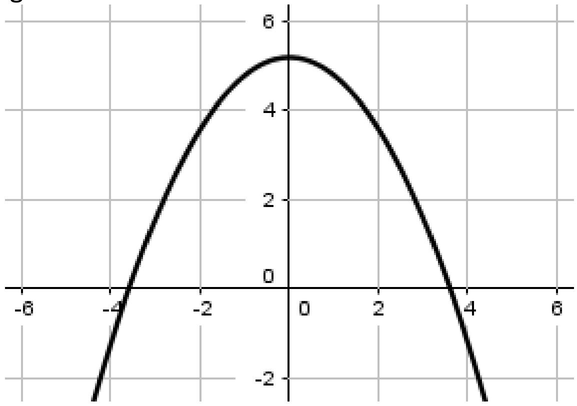
Key points:

The line $x = 0$ is the y-axis. The line $y = 0$ is the x-axis.	A graph crosses the y-axis when $x = 0$. A graph crosses the x-axis when $y = 0$.	Quadratics cross the y-axis once. They may cross the x-axis 0, 1 or 2 times.

A 'positive' quadratic is one where the coefficient of x^2 is positive. It has a *minimum*:



A 'negative' quadratic is one where the coefficient of x^2 is negative. It has a *minimum*:



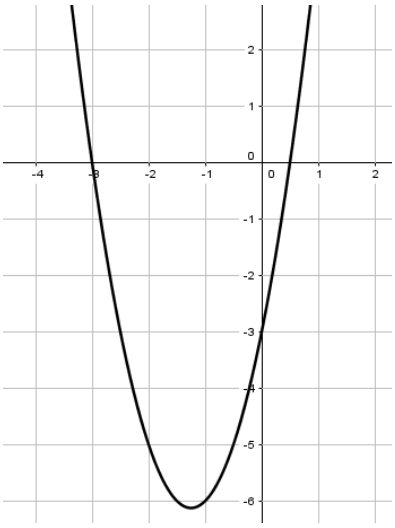
If it isn't obvious whether a quadratic is positive or negative (eg, there are brackets involved), rearrange it to check.

You are given:

$$y = (2x-1)(x+3)$$

The method:
Substitute in $x = 0$ to find the y-axis crossing point.
Substitute in $y = 0$ to find any x-axis crossing points.

You need to sketch:



The working:

y-axis crossing point:

$$x = 0 \quad \Longrightarrow \quad y = (2(0)-1)(0+3) = (-1)(3) = -3$$

$$Crossing\ point: \quad (\mathbf{0}, -\mathbf{3})$$

x-axis crossing points:

$$y = 0 \quad \Longrightarrow \quad 0 = (2x-1)(x+3)$$

$$\Longrightarrow \quad 2x - 1 = 0 \quad or \quad x + 3 = 0$$

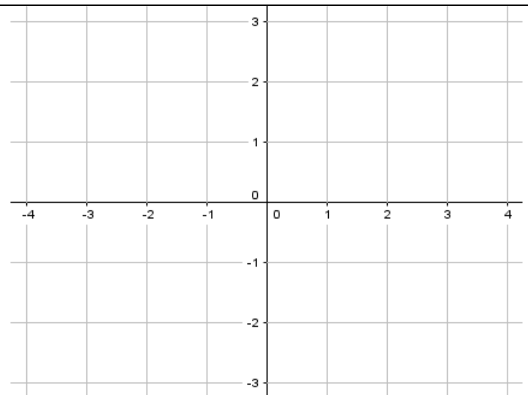
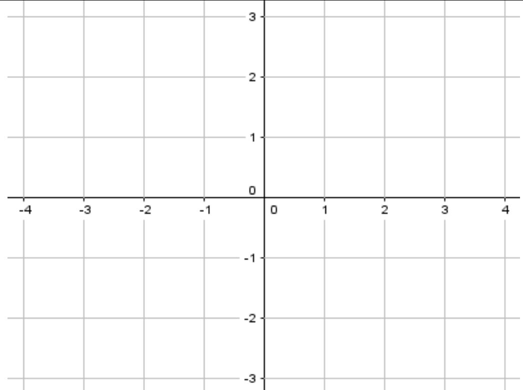
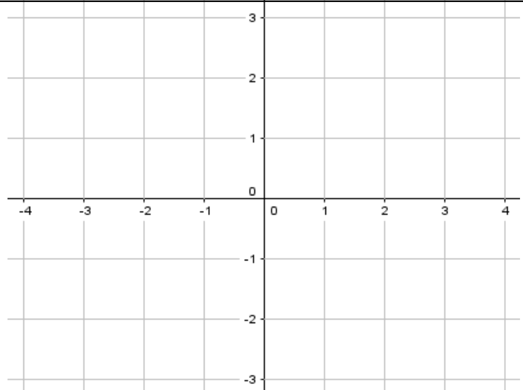
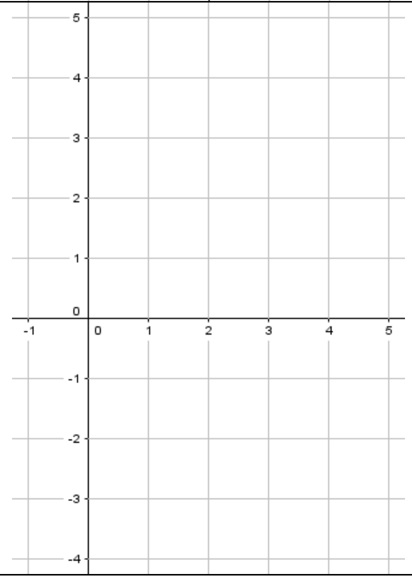
$$\Longrightarrow \quad x = 0.5 \quad or \quad x = -3$$

