

Specimen Paper 9MA0/01: Pure Mathematics Paper 1 Mark scheme

Question	Scheme	Marks	AOs
1 (a)	$\text{Area}(R) \approx \frac{1}{2} \times 0.5 \times [0.5 + 2(0.6742 + 0.8284 + 0.9686) + 1.0981]$	B1	1.1b
		<u>M1</u>	1.1b
	$\left\{ = \frac{1}{4} \times 6.5405 = 1.635125 \right\} = 1.635 \text{ (3 dp)}$	A1	1.1b
		(3)	
(b)	Any valid reason, for example <ul style="list-style-type: none"> • Increase the number of strips • Decrease the width of the strips • Use more trapezia between $x = 1$ and $x = 3$ 	B1	2.4
		(1)	
(c)(i)	$\left\{ \int_1^3 \frac{5x}{1 + \sqrt{x}} dx \right\} = 5("1.635") = 8.175$	B1ft	2.2a
(c)(ii)	$\left\{ \int_1^3 \left(6 + \frac{x}{1 + \sqrt{x}} \right) dx \right\} = 6(2) + ("1.635") = 13.635$	B1ft	2.2a
		(2)	
(6 marks)			
Question 1 Notes:			
(a)			
B1:	Outside brackets $\frac{1}{2} \times 0.5$ or $\frac{0.5}{2}$ or 0.25 or $\frac{1}{4}$		
M1:	For structure of trapezium rule [.....]. No errors are allowed, e.g. an omission of a y -ordinate or an extra y -ordinate or a repeated y -ordinate.		
A1:	Correct method leading to a correct answer only of 1.635		
(b)			
B1:	See scheme		
(c)			
B1:	8.175 or a value which is $5 \times$ their answer to part (a) Note: Allow B1ft for 8.176 (to 3 dp) which is found from $5(1.63125) = 8.175625$ Note: Do not allow an answer of 8.1886... which is found directly from integration		
(d)			
B1:	13.635 or a value which is $12 +$ their answer to part (a) Note: Do not allow an answer of 13.6377... which is found directly from integration		

Question	Scheme	Marks	AOs
2 (a)	$(4 + 5x)^{\frac{1}{2}} = (4)^{\frac{1}{2}} \left(1 + \frac{5x}{4}\right)^{\frac{1}{2}} = 2 \left(1 + \frac{5x}{4}\right)^{\frac{1}{2}}$	B1	1.1b
	$= \{2\} \left[1 + \left(\frac{1}{2}\right)\left(\frac{5x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!} \left(\frac{5x}{4}\right)^2 + \dots \right]$	M1	1.1b
	$= 2 + \frac{5}{4}x - \frac{25}{64}x^2 + \dots$	A1	2.1
		(4)	
(b)(i)	$\left\{ x = \frac{1}{10} \Rightarrow \right\} (4 + 5(0.1))^{\frac{1}{2}}$	M1	1.1b
	$= \sqrt{4.5} = \frac{3}{2}\sqrt{2} \text{ or } \frac{3}{\sqrt{2}}$		
	$\frac{3}{2}\sqrt{2} \text{ or } 1.5\sqrt{2} \text{ or } \frac{3}{\sqrt{2}} = 2 + \frac{5}{4}\left(\frac{1}{10}\right) - \frac{25}{64}\left(\frac{1}{10}\right)^2 + \dots \{= 2.121\dots\}$ $\Rightarrow \frac{3}{2}\sqrt{2} = \frac{543}{256} \text{ or } \frac{3}{\sqrt{2}} = \frac{543}{256} \Rightarrow \sqrt{2} = \dots$	M1	3.1a
	So, $\sqrt{2} = \frac{181}{128} \text{ or } \sqrt{2} = \frac{256}{181}$	A1	1.1b
(b)(ii)	$x = \frac{1}{10}$ satisfies $ x < \frac{4}{5}$ (o.e.), so the approximation is valid.	B1	2.3
		(4)	
(8 marks)			

