Counting Fish

Knowing how many fish are in a lake is very important both to fish farmers and to conservationists.

Unfortunately, any method for actually counting the fish would involve both prohibitive cost and considerable disruption.

Fortunately, the precise number is not really that important, so we can use relative frequency to produce an estimate for the total:

Recall that relative frequency is a good indicator of probability, and probability is a good indicator of relative frequency, especially for large numbers.

Step 1: Tagging
A certain number of fish are caught, tagged, and returned to the lake. This means they can be easily identified if they are caught again.

Step 2: Sampling
Once the tagged fish have had time to mingle with the rest, a new sample of fish are caught. Provided your samples are large enough, there should be a certain number of tagged fish in this sample.

Step 3: Comparing proportions
The proportion of tagged fish in the sample is our best indicator of the proportion of tagged fish in the whole population. So we use the numbers we know to predict the one we don't – the total population. For example, if 10% of the sample were tagged, we would predict that 10% of the population are tagged. If, say, 500 were tagged, that means the population will be around 5000.

A researcher catches 200 fish from a lake, tags them and returns them.
Later, a sample of 1000 fish is caught and examined. 80 of the fish had tags on them.

1. What proportion of the sample were tagged?

2. Assuming this is representative of the whole lake population, how many fish are likely to be in the entire population?

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A researcher catches 200 fish from a lake, tags them and returns them.
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1. What proportion of the sample were tagged?

$$80 \; out \; of \; 1000 = \frac{80}{1000} = \frac{8}{100} = \mathbf{8}\% \; or \; \frac{\mathbf{2}}{\mathbf{25}} \; or \; \mathbf{0.08}$$

2. Assuming this is representative of the whole lake population, how many fish are likely to be in the entire population?

$$If \; 8\% \; of \; all \; fish \; are \; assumed \; to \; be \; tagged, and \; 200 \; were \; tagged:$$

$$8\% \; of \; Total = 200 \quad \Rightarrow \quad 0.08P = 200 \quad \Rightarrow \quad P = \frac{200}{0.08} = \mathbf{2500 \; fish}$$