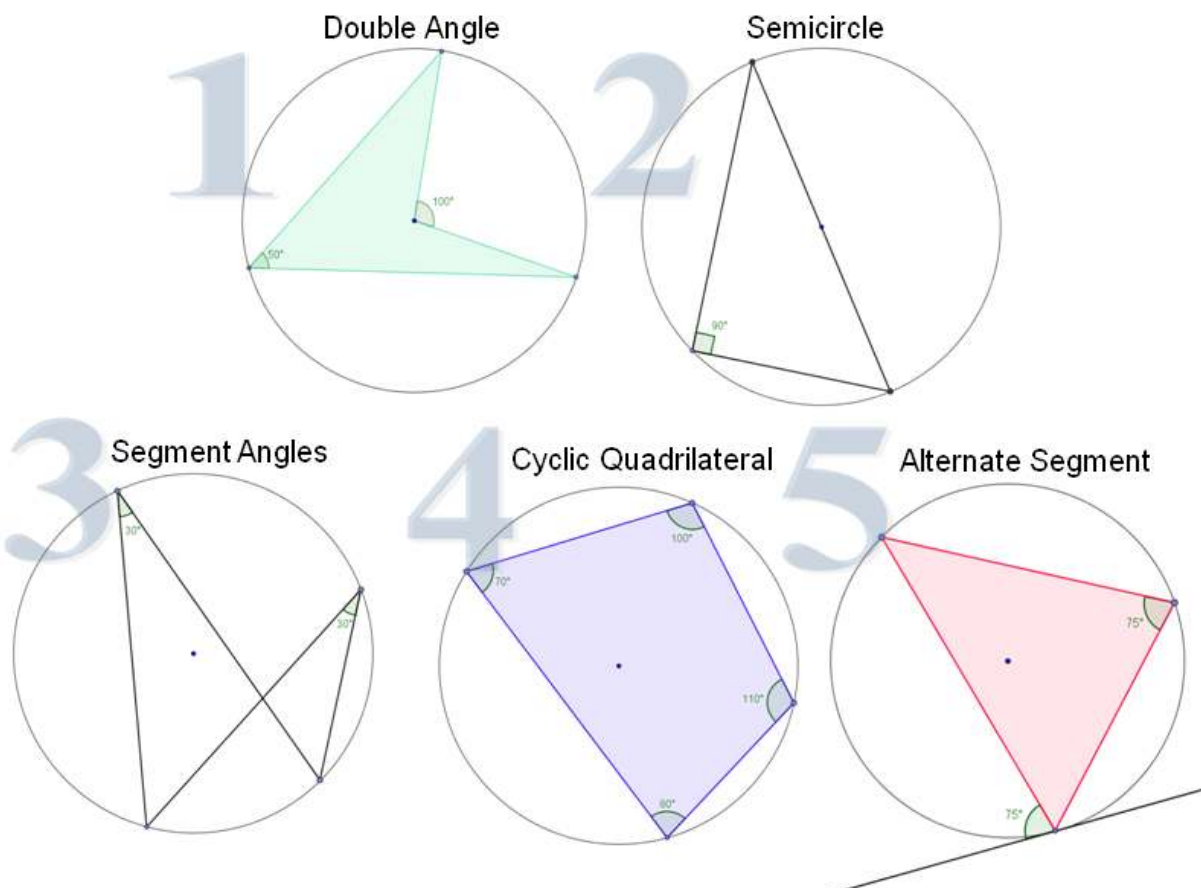# Circle Theorems

Circle Facts:

1. Any triangle with two points on the edge of a circle and one in the middle will be isosceles.

2. The tangent to a circle is perpendicular to the radius at the point of contact.

3. The triangle produced by two tangents to a circle and the chord between them is isosceles.

4. If a radius bisects a chord, it does so at right angles, and if it cuts it at right angles it bisects it.

Circle Theorems:

1. Double Angle: The angle made at the centre of a circle is twice the angle made at the edge.

2. Semicircle: The angle in a semicircle is a right angle.

3. Segment Angles: Angles in the same segment are equal.

4. Cyclic Quadrilateral: Opposite angles of a cyclic quadrilateral add up to 180°.

5. Alternate segment: The angle between a chord and the tangent at the point of contact is equal to the angle in the alternate segment.