Question 2 Notes:	
(a)	
B1:	Manipulates $(4 + 5x)^{\frac{1}{2}}$ by taking out a factor of $(4)^{\frac{1}{2}}$ or 2
M1:	Expands $(... + \lambda x)^{\frac{1}{2}}$ to give at least 2 terms which can be simplified or un-simplified, E.g. $1 + \left(\frac{1}{2}\right)(\lambda x)$ or $\left(\frac{1}{2}\right)(\lambda x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ or $1 + ... + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ where λ is a numerical value and where $\lambda \neq 1$.
A1ft:	A correct simplified or un-simplified $1 + \left(\frac{1}{2}\right)(\lambda x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ expansion with consistent (λx)
A1:	Fully correct solution leading to $2 + \frac{5}{4}x + kx^2$, where $k = -\frac{25}{64}$
(b)(i)	
M1:	Attempts to substitute $x = \frac{1}{10}$ or 0.1 into $(4 + 5x)^{\frac{1}{2}}$
M1:	A complete method of finding an approximate value for $\sqrt{2}$. E.g. <ul style="list-style-type: none"> • substituting $x = \frac{1}{10}$ or 0.1 into their part (a) binomial expansion and equating the result to an expression of the form $\alpha\sqrt{2}$ or $\frac{\beta}{\sqrt{2}}$; $\alpha, \beta \neq 0$ • followed by re-arranging to give $\sqrt{2} = ...$
A1:	$\frac{181}{128}$ or any equivalent fraction , e.g. $\frac{362}{256}$ or $\frac{543}{384}$ Also allow $\frac{256}{181}$ or any equivalent fraction
(b)(ii)	
B1:	Explains that the approximation is valid because $x = \frac{1}{10}$ satisfies $ x < \frac{4}{5}$

Question	Scheme	Marks	AOs
3 (a)	$a_1 = 3, a_2 = 0, a_3 = 1.5, a_4 = 3$	M1	1.1b
	$\sum_{r=1}^{100} a_r = 33(4.5) + 3$	M1	2.2a
	$= 151.5$	A1	1.1b
		(3)	
(b)	$\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = (2)(151.5) - 3 = 300$	B1ft	2.2a
		(1)	

(4 marks)

Question 3 Notes:

(a)

M1: Uses the formula $a_{n+1} = \frac{a_n - 3}{a_n - 2}$, with $a_1 = 3$ to generate values for a_2, a_3 and a_4

M1: Finds $a_4 = 3$ and deduces $\sum_{r=1}^{100} a_r = 33("3" + "0" + "1.5") + "3"$

A1: which leads to a correct answer of 151.5

(b)

B1ft: Follow through on their periodic function. Deduces that either

- $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = (2)("151.5") - 3 = 300$

- $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = "151.5" + (33)("3" + "0" + "1.5") = 151.5 + 148.5 = 300$

Question	Scheme	Marks	AOs
4 (a)	$\overrightarrow{OA} = \mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OC} = 2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}$		
	$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA} = (2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}) + (-3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$ or $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC} = (\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) + (-2\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$	M1	3.1a
	So $\overrightarrow{OD} = -\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$	A1	1.1b
		(2)	
(b)	$\left\{ \overrightarrow{AB} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k} \Rightarrow \right\} \left \overrightarrow{AB} \right = \sqrt{(3)^2 + (-4)^2 + (5)^2} \left\{ = \sqrt{50} = 5\sqrt{2} \right\}$	M1	1.1b
	As $\left \overrightarrow{AX} \right = 10\sqrt{2}$ then $\left \overrightarrow{AX} \right = 2 \left \overrightarrow{AB} \right \Rightarrow \overrightarrow{AX} = 2 \overrightarrow{AB}$		
	$\overrightarrow{OX} = \overrightarrow{OA} + 2\overrightarrow{AB} = (\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) + 2(3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ or $\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{AB} = (4 + 3\mathbf{j} + 3\mathbf{k}) + (3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$	M1	3.1a
	So $\overrightarrow{OX} = 7\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ only	A1	1.1b
		(3)	
(5 marks)			
Question 4 Notes:			
(a)			
M1:	A complete method for finding the position vector of D		
A1:	$-\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$ or $\begin{pmatrix} -1 \\ 14 \\ 4 \end{pmatrix}$		
(b)			
M1:	A complete attempt to find $\left \overrightarrow{AB} \right $ or $\left \overrightarrow{BA} \right $		
M1:	A complete process for finding the position vector of X		
A1:	$7\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ or $\begin{pmatrix} 7 \\ -1 \\ 8 \end{pmatrix}$		

