Solving Quadratics by Factorising

By **rearranging** if necessary (to make one side of the equation zero), and **factorising** (to write as two things multiplied), you can find the **two possible solutions*** of a quadratic.

Example: Solve $x^2 - 7x - 30 = 0$

Two numbers that multiply to make -30 but add to make -7:

Negative product means one +ve and one -ve. Negative sum means the larger one is –ve, so 3 and -10:

$$x^2 - 7x - 30 = 0$$
$$(x + 3)(x - 10) = 0$$

Since they multiply to make 0:

$$x + 3 = 0 \quad or \quad x - 10 = 0$$
$$\boldsymbol{x = -3 \quad or \quad x = 10}$$

A: Single Term Factorising	**B: Double Bracket Factorising**
1. $x^2 - 5x = 0$	1. $x^2 + 8x + 7 = 0$
2. $3x^2 + 4x = 0$	2. $x^2 + 12x + 11 = 0$
3. $5x^2 = 10x$	3. $x^2 + 7x + 12 = 0$
4. $7x - 6x^2 = 0$	4. $x^2 + 14x + 48 = 0$
5. $2x^2 + 4x = 3x^2$	5. $x^2 + 19x + 48 = 0$
6. $x(x - 3) = 2x(x + 2)$	6. $x^2 + 13x + 36 = 0$
C: Harder Double Bracket Factorising	**D: Disguised Double Bracket Factorising**
1. $x^2 - 36 = 0$	1. $x^2 - 5x + 4 = 0$
2. $x^2 + 4x - 32 = 0$	2. $(x + 1)^2 - 5(x + 1) + 4 = 0$
3. $x^2 - 11x + 28 = 0$	3. $x^4 - 5x^2 + 4 = 0$
4. $x^2 - 9x + 14 = 0$	4. $x - 5\sqrt{x} + 4 = 0$
5. $x^2 - 3x - 10 = 0$	5. $\frac{1}{x^2} - \frac{5}{x} + 4 = 0$
6. $x^2 + 10x - 39 = 0$	6. $x - 5 + \frac{4}{x} = 0$

**The two solutions of the quadratic may not really exist (they would involve the square root of a negative) or they may be identical to each other, but even though this means you may have 0, 1 or 2 solutions you should always look for two so as to avoid missing any out.*

Solving Quadratics by Factorising SOLUTIONS

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Example: Solve $x^2 - 7x - 30 = 0$

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$$\boldsymbol{x = -3 \quad or \quad x = 10}$$

A: Single Term Factorising

1. $x^2 - 5x = 0$
   $x(x - 5) = 0 \implies \boldsymbol{x = 0 \ or \ x = 5}$
2. $3x^2 + 4x = 0$
   $x(3x + 4) = 0 \implies \boldsymbol{x = 0 \ or \ x = -\frac{4}{3}}$
3. $5x^2 = 10x \implies x^2 = 2x \implies x^2 - 2x = 0$
   $x(x - 2) = 0 \implies \boldsymbol{x = 0 \ or \ x = 2}$
4. $7x - 6x^2 = 0$
   $x(7 - 6x) = 0 \implies \boldsymbol{x = 0 \ or \ x = \frac{7}{6}}$
5. $2x^2 + 4x = 3x^2 \implies -x^2 + 4x = 0$
   $-x(x - 4) = 0 \implies \boldsymbol{x = 0 \ or \ x = 4}$
6. $x(x - 3) = 2x(x + 2)$
   $\implies x^2 - 3x = 2x^2 + 4x \implies 0 = x^2 + 7x$
   $x(x + 7) = 0 \implies \boldsymbol{x = 0 \ or \ x = -7}$

B: Double Bracket Factorising

1. $x^2 + 8x + 7 = 0$
   $(x + 1)(x + 7) = 0 \implies \boldsymbol{x = -1 \ or \ x = -7}$
2. $x^2 + 12x + 11 = 0$
   $(x + 11)(x + 1) = 0 \implies \boldsymbol{x = -11 \ or \ x = -1}$
3. $x^2 + 7x + 12 = 0$
   $(x + 3)(x + 4) = 0 \implies \boldsymbol{x = -3 \ or \ x = -4}$
4. $x^2 + 14x + 48 = 0$
   $(x + 6)(x + 8) = 0 \implies \boldsymbol{x = -6 \ or \ x = -8}$
5. $x^2 + 19x + 48 = 0$
   $(x + 16)(x + 3) = 0 \implies \boldsymbol{x = -16 \ or \ x = -3}$
6. $x^2 + 13x + 36 = 0$
   $(x + 4)(x + 9) = 0 \implies \boldsymbol{x = -4 \ or \ x = -9}$

C: Harder Double Bracket Factorising

1. $x^2 - 36 = 0$
   $(x + 6)(x - 6) = 0 \implies \boldsymbol{x = -6 \ or \ x = 6}$
2. $x^2 + 4x - 32 = 0$
   $(x + 8)(x - 4) = 0 \implies \boldsymbol{x = -8 \ or \ x = 4}$
3. $x^2 - 11x + 28 = 0$
   $(x - 7)(x - 4) = 0 \implies \boldsymbol{x = 7 \ or \ x = 4}$
4. $x^2 - 9x + 14 = 0$
   $(x - 2)(x - 7) = 0 \implies \boldsymbol{x = 2 \ or \ x = 7}$
5. $x^2 - 3x - 10 = 0$
   $(x - 5)(x + 2) = 0 \implies \boldsymbol{x = 5 \ or \ x = -2}$
6. $x^2 + 10x - 39 = 0$
   $(x + 13)(x - 3) = 0 \implies \boldsymbol{x = -13 \ or \ x = 3}$

D: Disguised Double Bracket Factorising

1. $x^2 - 5x + 4 = 0$
   $(x - 1)(x - 4) = 0 \implies \boldsymbol{x = 1 \ or \ x = 4}$
2. $(x + 1)^2 - 5(x + 1) + 4 = 0$
   $\big((x + 1) - 1\big)\big((x + 1) - 4\big) = 0$
   $\implies x + 1 = 1 \ or \ x + 1 = 4 \implies \boldsymbol{x = 0 \ or \ x = 3}$
3. $x^4 - 5x^2 + 4 = 0$
   $(x^2 - 1)(x^2 - 4) = 0$
   $x^2 = 1 \ or \ x^2 = 4 \implies \boldsymbol{x = \pm 1 \ or \ x = \pm 2}$
4. $x - 5\sqrt{x} + 4 = 0$
   $\big(\sqrt{x} - 1\big)\big(\sqrt{x} - 4\big) = 0$
   $\sqrt{x} = 1 \ or \ \sqrt{x} = 4 \implies \boldsymbol{x = 1 \ or \ x = 16}$
5. $\frac{1}{x^2} - \frac{5}{x} + 4 = 0$
   $\left(\frac{1}{x} - 1\right)\left(\frac{1}{x} - 4\right) = 0$
   $\frac{1}{x} = 1 \ or \ \frac{1}{x} = 4 \implies \boldsymbol{x = 1 \ or \ x = \frac{1}{4}}$
6. $x - 5 + \frac{4}{x} = 0$
   Multiplying both sides by x gives $x^2 - 5x + 4 = 0$
   $(x - 1)(x - 4) = 0 \implies \boldsymbol{x = 1 \ or \ x = 4}$