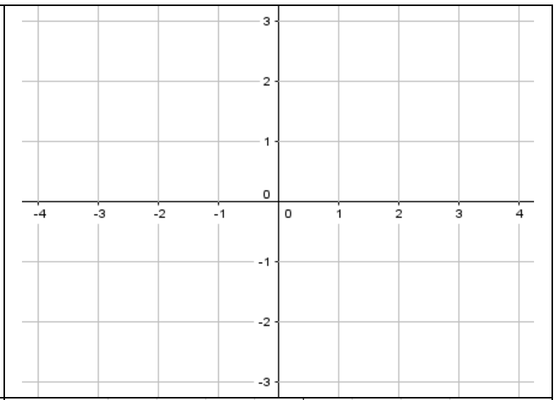
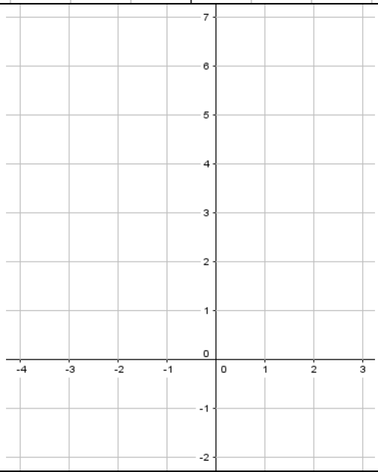
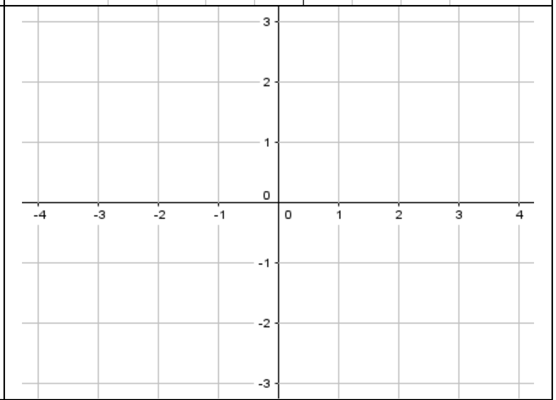
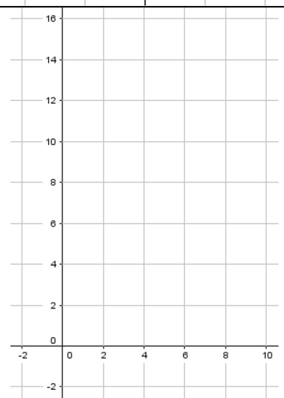
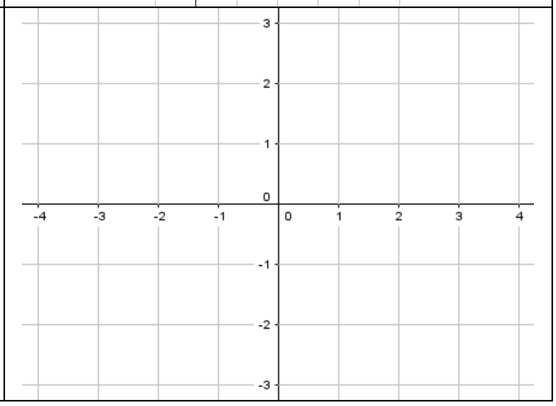
$$Crossing\ points: \quad (\mathbf{0.5}, \mathbf{0}) \;\; \boldsymbol{and} \;\; (-\mathbf{3}, \mathbf{0})$$

Since we can see that the quadratic is positive, we know the overall shape, and now we know where it crosses the axes, so we can sketch it as shown.