Question	Scheme	Marks	AOs
5 (a)(i)	$f(x) = x^3 + ax^2 - ax + 48, x \in \mathbb{R}$		
	$f(-6) = (-6)^3 + a(-6)^2 - a(-6) + 48$	M1	1.1b
	$= -216 + 36a + 6a + 48 = 0 \Rightarrow 42a = 168 \Rightarrow a = 4 *$	A1*	1.1b
(a)(ii)	Hence, $f(x) = (x + 6)(x^2 - 2x + 8)$	M1	2.2a
		A1	1.1b
		(4)	
(b)	$2\log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$		
	E.g.		
	<ul style="list-style-type: none"> $\log_2(x + 2)^2 + \log_2 x - \log_2(x - 6) = 3$ $2\log_2(x + 2) + \log_2\left(\frac{x}{x - 6}\right) = 3$ 	M1	1.2
	$\log_2\left(\frac{x(x + 2)^2}{(x - 6)}\right) = 3 \quad \left[\text{or } \log_2(x(x + 2)^2) = \log_2(8(x - 6)) \right]$	M1	1.1b
	$\left(\frac{x(x + 2)^2}{(x - 6)}\right) = 2^3 \quad \left\{ \text{i.e. } \log_2 a = 3 \Rightarrow a = 2^3 \text{ or } 8 \right\}$	B1	1.1b
	$x(x + 2)^2 = 8(x - 6) \Rightarrow x(x^2 + 4x + 4) = 8x - 48$		
	$\Rightarrow x^3 + 4x^2 + 4x = 8x - 48 \Rightarrow x^3 + 4x^2 - 4x + 48 = 0 *$	A1 *	2.1
		(4)	
(c)	$2\log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3 \Rightarrow x^3 + 4x^2 - 4x + 48 = 0$		
	$\Rightarrow (x + 6)(x^2 - 2x + 8) = 0$		
	Reason 1: E.g.		
	<ul style="list-style-type: none"> $\log_2 x$ is not defined when $x = -6$ $\log_2(x - 6)$ is not defined when $x = -6$ $x = -6$, but $\log_2 x$ is only defined for $x > 0$ 		
	Reason 2:		
	<ul style="list-style-type: none"> $b^2 - 4ac = -28 < 0$, so $(x^2 - 2x + 8) = 0$ has no (real) roots 		
At least one of Reason 1 or Reason 2	B1	2.4	
Both Reason 1 and Reason 2	B1	2.1	
	(2)		
			(10 marks)

Question 5 Notes:**(a)(i)****M1:** Applies $f(-6)$ **A1*:** Applies $f(-6) = 0$ to show that $a = 4$ **(a)(ii)****M1:** Deduces $(x + 6)$ is a factor of $f(x)$ and attempts to find a quadratic factor of $f(x)$ by either equating coefficients or by algebraic long division**A1:** $(x + 6)(x^2 - 2x + 8)$ **(b)****M1:** Evidence of applying a correct law of logarithms**M1:** Uses correct laws of logarithms to give either

- an expression of the form $\log_2(\mathbf{h}(x)) = k$, where k is a constant
- an expression of the form $\log_2(\mathbf{g}(x)) = \log_2(\mathbf{h}(x))$

