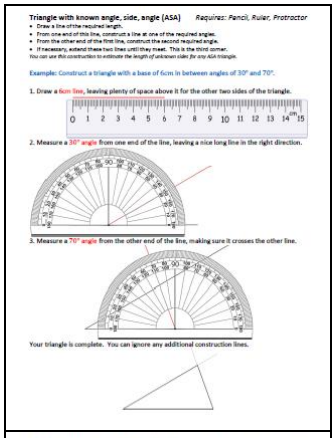
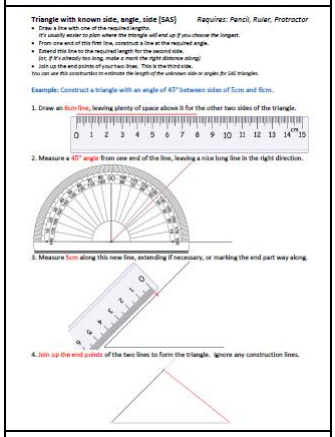
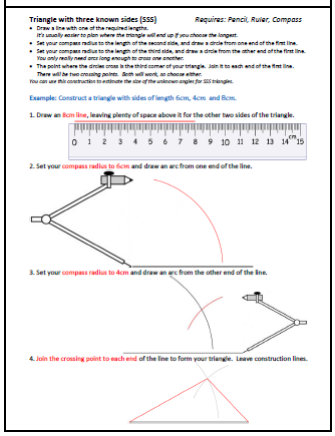
Mathematical Construction

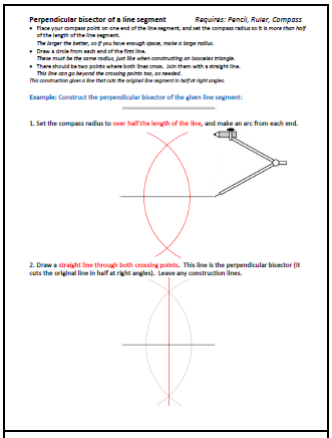


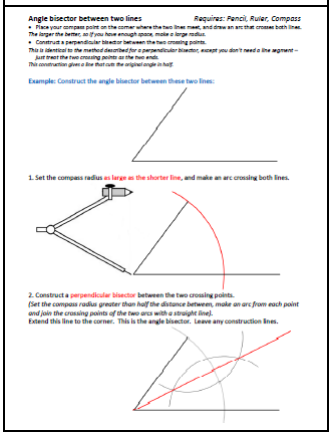




↑ Full illustrated instructions for the three triangles:

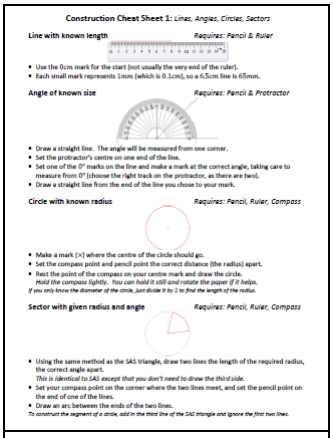
- **ASA**
- **SAS**
- **SSS**

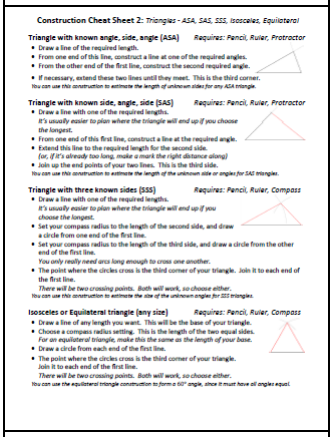


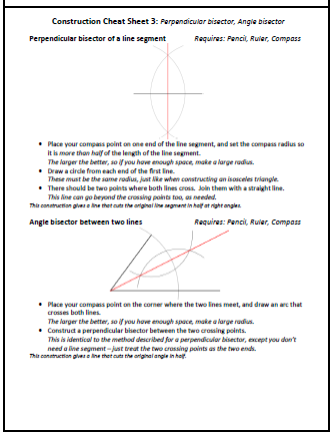


↑ Full illustrated instructions for the two bisectors:

- **Perpendicular bisector**
- **Angle bisector**

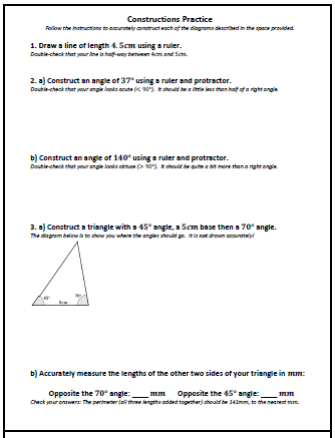


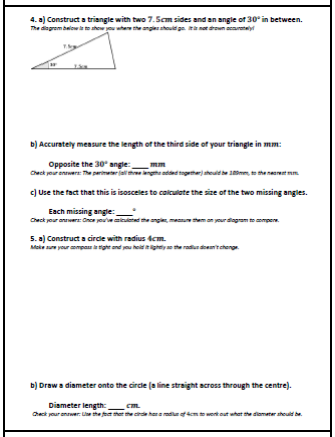


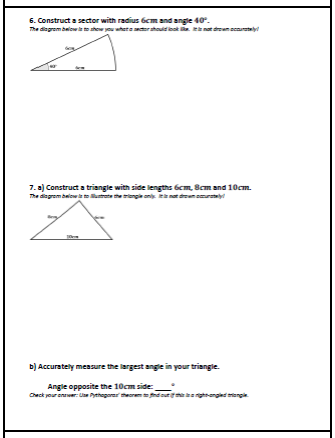


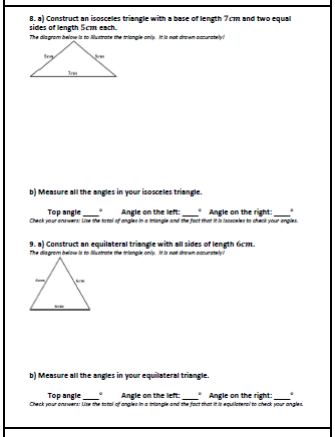
↑ Summarised instructions as cheat sheets for:

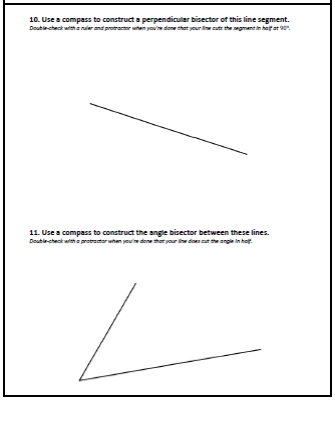
- **Lines**
- **Angles**
- **Circles**
- **Sectors**
- **Triangles**
- **Bisectors**











Practice →

- **Lines**
- **Angles**
- **Circles**
- **Sectors**
- **Triangles**
- **Bisectors**

*Note: These documents have been designed to be printed on **A4 paper**:*

Illustrated instructions are scaled to print out accurately, and questions have been designed to leave sufficient space for the required constructions.

Triangle with known angle, side, angle (ASA)

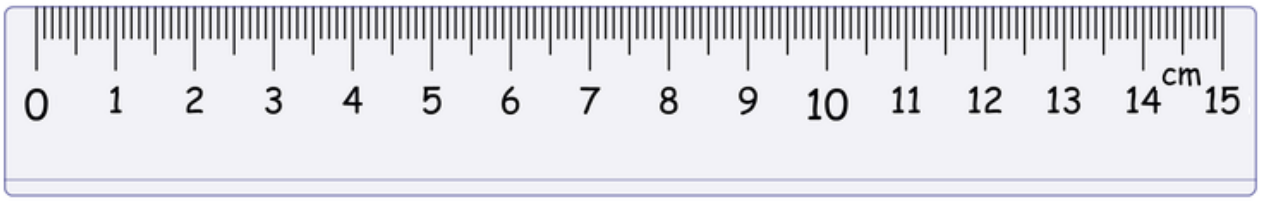
Requires: Pencil, Ruler, Protractor

- Draw a line of the required length.
- From one end of this line, construct a line at one of the required angles.
- From the other end of the first line, construct the second required angle.
- If necessary, extend these two lines until they meet. This is the third corner.

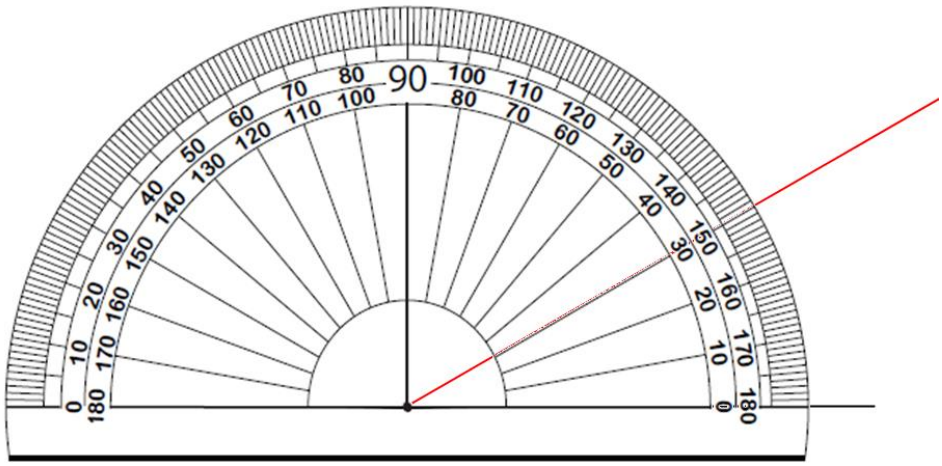
You can use this construction to estimate the length of unknown sides for any ASA triangle.

Example: Construct a triangle with a base of $6cm$ in between angles of $30°$ and $70°$.

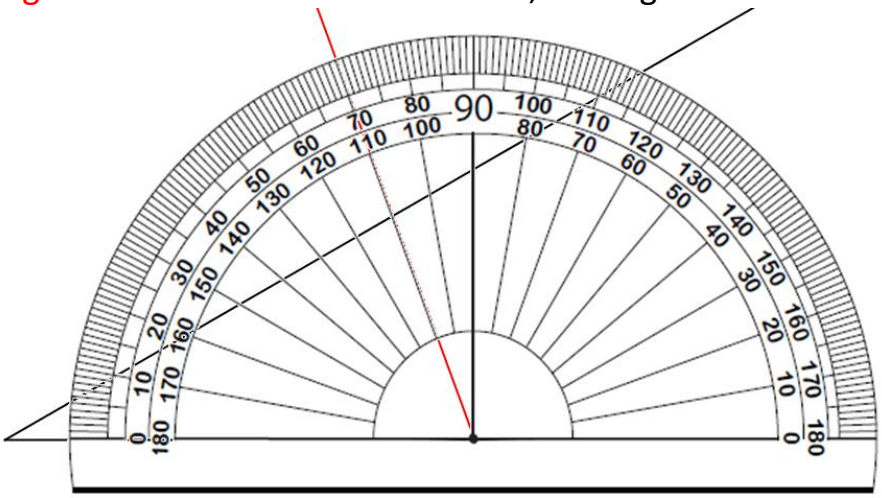
1. Draw a 6cm line, leaving plenty of space above it for the other two sides of the triangle.



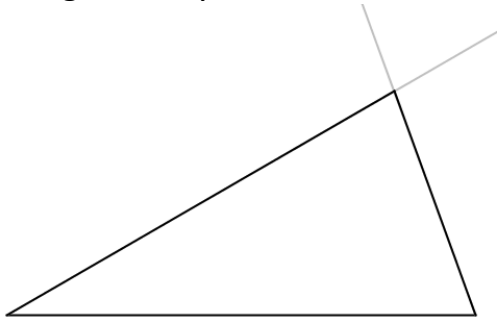
2. Measure a $30°$ angle from one end of the line, leaving a nice long line in the right direction.



