

Algebraic Fractions

Section A: Simplifying algebraic fractions

A numerical fraction is simplified by dividing the numerator and denominator both by their highest common factor (HCF).

Eg:

$$\frac{48}{60} = \frac{12 \times 4}{12 \times 5} = \frac{4}{5}$$

(\div numerator and denominator by 12)

The process is identical for an algebraic fraction, only the HCF can be an *expression* rather than just a number.

Eg:

$$\frac{4x + 6x^2}{18x^3} = \frac{2x(2 + 3x)}{2x(9x^2)} = \frac{2 + 3x}{9x^2}$$

(\div numerator and denominator by $2x$)

*Note: to be certain of finding the highest common factor, you must first **factorise fully**.*

1. $\frac{12x}{40x} =$	2. $\frac{2x^7}{x^8} =$
3. $\frac{4(x-3)}{6(x-3)} =$	4. $\frac{(x+2)(x-5)}{(x-5)(x+5)} =$
5. $\frac{13x^2 - 5x}{4x + 6x^3} =$	6. $\frac{5x^2 - 10x}{x^2 - 6x + 8} =$
7. $\frac{x^2 + 5x + 4}{x^2 - 16} =$	8. $\frac{2x^2 + 2x - 12}{4x^2 - 16} =$

Section B: Adding and subtracting algebraic fractions

Convert both to fractions with the same denominator (usually the lowest common multiple (LCM) of each). Then add or subtract the numerators.

Eg:

$$\frac{17}{18} + \frac{4}{27} = \frac{51}{54} + \frac{8}{54} = \frac{59}{54}$$

(LCM of denominators is 54)

The process is identical for an algebraic fraction, only the LCM can be an *expression* rather than just a number.

Eg:

$$\frac{x}{x+3} + \frac{2}{x} = \frac{x^2}{x(x+3)} + \frac{2(x+3)}{x(x+3)}$$

$$= \frac{x^2 + 2x + 6}{x(x+3)}$$

(LCM of denominators is $x(x+3)$)

*Note: as with numerical fractions you should check if the fraction **simplifies** at the end.*

1. $\frac{5}{x} + \frac{9}{2x} =$	2. $\frac{6x}{x+1} + \frac{7}{x+1} =$
3. $\frac{4x}{2x-1} - \frac{2}{2x-1} =$	4. $\frac{4}{x} - \frac{3}{y} =$
5. $\frac{3}{x+1} + \frac{5}{(x+1)(x-5)} =$	6. $\frac{2}{x^2-9} + \frac{3}{x-3} =$
7. $\frac{a}{b} - \frac{c}{d} =$	8. $\frac{p}{q^2r} + \frac{r}{3rq} =$

Section C: Multiplying and dividing algebraic fractions

<p>A numerical fraction is multiplied by simply multiplying the numerators and multiplying the denominators.</p> <p>Eg:</p> $\frac{10}{11} \times \frac{4}{5} = \frac{40}{55} = \frac{\mathbf{8}}{\mathbf{11}}$	<p>The process is identical for an algebraic fraction. Often it is easier since multiplying expressions simply involves writing them next to one another.</p> <p>Eg:</p> $\frac{12x}{x+2} \times \frac{x-1}{x^2} = \frac{12x(x-1)}{x^2(x+2)} = \frac{\mathbf{12(x-1)}}{\mathbf{x(x+2)}}$
<p>To divide a numerical fraction by something, multiply by its reciprocal (flip the fraction you want to divide by).</p> <p>Eg:</p> $\frac{10}{11} \div \frac{4}{5} = \frac{10}{11} \times \frac{5}{4} = \frac{50}{44} = \frac{\mathbf{25}}{\mathbf{22}}$ <p>(flip the second fraction and multiply)</p>	<p>To divide an algebraic fraction, flip and multiply just like numerical fractions:</p> <p>Eg:</p> $\begin{aligned} \frac{12x}{x+2} \div \frac{x-1}{x^2} &= \frac{12x}{x+2} \times \frac{x^2}{x-1} \\ &= \frac{\mathbf{12x^3}}{\mathbf{(x+2)(x-1)}} \end{aligned}$

