Limits of a Sequence: Investigation

For this investigation you will need to use a spreadsheet such as Microsoft Excel or Google Spreadsheets (which you can access through a web browser). You could use a calculator, but you'll find it much easier with a spreadsheet.

1. Make up a sequence in the following format:
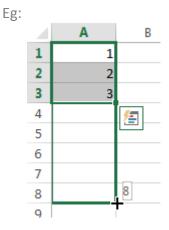
$$T(n) = \frac{an + b}{cn + d}$$

Eg:

$$T(n) = \frac{-3n + 4}{0.5n}$$

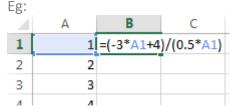where a, b, c and d can be any number, and n is the term number.

2. Make a column of whole numbers from 1 to 20 in your spreadsheet.

Hint: You can enter the first few, then highlight the cells and drag down from the bottom right corner to auto-complete the list.
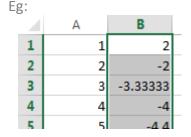
Eg:

3. Make a formula in a cell beside your list which calculates the nth term of your chosen sequence.

*Hint: Remember that Excel interprets * as ×, \ as ÷, and follows the standard mathematical order of operations (BODMAS).*

Eg:

4. Copy this formula down the column, to calculate the first 20 terms of your sequence.

Hint: You can use the same auto-complete method as for writing the first 20 numbers: click and drag the cell from the bottom right corner. Alternatively, double-click the bottom right corner to automatically fill down level with the column to the left. You can also copy and paste the cell, and the formula should automatically update to refer to the number immediately to the left every time.

Eg:

5. Look again at your n^{th} term rule. What should the limit of your sequence be? _____
Compare this limit with the 20th term in the sequence. How close is it? _____
Does the sequence appear to be approaching the limit? _____
What value of n do you need to go up to for the gap to be too small to measure? _____

*Hint: You don't have to list every term – you can change your original list (eg, type 1000 into the first cell instead of 1) and your formula will tell you the 1000th term. Try larger and larger numbers to see if it ever **appears** to reach the limit. (Remember, in reality, these sequences will never reach the limit using only a finite number of terms, but they can get as close as you like.)*

Eg:

1599998	-5.99999
1599999	-5.99999
1600000	-6
1600001	-6