3. Measure a $70°$ angle from the other end of the line, making sure it crosses the other line.



Your triangle is complete. You can ignore any additional construction lines.



Triangle with known side, angle, side (SAS)

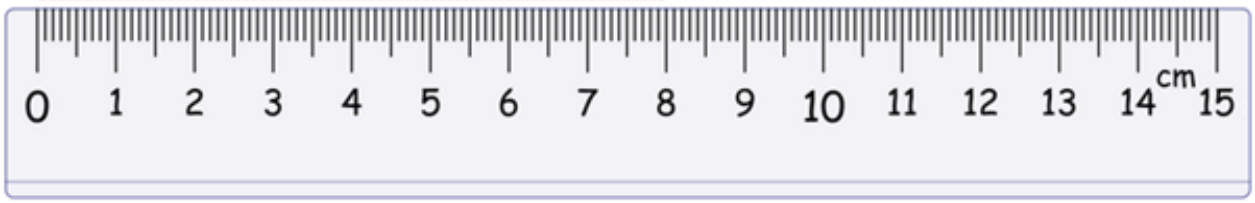
Requires: Pencil, Ruler, Protractor

- Draw a line with one of the required lengths.
  It's usually easier to plan where the triangle will end up if you choose the longest.
- From one end of this first line, construct a line at the required angle.
- Extend this line to the required length for the second side.
  (or, if it's already too long, make a mark the right distance along)
- Join up the end points of your two lines. This is the third side.

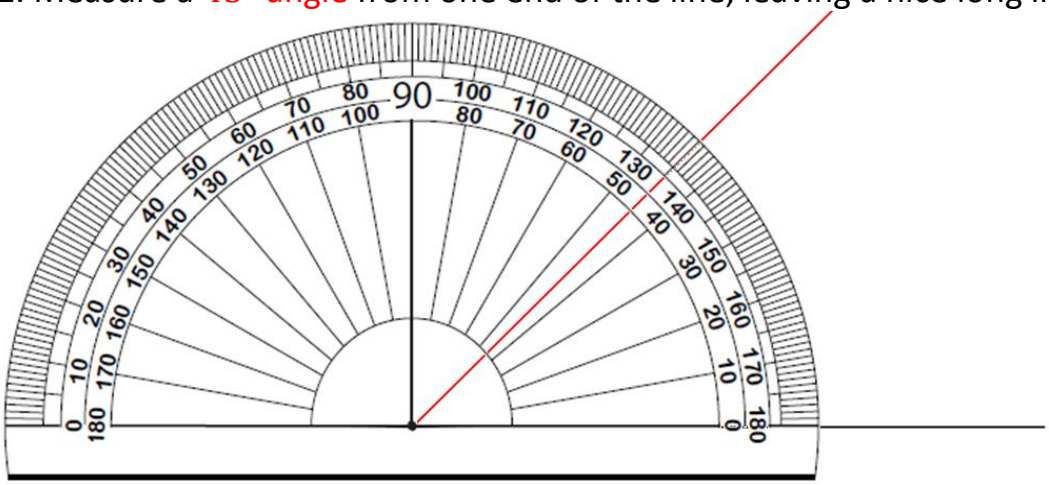
You can use this construction to estimate the length of the unknown side or angles for SAS triangles.

Example: Construct a triangle with an angle of $45°$ between sides of $5cm$ and $8cm$.

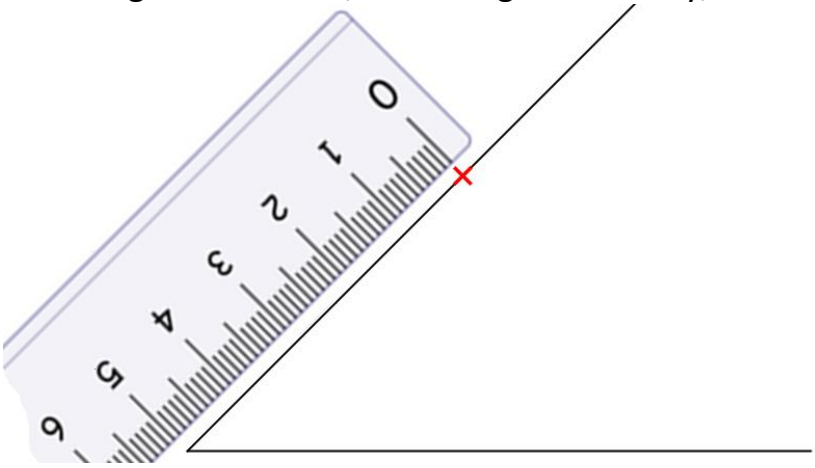
1. Draw an 8cm line, leaving plenty of space above it for the other two sides of the triangle.



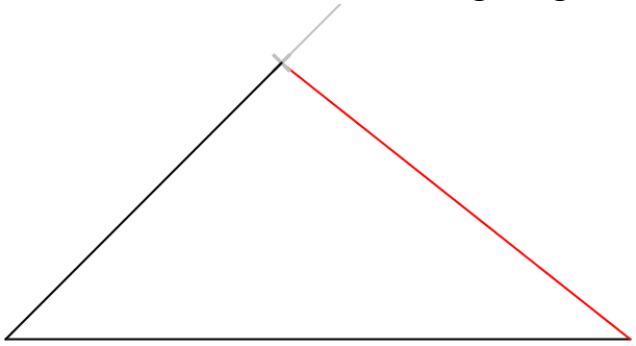
2. Measure a $45°$ angle from one end of the line, leaving a nice long line in the right direction.



