Logarithms

A logarithm is a function which does the opposite of raising a base to your number.

Note: The opposite of x^3 is the function $\sqrt[3]{x}$, but the opposite of 3^x is the function $\log_3 x$.

Consider the formula:

$$A = b^x$$

- The number $\boldsymbol{b}$ is called the **base**.
- The number $\boldsymbol{x}$ is called the **logarithm** (which is just another word for power or index).

The function which reverses this (makes x the subject of the formula) is the **log function**:

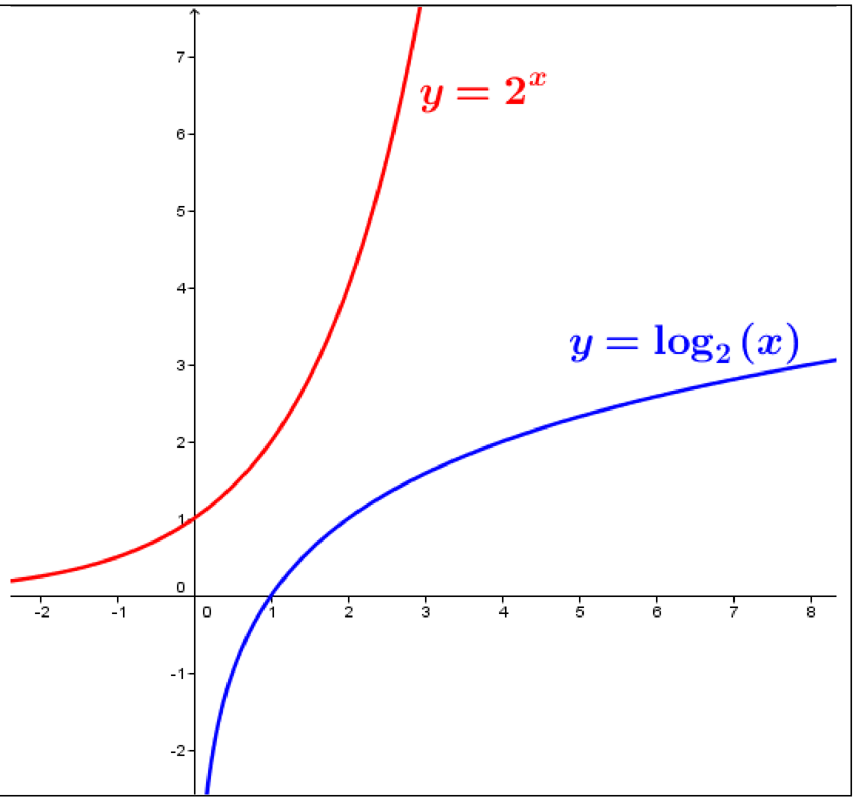
$$\log_b A = x$$

Since log base b reverses raising b to a power, the two functions can cancel each other out:

$$\log_b b^x = x = b^{\log_b x}$$

An exponential function such as $y = 2^x$ increases, and it does so at an ever-increasing rate.

A logarithmic function such as $y = \log_2 x$ also increases, but at an ever-decreasing rate.



Logarithms Questions

You may find these index laws useful:

$a^x a^y = a^{x+y}$ $\qquad \frac{a^x}{a^y} = a^{x-y}$ $\qquad (a^x)^y = a^{xy}$ $\qquad a^{-x} = \frac{1}{a^x}$ $\qquad a^{\frac{1}{n}} = \sqrt[n]{a}$

Recall that $\log_5 125 = ?$ means the same as $5^? = 125$.
Example:

$$\log_{10} 1000 = \log_{10} 10^3 = 3$$

Answer the questions below. Two of the questions below are impossible…

$\log_4 256 =$ $\qquad \log_2 \frac{1}{16} =$ $\qquad \log_{49} 7 =$ $\qquad \log_9 9 =$

$\log_{17} 1 =$ $\qquad \log_{\frac{1}{2}} 32 =$ $\qquad \log_{16} 0 =$ $\qquad \log_2 -4 =$

Recall that $\log_n p = q$ is equivalent to $n^q = p$.
Example:

$$\log_6 36 = 2 \quad \Longrightarrow \quad 6^2 = 36$$

Rewrite the following statements in exponential form.

$\log_2 32 = 5 \quad \Longrightarrow$ $\qquad \log_{10} 100 = 2 \quad \Longrightarrow$

$\log_{0.25} 16 = -2 \quad \Longrightarrow$ $\qquad \log_5 0.2 = -1 \quad \Longrightarrow$

Common bases for logarithms:

Base:	**Used by:**	**Useful because:**
10	Astronomers	Indicates the number of digits
2	Programmers	Indicates the number of digits in binary
e ($\approx 2.71828\ldots$)	Mathematicians	$y = e^x$ has the same gradient as y value

$\log_{10} 1234 \approx 3.091$ (between 10^3 and 10^4, so 4 digits)
$\log_2 1234 \approx 10.269$ (between 2^{10} and 2^{11}, so 11 digits: 10011010010)
$\log_e 1234 = \ln 1234 \approx 7.118\ldots$ ($y = e^x$ has gradient 1234 at (7.118,1234))

Logarithms Questions SOLUTIONS

You may find these index laws useful:

$a^x a^y = a^{x+y}$ $\quad \frac{a^x}{a^y} = a^{x-y}$ $\quad (a^x)^y = a^{xy}$ $\quad a^{-x} = \frac{1}{a^x}$ $\quad a^{\frac{1}{n}} = \sqrt[n]{a}$

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$\log_4 256 = 4$ $\quad \log_2 \frac{1}{16} = -4$ $\quad \log_{49} 7 = \frac{1}{2}$ $\quad \log_9 9 = 1$

$\log_{17} 1 = 0$ $\quad \log_{\frac{1}{2}} 32 = -5$ $\quad \log_{16} 0 = *$ $\quad \log_2 -4 = *$

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Rewrite the following statements in exponential form.

$\log_2 32 = 5 \quad \Longrightarrow \quad 2^5 = 32$ $\qquad \log_{10} 100 = 2 \quad \Longrightarrow \quad 10^2 = 100$

$\log_{0.25} 16 = -2 \quad \Longrightarrow \quad 0.25^{-2} = 16$ $\qquad \log_5 0.2 = -1 \quad \Longrightarrow \quad 5^{-1} = 0.2$

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