Double Angle

Semicircle

Segment Angles

Cyclic Quadrilateral

Alternate Segment
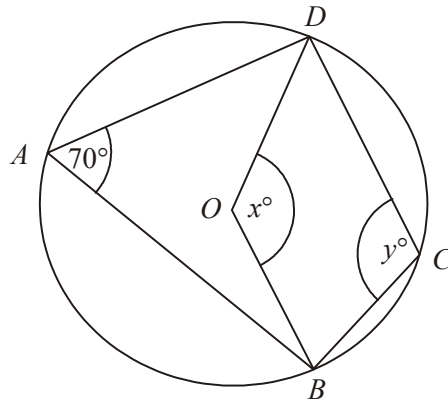
Circle Theorems Questions

1.

Diagram **NOT** accurately drawn

In the diagram, *A*, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
Angle *BAD* = 70°.
Angle *BOD* = *x*°.
Angle *BCD* = *y*°.

(a) (i) Work out the value of *x*.

$x =$ ....

(ii) Give a reason for your answer.

....

....

(2)

(b) (i) Work out the value of *y*.

$y =$ ....

(ii) Give a reason for your answer.

....

....
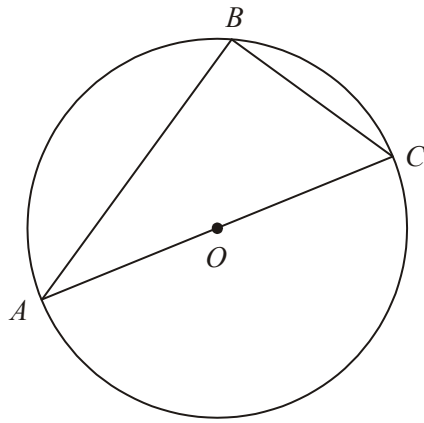
(2)
(Total 4 marks)

2.

Diagram **NOT** accurately drawn

A, *B* and *C* are points on the circumference of a circle, centre *O*.
AC is a diameter of the circle.

(a)　(i)　Write down the size of angle *ABC*.

..........................°

　　(ii)　Give a reason for your answer.

....

....
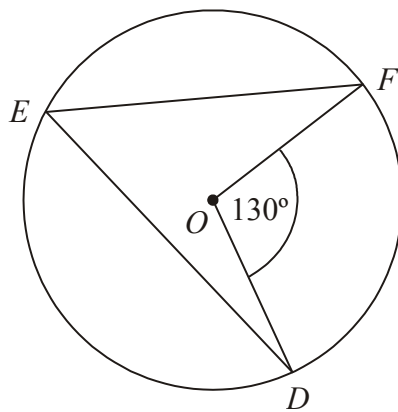
(2)

Diagram **NOT** accurately drawn

D, *E* and *F* are points on the circumference of a circle, centre *O*.
Angle *DOF* = 130°.

(b)　(i)　Work out the size of angle *DEF*.

..........................°

　　(ii)　Give a reason for your answer.

....

....

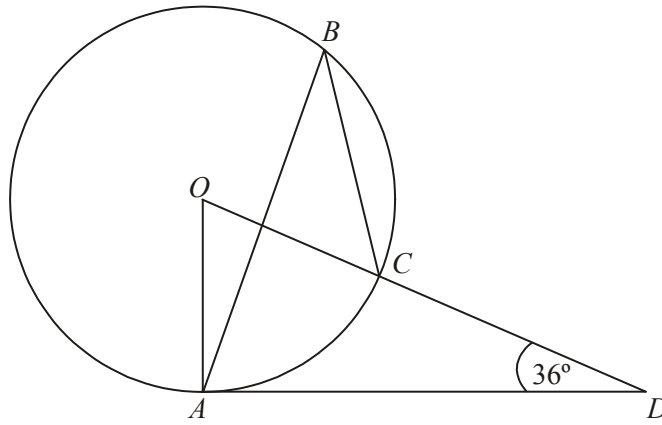**(2)**
(Total 4 marks)

3.

Diagram **NOT** accurately drawn

The diagram shows a circle centre *O*.
A, *B* and *C* are points on the circumference.

DCO is a straight line.
DA is a tangent to the circle.