3. Measure 5cm along this new line, extending if necessary, or marking the end part way along.



4. Join up the end points of the two lines to form the triangle. Ignore any construction lines.



Triangle with three known sides (SSS)

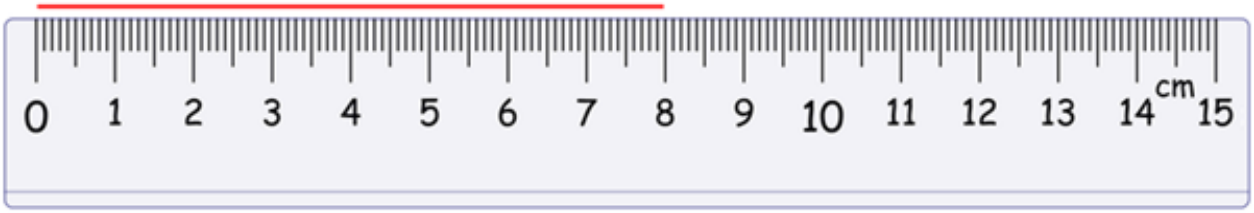
Requires: Pencil, Ruler, Compass

- Draw a line with one of the required lengths.
  It's usually easier to plan where the triangle will end up if you choose the longest.
- Set your compass radius to the length of the second side, and draw a circle from one end of the first line.
- Set your compass radius to the length of the third side, and draw a circle from the other end of the first line.
  You only really need arcs long enough to cross one another.
- The point where the circles cross is the third corner of your triangle. Join it to each end of the first line.
  There will be two crossing points. Both will work, so choose either.

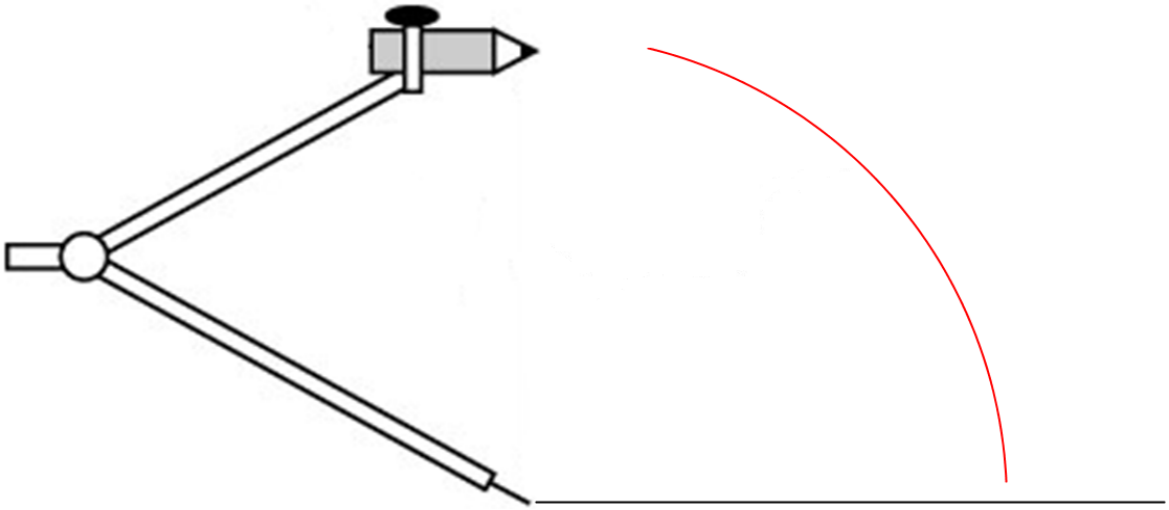
You can use this construction to estimate the size of the unknown angles for SSS triangles.

Example: Construct a triangle with sides of length $6cm$, $4cm$ and $8cm$.

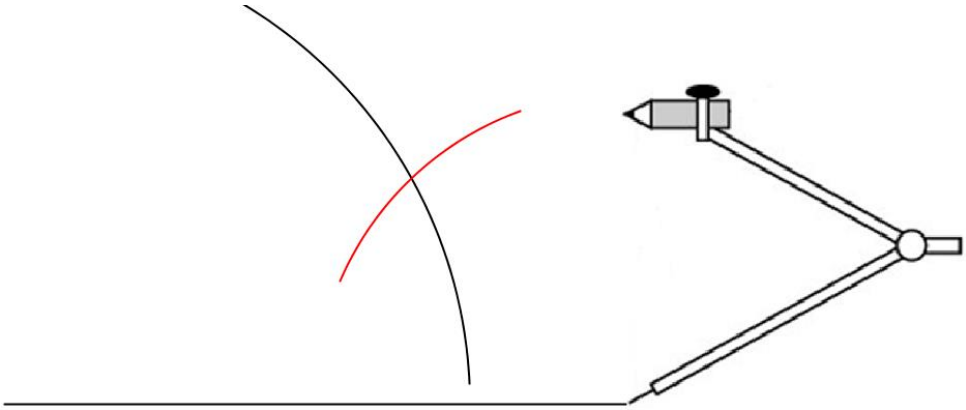
1. Draw an 8cm line, leaving plenty of space above it for the other two sides of the triangle.



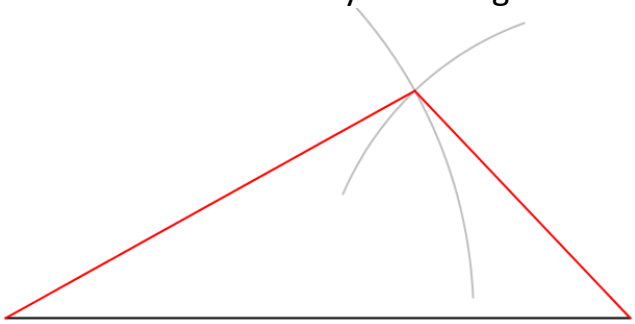
2. Set your compass radius to $6cm$ and draw an arc from one end of the line.