Note:

<p>1.</p> $\frac{3}{x} \times \frac{2}{x} =$	<p>2.</p> $\frac{x}{4} \times \frac{12}{x} =$
<p>3.</p> $\frac{9}{x-1} \times \frac{x}{x-1} =$	<p>4.</p> $\frac{2x}{2x+1} \times \frac{x+1}{x} =$
<p>5.</p> $\frac{(x+1)^2}{4x} \div \frac{x-4}{2x} =$	<p>6.</p> $\frac{1-y}{x+2} \div \frac{2-x}{y+1} =$
<p>7.</p> $\frac{x^2+3x+2}{x+3} \times \frac{x^2-9}{x+1}$	<p>8.</p> $\frac{abc}{def} \div \frac{a^2b^2c^2}{d^2e^2f^2} =$

Algebraic Fractions **SOLUTIONS**

Section A: Simplifying algebraic fractions

1. $\frac{12x}{40x} = \frac{3}{10}$	2. $\frac{2x^7}{x^8} = \frac{2}{x}$
3. $\frac{4(x-3)}{6(x-3)} = \frac{2}{3}$	4. $\frac{(x+2)(x-5)}{(x-5)(x+5)} = \frac{x+2}{x+5}$
5. $\frac{13x^2 - 5x}{4x + 6x^3} = \frac{13x - 5}{2(2 + 3x)}$	6. $\frac{5x^2 - 10x}{x^2 - 6x + 8} = \frac{5}{x - 4}$
7. $\frac{x^2 + 5x + 4}{x^2 - 16} = \frac{x + 1}{x - 4}$	8. $\frac{2x^2 + 2x - 12}{4x^2 - 16} = \frac{2(x - 2)}{4(x + 2)}$

Section B: Adding and subtracting algebraic fractions

1. $\frac{5}{x} + \frac{9}{2x} = \frac{19}{2x}$	2. $\frac{6x}{x+1} + \frac{7}{x+1} = \frac{6x+7}{x+1}$
3. $\frac{4x}{2x-1} - \frac{2}{2x-1} = 2$	4. $\frac{4}{x} - \frac{3}{y} = \frac{4y - 3x}{xy}$
5. $\frac{3}{x+1} + \frac{5}{(x+1)(x-5)} = \frac{3x-10}{(x+1)(x-5)}$	6. $\frac{2}{x^2-9} + \frac{3}{x-3} = \frac{3x+11}{(x+3)(x-3)}$
7. $\frac{a}{b} - \frac{c}{d} = \frac{ad - cb}{bd}$	8. $\frac{p}{q^2r} + \frac{r}{3rq} = \frac{3p + qr}{3q^2r}$

Section C: Multiplying and dividing algebraic fractions

1. $\frac{3}{x} \times \frac{2}{x} = \frac{6}{x^2}$	2. $\frac{x}{4} \times \frac{12}{x} = 3$
3. $\frac{9}{x-1} \times \frac{x}{x-1} = \frac{9x}{x-1}$	4. $\frac{2x}{2x+1} \times \frac{x+1}{x} = \frac{2(x+1)}{2x+1}$
5. $\frac{(x+1)^2}{4x} \div \frac{x-4}{2x} = \frac{(x+1)^2}{2(x-4)}$	6. $\frac{1-y}{x+2} \div \frac{2-x}{y+1} = \frac{(1-y)(y+1)}{(x+2)(2-x)}$
7. $\frac{x^2 + 3x + 2}{x+3} \times \frac{x^2 - 9}{x+1} = (x+2)(x-3)$	8. $\frac{abc}{def} \div \frac{a^2b^2c^2}{d^2e^2f^2} = \frac{def}{abc}$