C3 Essentials: *Summary of AQA Core 3 content not provided in the formula book*

Functions

A function can be many to one or one to one.
If it is to have an inverse, it must be one to one.

The domain is the set of input values it takes.
The range is the set of output values it generates.

The composite function $fg(x) = f\big(g(x)\big)$.
The inverse f^{-1} has: $ff^{-1}(x) = f^{-1}f(x) = x$.

Modulus function

The function $|f(x)| = f(x)$ when $f(x) > 0$
and $|f(x)| = -f(x)$ when $f(x) < 0$

Inverse trig functions

	Domain	Range
$\sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$
$\cos^{-1} x$	$-1 \le x \le 1$	$0 \le \cos^{-1} x \le \pi$
$\tan^{-1} x$	$x \in \mathbb{R}$	$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

$$\sec x = \frac{1}{\cos x} \quad \operatorname{cosec} x = \frac{1}{\sin x} \quad \cot x = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

e^x and $\ln x$

e^x and $\ln x$ are inverse functions $\implies$ $e^{\ln x} = x = \ln e^x$

$\frac{d}{dx}(e^x) = e^x$	$\int e^x \, dx = e^x + C$
$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$	$\int e^{ax+b} \, dx = \frac{e^{ax+b}}{a} + C$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x} \, dx = \ln x + C$

Chain rule

$$\frac{d}{dx} fg(x) = f'\big(g(x)\big)g'(x)$$

Trig calculus

$\frac{d}{dx}(\sin x) = \cos x$	$\int \sin x \; dx = -\cos x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \cos x \; dx = \sin x + C$

Product rule

$$\frac{d}{dx} uv = uv' + vu'$$

Quotient rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

Integration

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C \qquad \int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$$

Integration by parts

$$\int u\frac{dv}{dx} \, dx = uv - \int v\frac{du}{dx} \, dx$$

Volume of revolution

$$V = \int_a^b \pi y^2 \, dx$$