3. Set your compass radius to 4cm and draw an arc from the other end of the line.



4. Join the crossing point to each end of the line to form your triangle. Leave construction lines.



Perpendicular bisector of a line segment

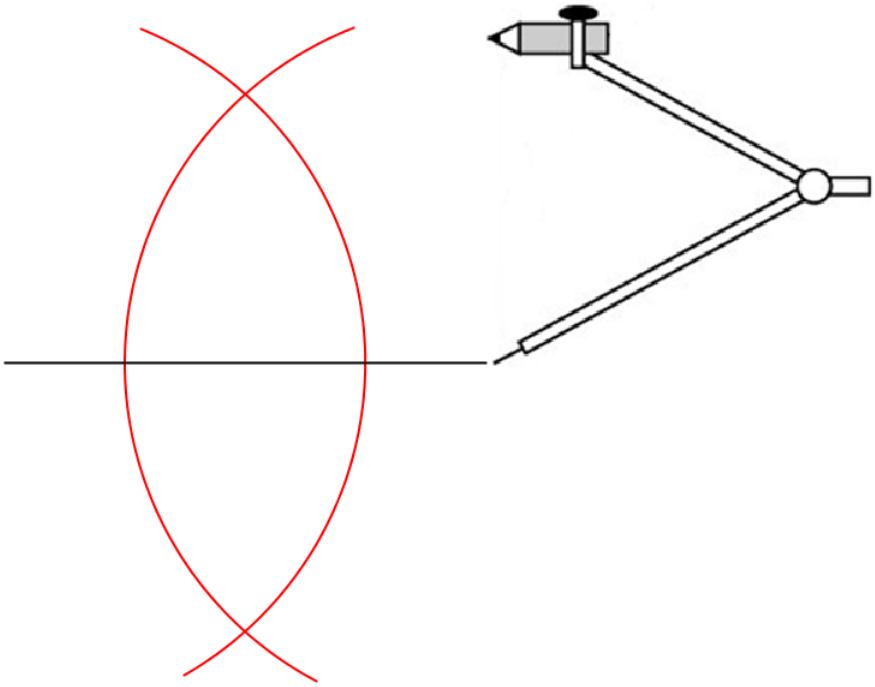
Requires: Pencil, Ruler, Compass

- Place your compass point on one end of the line segment, and set the compass radius so it is *more than half* of the length of the line segment.
  The larger the better, so if you have enough space, make a large radius.
- Draw a circle from each end of the first line.
  These must be the same radius, just like when constructing an isosceles triangle.
- There should be two points where both lines cross. Join them with a straight line.
  This line can go beyond the crossing points too, as needed.

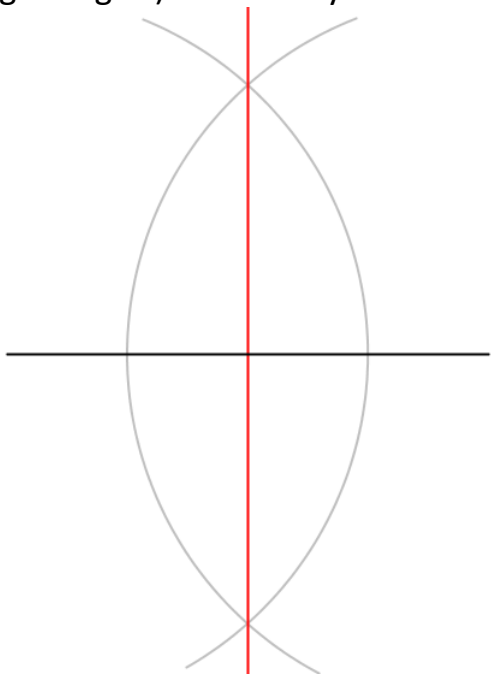
This construction gives a line that cuts the original line segment in half at right angles.

Example: Construct the perpendicular bisector of the given line segment:

1. Set the compass radius to over half the length of the line, and make an arc from each end.



2. Draw a straight line through both crossing points. This line is the perpendicular bisector (it cuts the original line in half at right angles). Leave any construction lines.



Angle bisector between two lines

Requires: Pencil, Ruler, Compass

- Place your compass point on the corner where the two lines meet, and draw an arc that crosses both lines.

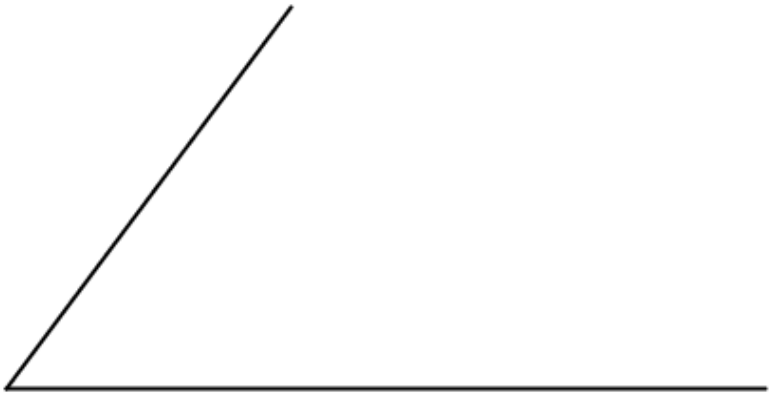
The larger the better, so if you have enough space, make a large radius.

- Construct a perpendicular bisector between the two crossing points.

