Moving Ship Bearings Problem

At 0800 you are in your coastguard patrol boat, moored at Beach End Buoy 2 miles east and 1 mile south of the Hook Lighthouse.
You see a ship which doesn't respond to radio hails, on a bearing of $045°$ from you.
You radio the lighthouse, and they confirm the sighting, telling you that the ship is due east of them.

You set off due south, travelling at a speed of $6mph$ due south.
You take a second reading half an hour later. The vessel is still on a bearing of $045°$ from you.
The lighthouse master reports that the unidentified ship is now in line with Beach End Buoy from the lighthouse.

1. Plot the course of the coastguard patrol boat on squared paper, taking a scale of $1cm$ per $1\ mile$, labelling each point with a timestamp (the time at which this position was correct) and showing with a line the likely course of the journey.

2. Complete the diagram to show the two known positions of the unidentified ship (include timestamp), and use a line to illustrate projected course. Use your diagram to **calculate** the speed and heading of the ship.

3. Assuming the unidentified ship continues on its current course, and the speed of each vessel remains the same, what heading should the coastguard patrol boat take now (at 0830) in order to intercept it? And at what time should the coastguard patrol boat expect to reach the ship?

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Moving Ship Bearings Problem SOLUTIONS

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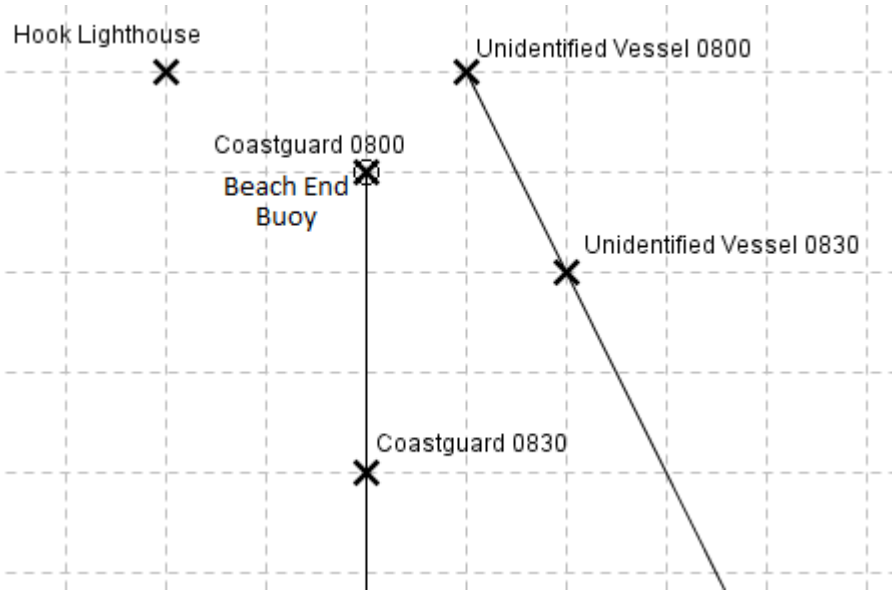

Considering the right-angled triangle whose hypotenuse is the line from the vessel's position at 0800 to that at 0830:
Speed: using Pythagoras
The unidentified vessel travels 1 mile east and 2 miles south in half an hour, which is: $\sqrt{1^2 + 2^2} = \sqrt{5}$.
$\sqrt{5}$ miles in half an hour is $2\sqrt{5}\ mph$ or roughly $\mathbf{4.47mph}$.
Heading: using trigonometry
The bearing from north is $180° - x°$ where $x°$ is the angle between the vessel's course and south.
$\tan x = \frac{1}{2} \implies x = \tan^{-1} 0.5 = 26.6° \implies heading: 180 - 26.6 = 153.4°$ or approximately $\mathbf{153°}$.

3. Assuming the unidentified ship continues on its current course, and the speed of each vessel remains the same, what heading should the coastguard patrol boat take now (at 0830) in order to intercept it? And at what time should the coastguard patrol boat expect to reach the ship?

This potentially tricky question can be solved for this particular situation by extrapolating the unidentified vessel's position. In half an hour more, it will be due east of the coastguard's current position, exactly 3 miles away. Since the coastguard's boat travels at $6mph$, if he heads due east (on a bearing of $090°$) from 0830 he will intercept the vessel in half an hour (0900).