B1: Evidence in their working of $\log_2 a = 3 \Rightarrow a = 2^3$ or 8**A1*:** Correctly proves $x^3 + 4x^3 - 4x + 48 = 0$ with no errors seen**(c)****B1:** See scheme**B1:** See scheme

Question	Scheme	Marks	AOs
6 (a)	Attempts to use an appropriate model; e.g. $y = A(3-x)(3+x)$ or $y = A(9-x^2)$	M1	3.3
	e.g. $y = A(9-x^2)$ Substitutes $x = 0, y = 5 \Rightarrow 5 = A(9-0) \Rightarrow A = \frac{5}{9}$	M1	3.1b
	$y = \frac{5}{9}(9-x^2)$ or $y = \frac{5}{9}(3-x)(3+x), \{-3 \leq x \leq 3\}$	A1	1.1b
		(3)	
(b)	Substitutes $x = \frac{2.4}{2}$ into their $y = \frac{5}{9}(9-x^2)$	M1	3.4
	$y = \frac{5}{9}(9-x^2) = 4.2 > 4.1 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		(2)	
(b) Alt 1	$4.1 = \frac{5}{9}(9-x^2) \Rightarrow x = \frac{9\sqrt{2}}{10}$, so maximum width = $2\left(\frac{9\sqrt{2}}{10}\right)$	M1	3.4
	$= 2.545... > 2.4 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		(2)	
(c)	E.g. <ul style="list-style-type: none"> Coach needs to enter through the centre of the tunnel. This will only be possible if it is a one-way tunnel In real-life the road may be cambered (and not horizontal) The quadratic curve BCA is modelled for the entrance to the tunnel but we do not know if this curve is valid throughout the entire length of the tunnel There may be overhead lights in the tunnel which may block the path of the coach 	B1	3.5b
		(1)	
(6 marks)			
Question 6 Notes:			
(a)			
M1:	Translates the given situation into an appropriate quadratic model – see scheme		
M1:	Applies the maximum height constraint in an attempt to find the equation of the model – see scheme		
A1:	Finds a suitable equation – see scheme		
(b)			
M1:	See scheme		
A1:	Applies a fully correct argument to infer {by assuming that curve BCA is quadratic and the given measurements are correct}, that is possible for the coach to enter the tunnel		
(c)			
B1:	See scheme		

Question	Scheme	Marks	AOs
7	$\left\{ \int x e^{2x} dx \right\}, \left\{ \begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^{2x} \Rightarrow v = \frac{1}{2} e^{2x} \end{array} \right\}$		
	$\left\{ \int x e^{2x} dx \right\} = \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} \{dx\}$	M1	3.1a
	$\left\{ \int 2e^{2x} - x e^{2x} dx \right\} = e^{2x} - \left(\frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} \{dx\} \right)$	M1	1.1b
	$= e^{2x} - \left(\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right)$	A1	1.1b
	$\text{Area}(R) = \int_0^2 2e^{2x} - x e^{2x} dx = \left[\frac{5}{4} e^{2x} - \frac{1}{2} x e^{2x} \right]_0^2$	M1	2.2a
	$= \left(\frac{5}{4} e^4 - e^4 \right) - \left(\frac{5}{4} e^{2(0)} - \frac{1}{2} (0) e^0 \right) = \frac{1}{4} e^4 - \frac{5}{4}$	A1	2.1
		(5)	
7 Alt 1	$\left\{ \int 2e^{2x} - x e^{2x} dx = \int (2-x)e^{2x} dx \right\}, \left\{ \begin{array}{l} u = 2-x \Rightarrow \frac{du}{dx} = -1 \\ \frac{dv}{dx} = e^{2x} \Rightarrow v = \frac{1}{2} e^{2x} \end{array} \right\}$		
	$= \frac{1}{2} (2-x) e^{2x} - \int -\frac{1}{2} e^{2x} \{dx\}$	M1	3.1a
	$= \frac{1}{2} (2-x) e^{2x} + \frac{1}{4} e^{2x}$	M1	1.1b
	$= \frac{1}{2} (2-x) e^{2x} + \frac{1}{4} e^{2x}$	A1	1.1b
	$\left\{ \text{Area}(R) = \int_0^2 (2-x) e^{2x} dx = \right\} \left[\frac{1}{2} (2-x) e^{2x} + \frac{1}{4} e^{2x} \right]_0^2$	M1	2.2a
	$= \left(0 + \frac{1}{4} e^4 \right) - \left(\frac{1}{2} (2) e^0 + \frac{1}{4} e^0 \right) = \frac{1}{4} e^4 - \frac{5}{4}$	A1	2.1
		(5)	
(5 marks)			