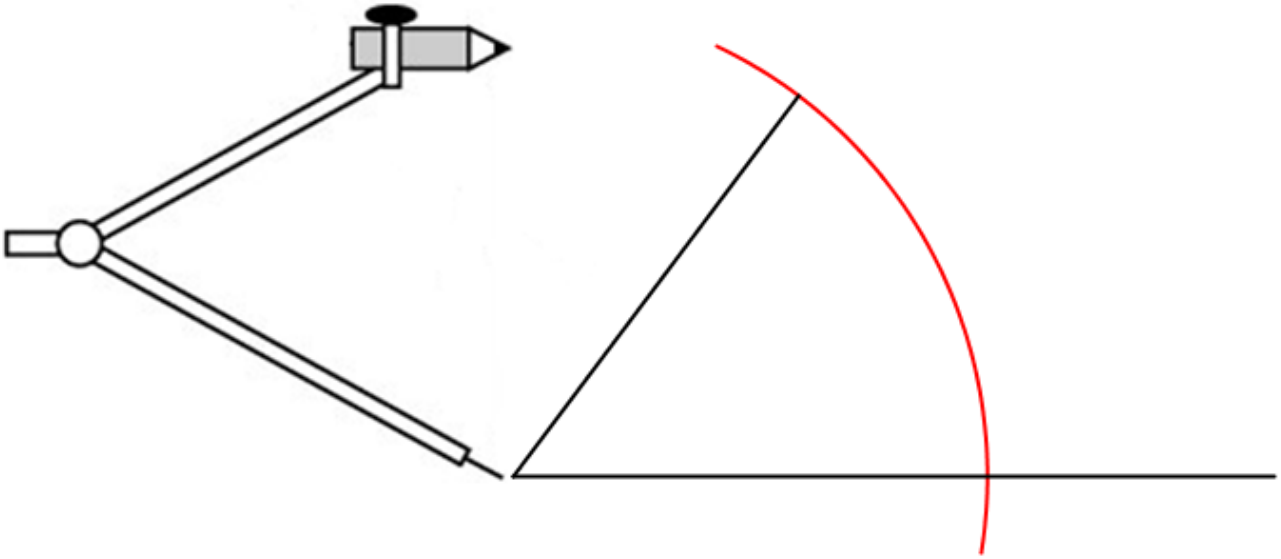
This is identical to the method described for a perpendicular bisector, except you don't need a line segment – just treat the two crossing points as the two ends.

This construction gives a line that cuts the original angle in half.

Example: Construct the angle bisector between these two lines:



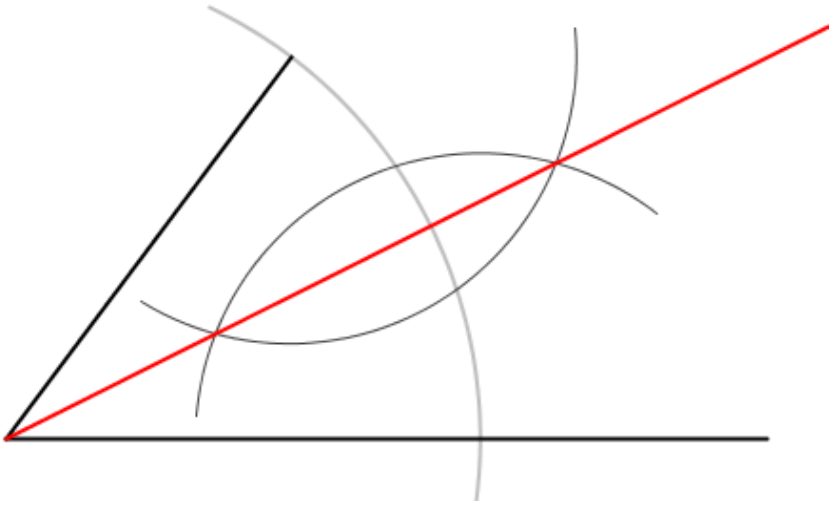
1. Set the compass radius as large as the shorter line, and make an arc crossing both lines.



2. Construct a perpendicular bisector between the two crossing points.

(Set the compass radius greater than half the distance between, make an arc from each point and join the crossing points of the two arcs with a straight line).

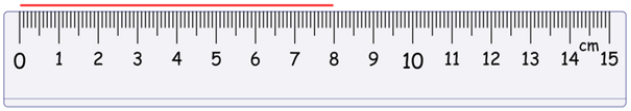
Extend this line to the corner. This is the angle bisector. Leave any construction lines.



Construction Cheat Sheet 1: *Lines, Angles, Circles, Sectors*

Line with known length

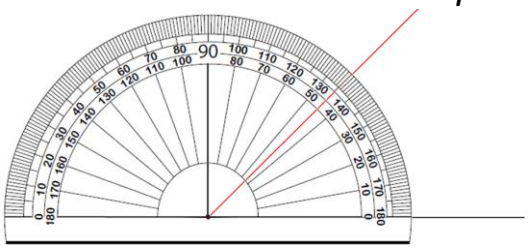
Requires: Pencil & Ruler



- Use the $0cm$ mark for the start (not usually the very end of the ruler).
- Each small mark represents $1mm$ (which is $0.1cm$), so a $6.5cm$ line is $65mm$.

Angle of known size

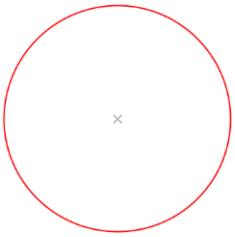
Requires: Pencil & Protractor



- Draw a straight line. The angle will be measured from one corner.
- Set the protractor's centre on one end of the line.
- Set one of the $0°$ marks on the line and make a mark at the correct angle, taking care to measure from $0°$ (choose the right track on the protractor, as there are two).
- Draw a straight line from the end of the line you chose to your mark.

Circle with known radius

Requires: Pencil, Ruler, Compass



- Make a mark ($\times$) where the centre of the circle should go.
- Set the compass point and pencil point the correct distance (the radius) apart.
- Rest the point of the compass on your centre mark and draw the circle.
  Hold the compass lightly. You can hold it still and rotate the paper if it helps.

If you only know the diameter of the circle, just divide it by 2 to find the length of the radius.

