The Difference of Two Squares

Task A

Answer the following questions:

1 a) $4 \times 4 =$ b) $3 \times 5 =$

2 a) $5 \times 5 =$ b) $4 \times 6 =$

3 a) $10 \times 10 =$ b) $9 \times 11 =$

4 a) $50 \times 50 =$ b) $49 \times 51 =$

What do you notice?

Task B

Choose two one-digit numbers. Call the largest x, and the smallest y:

$x =$ $y =$

Work out their squares:

$x^2 =$ $y^2 =$

Work out the **difference of the two squares**:

$x^2 - y^2 =$

Next, work out the following two numbers: (the original two numbers' *sum*, and *difference*):

$x + y =$ $x - y =$

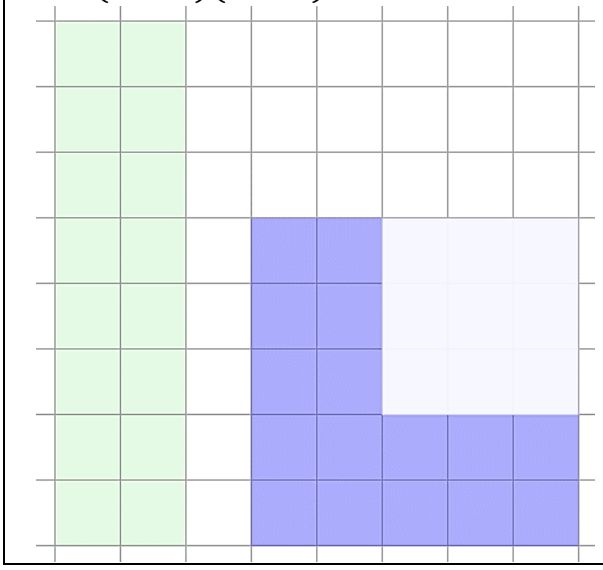
Finally, work out the product of these, then compare it to the difference of the two squares:

$(x + y)(x - y) =$ $x^2 - y^2 =$

What do you notice? *This result is known as the* ***difference of two squares***.

Task C

$A = (5 + 3)(5 - 3)$ $A = 5^2 - 3^2$



Construct shapes of your own to show this idea:

Can you explain why the rectangle must always have the same area as the difference of squares?

Using the *difference of two squares* result

$$(x+y)(x-y) = x^2 - y^2$$

1. By writing 1994 as $2000 - 6$ and 2006 as $2000 + 6$, work out $1994 \times 2006 =$

2. Using the fact that $3124 \times 3126 = 9765624$, calculate $3125^2 =$

3. Two numbers, x and y have a sum of $x + y = 100$ and a difference of $x - y = 50$.
   Work out the difference of their squares $x^2 - y^2 =$

4. Multiply out the following brackets:

a) $(x-6)(x+6) =$

b) $(2x-5)(2x+5) =$

c) $\left(x-\sqrt{7}\right)\left(x+\sqrt{7}\right) =$

d) $\left(\sqrt{7}+\sqrt{3}\right)\left(\sqrt{7}-\sqrt{3}\right) =$

e) $(x-1)(x+1)(x^2+1) =$

5. Factorise the following expressions:

a) $x^2 - 9 =$

b) $x^2 - 100 =$

c) $49 - x^2 =$

d) $16x^2 - 25 =$

e) $36x^2 - 64y^2 =$

f) $x^4 - 1 =$

The Difference of Two Squares SOLUTIONS

Task A

Answer the following questions:

1 a) $4 \times 4 = 16$ b) $3 \times 5 = 15$

2 a) $5 \times 5 = 25$ b) $4 \times 6 = 24$

3 a) $10 \times 10 = 100$ b) $9 \times 11 = 99$

4 a) $50 \times 50 = 2500$ b) $49 \times 51 = 2499$

What do you notice?

The product of two numbers with a difference of two is always 1 less than the square of the number between.

