Tracing an algorithm

When a computer follows a computer program, it carries out one instruction at a time (albeit very rapidly). To understand how a program functions it can be helpful to manually *trace* the algorithm; that is, to write down the outcome of every instruction as it comes along, just as a computer would.

Line 10: $A = 1,\ B = \frac{1}{\sqrt{2}},\ C = \frac{1}{4},\ X = 1$
Line 20: Let $Y = A$
Line 30: Let $A = \frac{A+B}{2}$
Line 40: Let $B = \sqrt{BY}$
Line 50: Let $C = C - X(A-Y)^2$
Line 60: Let $X = 2X$
Line 70: Print $\frac{(A+B)^2}{4C}$
Line 80: If $X < 8$ GoTo Line 20
Line 90: End

A trace of this algorithm involves making a note of each update to any variables used by the program (and often the printed outputs as well). Complete the trace:

A	B	C	X	Y	Print
1	0.707107	0.25	1		
				1	
0.853553					

Can you see the aim? Designed by Yoshiaki Tamura and Yasumasa Kanada, in fewer than 20 passes it gives a result accurate to over 1 million decimal places!

Tracing an algorithm SOLUTIONS

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A	B	C	X	Y	Print
1	0.707107	0.25	1		
				1	
0.853553					
	0.840896				
		0.228553			
			2		
					3.14057925052217
				0.853553	
0.847225					
	0.847201				
		0.228473			
			4		
					3.14159264621354
				0.847225	
0.847213					
	0.847213				
		0.228473			
			8		
					3.14159265358979

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