Sector with given radius and angle

Requires: Pencil, Ruler, Compass

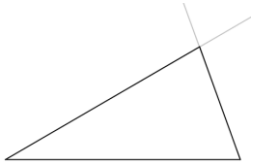
- Using the same method as the SAS triangle, draw two lines the length of the required radius, the correct angle apart.
  This is identical to SAS except that you don't need to draw the third side.
- Set your compass point on the corner where the two lines meet, and set the pencil point on the end of one of the lines.
- Draw an arc between the ends of the two lines.

To construct the segment of a circle, add in the third line of the SAS triangle and ignore the first two lines.

Construction Cheat Sheet 2: *Triangles - ASA, SAS, SSS, Isosceles, Equilateral*

Triangle with known angle, side, angle (ASA)

Requires: Pencil, Ruler, Protractor

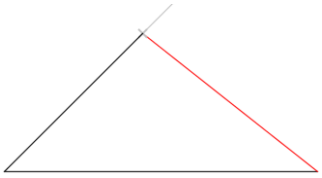


- Draw a line of the required length.
- From one end of this line, construct a line at one of the required angles.
- From the other end of the first line, construct the second required angle.
- If necessary, extend these two lines until they meet. This is the third corner.

You can use this construction to estimate the length of unknown sides for any ASA triangle.

Triangle with known side, angle, side (SAS)

Requires: Pencil, Ruler, Protractor

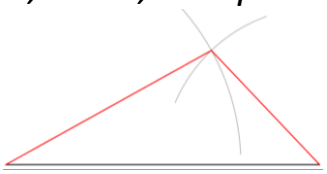


- Draw a line with one of the required lengths.
  It's usually easier to plan where the triangle will end up if you choose the longest.
- From one end of this first line, construct a line at the required angle.
- Extend this line to the required length for the second side.
  (or, if it's already too long, make a mark the right distance along)
- Join up the end points of your two lines. This is the third side.

You can use this construction to estimate the length of the unknown side or angles for SAS triangles.

Triangle with three known sides (SSS)

Requires: Pencil, Ruler, Compass

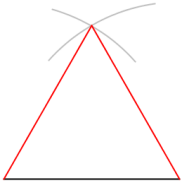


- Draw a line with one of the required lengths.
  It's usually easier to plan where the triangle will end up if you choose the longest.
- Set your compass radius to the length of the second side, and draw a circle from one end of the first line.
- Set your compass radius to the length of the third side, and draw a circle from the other end of the first line.
  You only really need arcs long enough to cross one another.
- The point where the circles cross is the third corner of your triangle. Join it to each end of the first line.
  There will be two crossing points. Both will work, so choose either.

You can use this construction to estimate the size of the unknown angles for SSS triangles.

Isosceles or Equilateral triangle (any size)

Requires: Pencil, Ruler, Compass



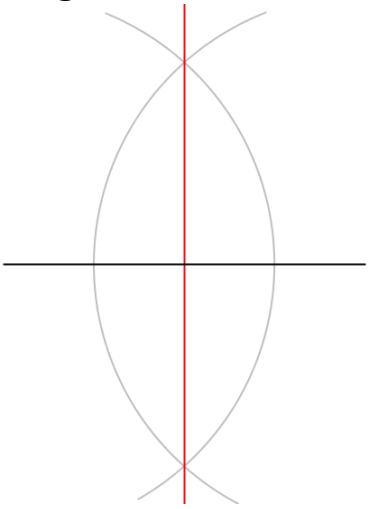
- Draw a line of any length you want. This will be the base of your triangle.
- Choose a compass radius setting. This is the length of the two equal sides.
  For an equilateral triangle, make this the same as the length of your base.
- Draw a circle from each end of the first line.
- The point where the circles cross is the third corner of your triangle.
  Join it to each end of the first line.
  There will be two crossing points. Both will work, so choose either.

You can use the equilateral triangle construction to form a $60°$ angle, since it must have all angles equal.

Construction Cheat Sheet 3: *Perpendicular bisector, Angle bisector*

Perpendicular bisector of a line segment

Requires: Pencil, Ruler, Compass

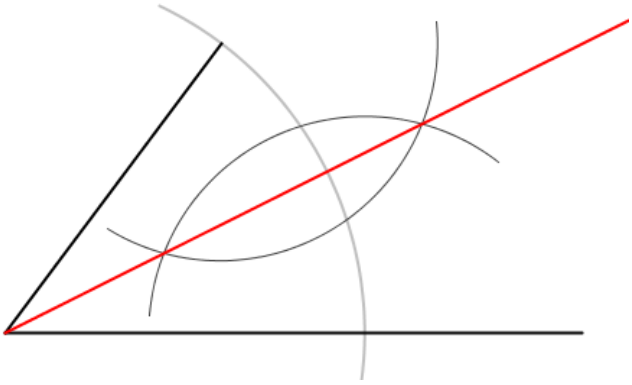


- Place your compass point on one end of the line segment, and set the compass radius so it is *more than half* of the length of the line segment.
  The larger the better, so if you have enough space, make a large radius.
- Draw a circle from each end of the first line.
  These must be the same radius, just like when constructing an isosceles triangle.
- There should be two points where both lines cross. Join them with a straight line.
  This line can go beyond the crossing points too, as needed.

This construction gives a line that cuts the original line segment in half at right angles.

Angle bisector between two lines

Requires: Pencil, Ruler, Compass



- Place your compass point on the corner where the two lines meet, and draw an arc that crosses both lines.
  The larger the better, so if you have enough space, make a large radius.
- Construct a perpendicular bisector between the two crossing points.
  This is identical to the method described for a perpendicular bisector, except you don't need a line segment – just treat the two crossing points as the two ends.

This construction gives a line that cuts the original angle in half.

Constructions Practice

Follow the instructions to accurately construct each of the diagrams described in the space provided.

1. Draw a line of length $4.5cm$ using a ruler.

Double-check that your line is half-way between $4cm$ and $5cm$.

2. a) Construct an angle of $37°$ using a ruler and protractor.