Angle *ADO* = 36°

(a)   Work out the size of angle *AOD*.

.................................°

(2)

(b)   (i)   Work out the size of angle *ABC*.

.................................°

(ii)   Give a reason for your answer.

....
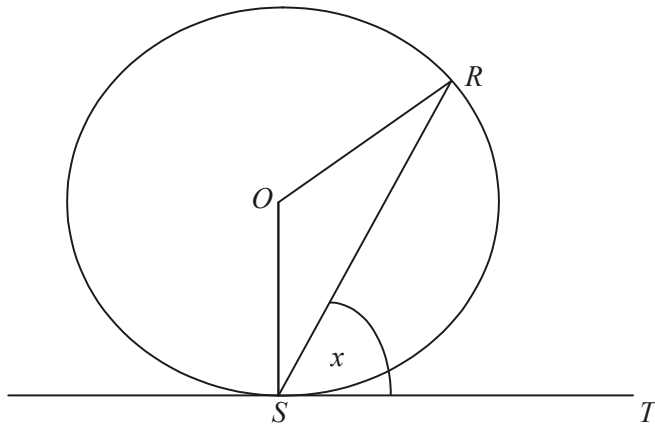
(3)

(Total 5 marks)

4.

Diagram **NOT** accurately drawn

R and *S* are two points on a circle, centre *O*.
TS is a tangent to the circle.
Angle *RST* = *x*.

Prove that angle *ROS* = 2*x*.
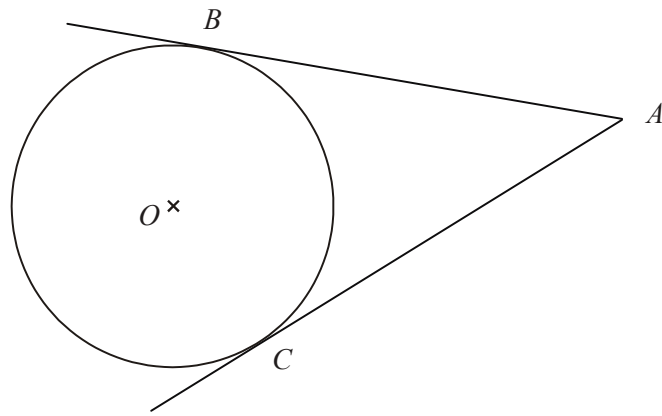You must give reasons for each stage of your working.

(Total 4 marks)

5.

Diagram **NOT** accurately drawn

B and *C* are points on a circle, centre *O*.
AB and *AC* are tangents to the circle.
Angle *BOC* = 130°.

Work out the size of angle *BAO*.