Task B

Choose two one-digit numbers. Call the largest x, and the smallest y:

$x =$ $y =$

Work out their squares:

$x^2 =$ $y^2 =$

Work out the **difference of the two squares**:

$x^2 - y^2 =$

Next, work out the following two numbers: (the original two numbers' *sum*, and *difference*):

$x + y =$ $x - y =$

Finally, work out the product of these, then compare it to the difference of the two squares:

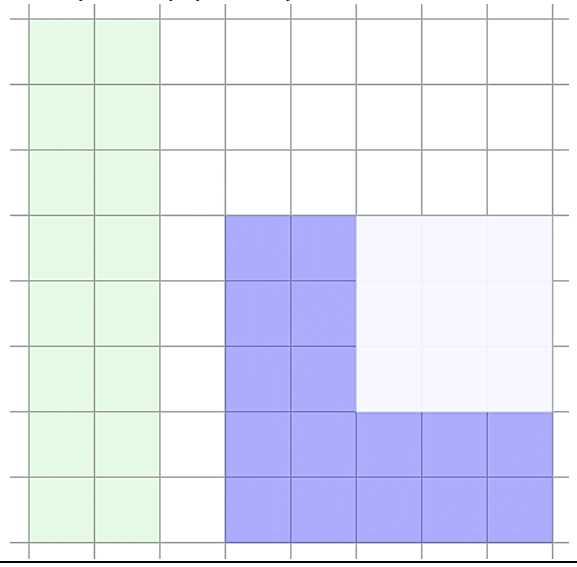
$(x + y)(x - y) =$ $x^2 - y^2 =$

What do you notice? *This result is known as the* ***difference of two squares***.

The product is equal to the difference of the two squares.

Task C

$A = (5 + 3)(5 - 3)$ $A = 5^2 - 3^2$



Construct shapes of your own to show this idea:

Can you explain why the rectangle must always have the same area as the difference of squares?

The section below the smaller square can be moved to sit above the left-hand section to form the rectangle.

Using the *difference of two squares* result SOLUTIONS

$$(x+y)(x-y) = x^2 - y^2$$

1. By writing 1994 as $2000-6$ and 2006 as $2000+6$, work out $1994 \times 2006 =$

$(2000-6)(2000+6) = 2000^2 - 36 = 4000000 - 36 = \mathbf{3999964}$

2. Using the fact that $3124 \times 3126 = 9765624$, calculate $3125^2 =$

$9765624 = (3125-1)(3125+1) = 3125^2 - 1 \implies 3125^2 = \mathbf{9765625}$

3. Two numbers, x and y have a sum of $x+y=100$ and a difference of $x-y=50$.
Work out the difference of their squares $x^2 - y^2 = (x+y)(x-y) = 100 \times 50 = \mathbf{5000}$

4. Multiply out the following brackets:

a) $(x-6)(x+6) = \boldsymbol{x^2 - 36}$

b) $(2x-5)(2x+5) = \boldsymbol{4x^2 - 25}$

c) $(x-\sqrt{7})(x+\sqrt{7}) = \boldsymbol{x^2 - 7}$

d) $(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3}) = 7 - 3 = \mathbf{4}$

e) $(x-1)(x+1)(x^2+1) = (x^2-1)(x^2+1) = \boldsymbol{x^4 - 1}$

5. Factorise the following expressions:

a) $x^2 - 9 = \boldsymbol{(x+3)(x-3)}$

b) $x^2 - 100 = \boldsymbol{(x+10)(x-10)}$

c) $49 - x^2 = \boldsymbol{(7-x)(7+x)}$

d) $16x^2 - 25 = \boldsymbol{(4x-5)(4x+5)}$

e) $36x^2 - 64y^2 = \boldsymbol{(6x-8y)(6x+8y)}$ *or* $\boldsymbol{4(3x-4y)(3x+4y)}$

f) $x^4 - 1 = (x^2+1)(x^2-1)$ *or even* $\boldsymbol{(x^2+1)(x+1)(x-1)}$