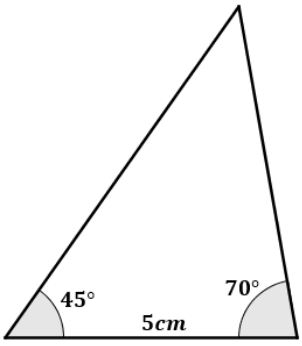
Double-check that your angle looks acute ($< 90°$). It should be a little less than half of a right angle.

b) Construct an angle of $140°$ using a ruler and protractor.

Double-check that your angle looks obtuse ($> 90°$). It should be quite a bit more than a right angle.

3. a) Construct a triangle with a $45°$ angle, a $5cm$ base then a $70°$ angle.

*The diagram below is to show you where the angles should go. It is **not** drawn accurately!*



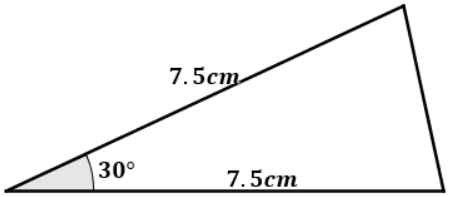
b) Accurately measure the lengths of the other two sides of your triangle in mm:

Opposite the $70°$ angle: ____ mm Opposite the $45°$ angle: ____ mm

Check your answers: The perimeter (all three lengths added together) should be 141mm, to the nearest mm.

4. a) Construct a triangle with two $7.5cm$ sides and an angle of $30°$ in between.

*The diagram below is to show you where the angles should go. It is **not** drawn accurately!*



b) Accurately measure the length of the third side of your triangle in mm:

Opposite the $30°$ angle: ____ mm

Check your answers: The perimeter (all three lengths added together) should be 189mm, to the nearest mm.

c) Use the fact that this is isosceles to *calculate* the size of the two missing angles.

Each missing angle: ____°

Check your answers: Once you've calculated the angles, measure them on your diagram to compare.

5. a) Construct a circle with radius $4cm$.

Make sure your compass is tight and you hold it lightly so the radius doesn't change.

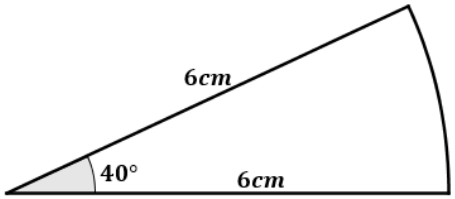
b) Draw a diameter onto the circle (a line straight across through the centre).

Diameter length: ____ cm.

Check your answer: Use the fact that the circle has a radius of $4cm$ to work out what the diameter should be.

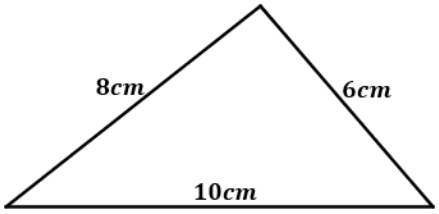
6. Construct a sector with radius $6cm$ and angle $40°$.

*The diagram below is to show you what a sector should look like. It is **not** drawn accurately!*



7. a) Construct a triangle with side lengths $6cm$, $8cm$ and $10cm$.

*The diagram below is to illustrate the triangle only. It is **not** drawn accurately!*



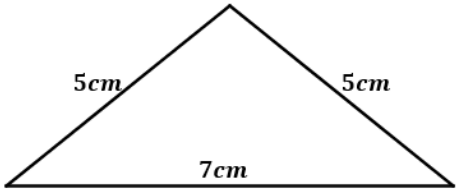
b) Accurately measure the largest angle in your triangle.

Angle opposite the $10cm$ side: ____°

Check your answer: Use Pythagoras' theorem to find out if this is a right-angled triangle.

8. a) Construct an isosceles triangle with a base of length $7cm$ and two equal sides of length $5cm$ each.

*The diagram below is to illustrate the triangle only. It is **not** drawn accurately!*



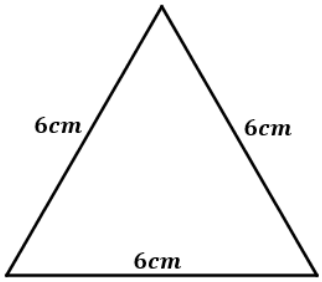
b) Measure all the angles in your isosceles triangle.

Top angle ____° Angle on the left: ____° Angle on the right: ____°

Check your answers: Use the total of angles in a triangle and the fact that it is isosceles to check your angles.

9. a) Construct an equilateral triangle with all sides of length $6cm$.

*The diagram below is to illustrate the triangle only. It is **not** drawn accurately!*



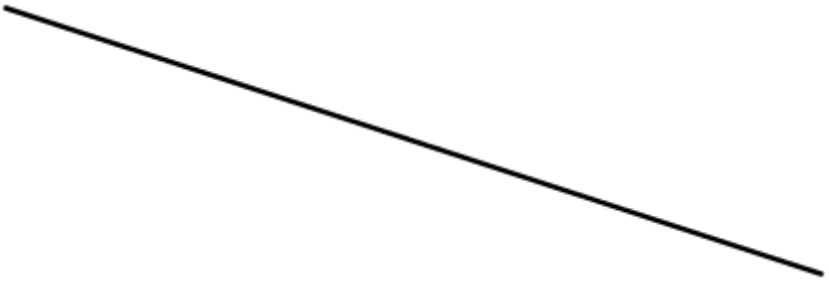
b) Measure all the angles in your equilateral triangle.

Top angle ____° Angle on the left: ____° Angle on the right: ____°

Check your answers: Use the total of angles in a triangle and the fact that it is equilateral to check your angles.

10. Use a compass to construct a perpendicular bisector of this line segment.

Double-check with a ruler and protractor when you're done that your line cuts the segment in half at $90°$.



11. Use a compass to construct the angle bisector between these lines.

Double-check with a protractor when you're done that your line does cut the angle in half.