Jodrell Bank

The year is 2415. The Earth has long since been abandoned by all but a hardy civilization of cosmophobes (the space-travel averse). While the majority of the planet's population is living a life of ease on the glistening shores of Europa's shining seas, these few surviving Earthlings have congregated (out of loneliness, and the fact that the Europans took Facebook with them) at Jodrell Bank observatory:

Having originally chosen the site for ease of calling Europa, the hour-long time-lag in phone conversations caused them to rethink their use of the 76m diameter satellite dish. Nowadays, it faces directly upwards and collects rainwater.

Like all satellite dishes, this dish is designed as a parabola (this bounces radio waves towards a central focus, effectively amplifying signals). This particular parabolic dish is based on the shape produced by a 360° rotation about the y-axis for the curve $y = \frac{1}{44}x^2$ between the x-axis and the line $y = 33$.
All measurements are in metres.

How much water, in cubes (cubic metres), would the satellite dish hold when full?

How much water will the dish collect a year, given the average rainfall of the Jodrell Bank region of 800mm a year (assuming no evaporation)?

How many people would this sustainably support?
(note: $1m^3 = 1000$ litres, and the average person uses 150 litres per day)

How long would it take before the dish overflows if the population of the colony is actually 40 people?

Jodrell Bank - Solutions

The year is 2415. The Earth has long since been abandoned by all but a hardy civilization of cosmophobes (the space-travel averse). While the majority of the planet's population is living a life of ease on the glistening shores of Europa's shining seas, these few surviving Earthlings have congregated (out of loneliness, and the fact that the Europans took Facebook with them) at Jodrell Bank observatory:

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How much water, in cubes (cubic metres), would the satellite dish hold when full?

$$\pi \int_0^{33} 44y\, dy = 44\pi \left[\frac{y^2}{2}\right]_0^{33} = 23958\pi = \mathbf{75266m^3\ to\ the\ nearest\ m^3}$$

How much water will the dish collect a year, given the average rainfall of the Jodrell Bank region of 800mm a year (assuming no evaporation)?

$$Area\ of\ opening = \pi r^2 = 38^2\pi = 1444\pi = 4536m^3\ to\ the\ nearest\ m^3$$

$$800mm = 0.8m\ so\ total = 4536 \times 0.8 = \mathbf{3629m^3\ to\ the\ nearest\ m^3}$$

How many people would this sustainably support?
(note: $1m^3 = 1000$ litres, and the average person uses 150 litres per day)

$150\ litres\ per\ day = 0.15m3\ per\ day = 54.75m3\ per\ year$

$$3629 \div 54.75 = \mathbf{66\ people\ to\ the\ nearest\ person}$$

How long would it take before the dish overflows if the population of the colony is actually 40 people?

$$40\ people\ use\ 54.75 \times 40 = 2190m^3\ per\ year$$

$$This\ leaves\ 3629 - 2190 = 1439m^3\ extra\ per\ year$$

$$The\ dish\ holds\ 75266m^3\ \ so\ \ \frac{75266}{1439} = \mathbf{52\ years\ to\ the\ nearest\ year}$$