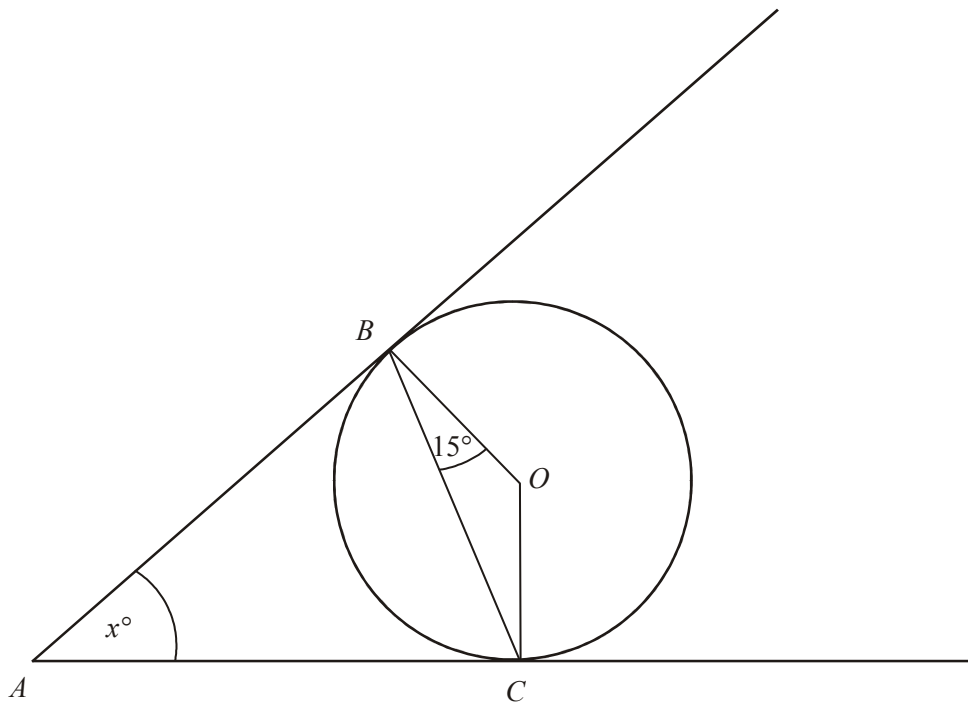.............................°

(Total 3 marks)

6.

Diagram **NOT** accurately drawn

B and *C* are two points on a circle, centre *O*.

Angle *OBC* = 15°.

AB and *AC* are tangents to the circle.

(a)   Calculate the size of the angle marked *x*°.

.....................°

(2)

(b)   Give reasons for your answer.

....

....

....
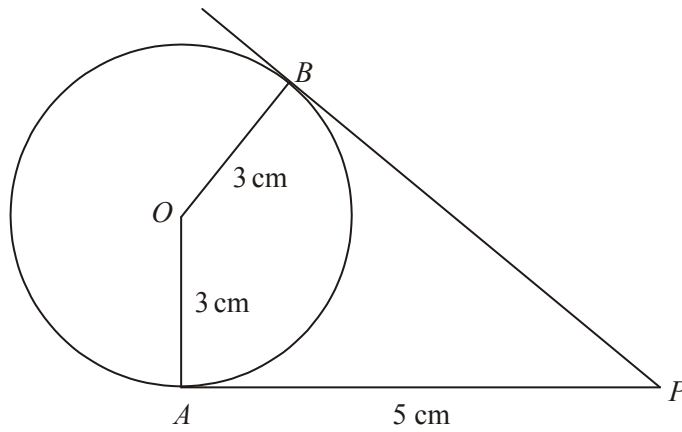
(2)
(Total 4 marks)

7.

Diagram **NOT** accurately drawn

A and *B* are points on a circle, centre *O*, radius 3 cm.

PA and *PB* are tangents to the circle.

PA = 5 cm.

(a)   Write down the size of the angle *OBP*.

..........................°

(1)

(b)   (i)   Write down the length of *PB*.

.... cm

(ii)   Give a reason for your answer.

....

....
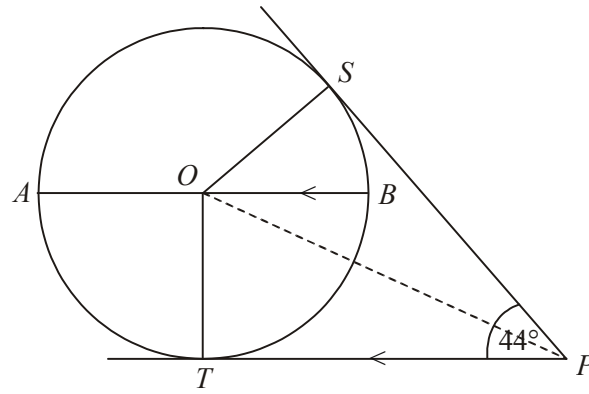
(2)
(Total 3 marks)

8.

Diagram **NOT** accurately drawn

The diagram shows a circle, centre *O*.
A, S, B and *T* are points on the circumference of the circle.

PT and *PS* are tangents to the circle.
AB is parallel to *TP*.

Angle *SPT* = 44°.

Work out the size of angle *SOB*.