Sketching Quadratics

By working out where the following quadratic curves cross the x- and y-axes, draw a sketch of each one on the grid.

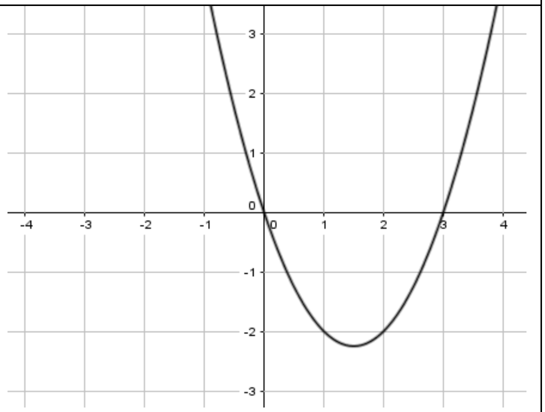
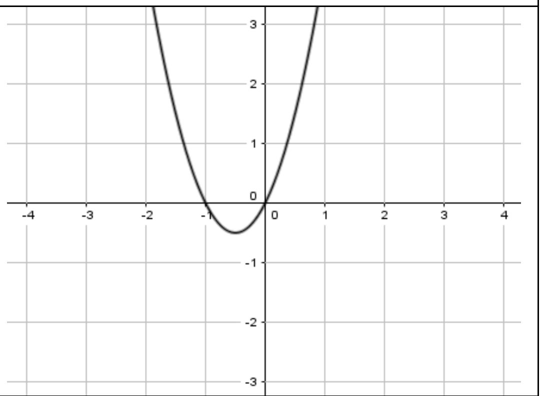
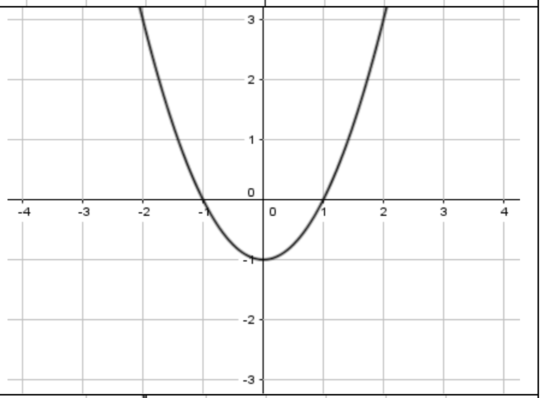
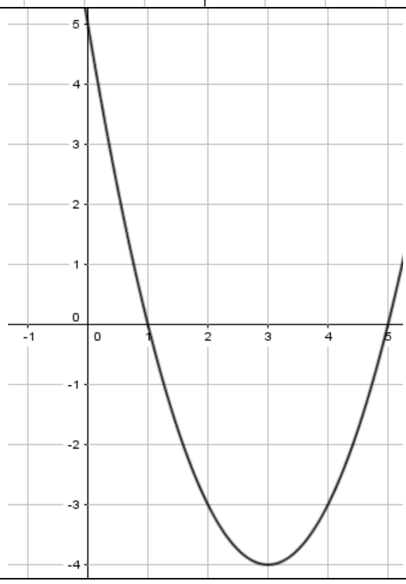
Quadratic Curve Equation	Sketch
$y = x(x-3)$	
$y = 2x(x+1)$	
$y = (x-1)(x+1)$	
$y = (x-1)(x-5)$	

$y = (x+1)(x+3)$	
$y = (x+3)(2-x)$	
$y = x^2$	
$y = (x-4)^2$	
$y = -x^2$	

Challenge: Sketch $y = x^2 + 4$. Note that some quadratics – such as this one – cannot be factorised.
In fact, it is not possible to find any real solutions to $x^2 + 4 = 0$ at all, so it may be necessary to use a table of values.

Sketching Quadratics SOLUTIONS

By working out where the following quadratic curves cross the x- and y-axes, draw a sketch of each one on the grid.