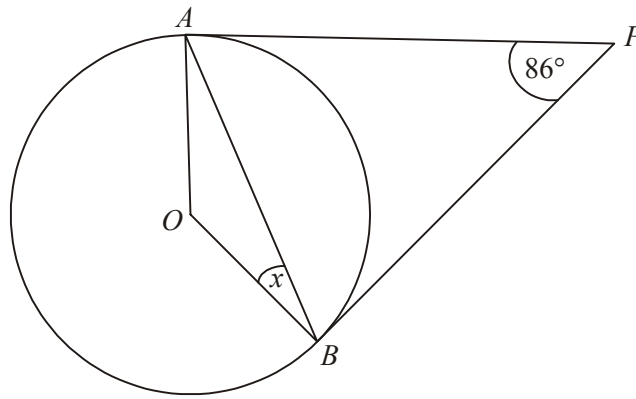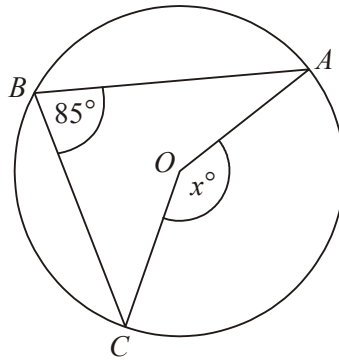....................................°

(Total 4 marks)

9.

Diagram **NOT** accurately drawn

A and *B* are points on the circumference of a circle, centre *O*.
PA and *PB* are tangents to the circle.
Angle *APB* is 86°.

Work out the size of the angle marked *x*.

..........................°

(Total 2 marks)

10.

In the diagram, *A*, *B* and *C* are points on the circumference of a circle, centre *O*.

Angle *ABC* = 85°.

(i)   Work out the size of the angle marked *x*°.

.................................°

(ii)   Give a reason for your answer.

....

....

(Total 2 marks)

Circle Theorem Hints	Circle Theorem Answers
Question 1 You will need circle theorems 1 and 4.	$x = 140°$ because the angle made at the centre of a circle is twice the angle made at the edge. $y = 110°$ because opposite angles of a cyclic quadrilateral add up to $180°$.
Question 2 You will need circle theorems 2 and 1.	$ABC = 90°$ because the angle in a semicircle is a right angle. $DEF = 65°$ because the angle made at the centre of a circle is twice the angle made at the edge.
Question 3 You will need circle fact 2 and circle theorem 1.	$AOD = 54°$ because $DAO = 90°$. $ABC = 27°$ because the angle made at the centre of a circle is twice the angle made at the edge.
Question 4 You will need circle facts 2 and 1.	$TSO = 90°$ because the tangent to a circle is perpendicular to the radius at the point of contact. Therefore angle $OSR = 90 - x$. Triangle SOR is isosceles since SO and OR are both the radius of the circle, therefore $ORS = 90 - x$ and $ROS = 180 - (180 - 2x) = 2x$.
Question 5 You will need circle facts 2 and 3.	Angle $ABO = 90°$ because AB is a tangent and BO is a radius. Similarly, $ACO = 90°$. The angles in the quadrilateral $ABOC$ must add up to $180°$, so since $BOC = 130°$, $BAC = 50°$. Triangle ABC is isosceles, and triangle BOC is isosceles. Since they are both symmetrical about AO, the line AO bisects the angle BAO. Therefore $BAO = 25°$.
Question 6 You will need circle facts 2 and 3.	Triangle BOC is isosceles since BO and CO are both the radius of the circle. Therefore $OCB = 15°$ and $BOC = 180 - 2(15) = 150°$. Since $ABO = 90°$ and $ACO = 90°$ (as AB and AC are tangents and BO and CO radii), the fourth angle in the quadrilateral $ABOC$ - the angle x - must add to $150°$, $90°$ and $90°$ to make $360°$, therefore $x = 30°$.
Question 7 You will need circle facts 2 and 3	$OBP = 90°$ because PB is a tangent to the circle at B, and BO is a radius. $PB = 5cm$ because triangle PBA is isosceles, therefore $PB = PA$.
Question 8 You will need circle fact 2.	Angle $PTO = 90°$, angle $PSO = 90°$ and angle $TPS = 44°$. The fourth angle in the quadrilateral $TPSO$ must therefore be $TOS = 136°$. Angle $TOB = 90°$ because line BA is parallel to line PT and interior angles on parallel lines add up to $180°$. Therefore $SOB = 136 - 90 = 46°$.
Question 9 You will need circle facts 2 and 3.	Since OAP and OBP are $90°$ (AP and BP are tangents to the circle, and OA and OB are radii), $AOB = 180 - 86 = 94°$. Triangle AOB is isosceles, so $x = \frac{180-94}{2} = 43°$.
Question 10 You will need circle theorem 1.	$x = 170°$ because the angle made at the centre of a circle is twice the angle made at the edge.