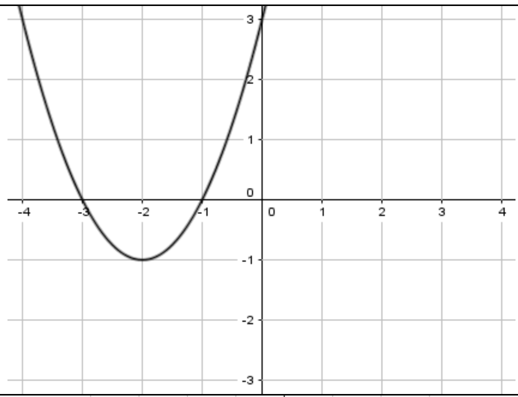
Quadratic Curve Equation	Sketch
$y = x(x-3)$ Factors: x and $x-3$ y-axis crossing point: $(0,0)$ x-axis crossing points: $(0,0)\ \ and\ \ (3,0)$	
$y = 2x(x+1)$ Factors: $2x$ and $x+1$ y-axis crossing point: $(0,0)$ x-axis crossing points: $(0,0)\ \ and\ \ (-1,0)$	
$y = (x-1)(x+1)$ Factors: $x-1$ and $x+1$ y-axis crossing point: $(0,-1)$ x-axis crossing points: $(1,0)\ \ and\ \ (-1,0)$	
$y = (x-1)(x-5)$ Factors: $x-1$ and $x-5$ y-axis crossing point: $(0,5)$ x-axis crossing points: $(1,0)\ \ and\ \ (-1,0)$	

$$y = (x+1)(x+3)$$

Factors: $x+1$ and $x+3$

y-axis crossing point: $(0,3)$

x-axis crossing points: $(-1,0)\ \ and\ \ (-3,0)$

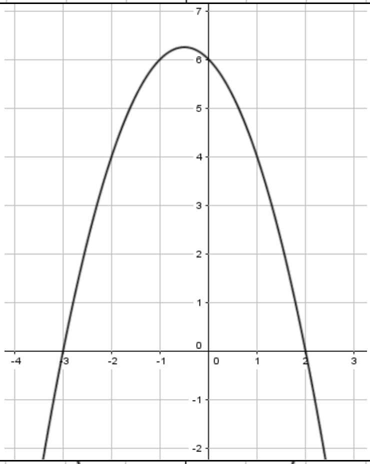


$$y = (x+3)(2-x)$$

Factors: $x+3$ and $2-x$

y-axis crossing point: $(0,6)$

x-axis crossing points: $(-3,0)\ \ and\ \ (2,0)$

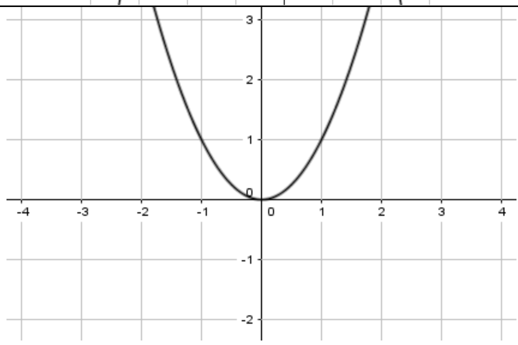


$$y = x^2$$

Factors: x (and x, a repeated factor)

y-axis crossing point: $(0,0)$

x-axis crossing point: $(0,0)$

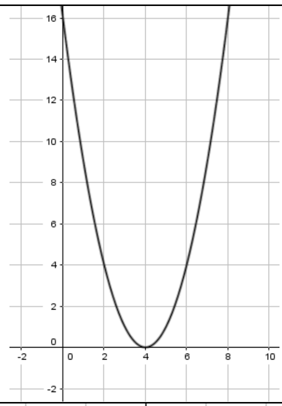


$$y = (x-4)^2$$

Factors: $x-4$ (and $x-4$, a repeated factor)

y-axis crossing point: $(0,16)$

x-axis crossing point: $(4,0)$

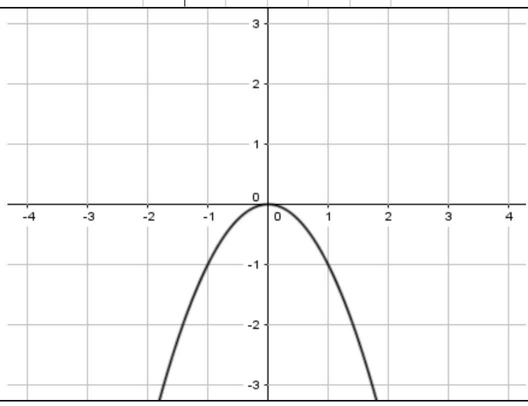


$$y = -x^2$$

Factors: x and $-x$

y-axis crossing point: $(0,0)$

x-axis crossing point: $(0,0)$



Challenge: Sketch $y = x^2 + 4$. Note that some quadratics – such as this one – cannot be factorised.
In fact, it is not possible to find any real solutions to $x^2 + 4 = 0$ at all, so it may be necessary to use a table of values.
The sketch crosses the y-axis at $(0,4)$, and this is the lowest point. It is symmetrical about the y-axis.