Question 7 Notes:	
M1:	<p>Attempts to solve the problem by recognising the need to apply a method of integration by parts on either xe^{2x} or $(2-x)e^{2x}$. Allow this mark for either</p> <ul style="list-style-type: none"> • $\pm xe^{2x} \rightarrow \pm \lambda xe^{2x} \pm \int \mu e^{2x} \{dx\}$ • $(2-x)e^{2x} \rightarrow \pm \lambda(2-x)e^{2x} \pm \int \mu e^{2x} \{dx\}$ <p>where $\lambda, \mu \neq 0$ are constants.</p>
M1:	<p>For either</p> <ul style="list-style-type: none"> • $2e^{2x} - xe^{2x} \rightarrow e^{2x} \pm \frac{1}{2}xe^{2x} \pm \int \frac{1}{2}e^{2x} \{dx\}$ • $(2-x)e^{2x} \rightarrow \pm \frac{1}{2}(2-x)e^{2x} \pm \int \frac{1}{2}e^{2x} \{dx\}$
A1:	<p>Correct integration which can be simplified or un-simplified. E.g.</p> <ul style="list-style-type: none"> • $2e^{2x} - xe^{2x} \rightarrow e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)$ • $2e^{2x} - xe^{2x} \rightarrow e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x}$ • $2e^{2x} - xe^{2x} \rightarrow \frac{5}{4}e^{2x} - \frac{1}{2}xe^{2x}$ • $(2-x)e^{2x} \rightarrow \frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x}$
M1:	<p>Deduces that the upper limit is 2 and uses limits of 2 and 0 on their integrated function</p>
A1:	<p>Correct proof leading to $pe^4 + q$, where $p = \frac{1}{4}$, $q = -\frac{5}{4}$</p>

Question	Scheme	Marks	AOs
8 (a)	Total amount = $\frac{2100(1 - (1.012)^{14})}{1 - 1.012}$ or $\frac{2100((1.012)^{14} - 1)}{1.012 - 1}$	M1	3.1b
	= 31806.9948 ... = 31800 (tonnes) (3 sf)	A1	1.1b
		(2)	
	Total Cost = 5.15(2000(14)) + 6.45(31806.9948... - (2000)(14))	M1	3.1b
		M1	1.1b
	= 5.15(28000) + 6.45(3806.9948...) = 144200 + 24555.116...		
	= 168755.116... = £169000 (nearest £1000)	A1	3.2a
	(3)		
(5 marks)			
Question 8 Notes:			
(a)			
M1:	Attempts to apply the correct geometric summation formula with either $n = 13$ or $n = 14$, $a = 2100$ and $r = 1.012$ (Condone $r = 1.12$)		
A1:	Correct answer of 31800 (tonnes)		
(b)			
M1:	Fully correct method to find the total cost		
M1:	For either		
	<ul style="list-style-type: none"> • $5.15(2000(14)) \{= 144200\}$ • $6.45("31806.9948..." - (2000)(14)) \{= 24555.116...\}$ • $5.15(2000(13)) \{= 133900\}$ • $6.45("29354.73794..." - (2000)(13)) \{= 21638.059...\}$ 		
A1:	Correct answer of £169000		
	Note: Using rounded answer in part (a) gives 168710 which becomes £169000 (nearest £1000)		

Question	Scheme	Marks	AOs
9	Gradient of chord = $\frac{(2(x+h)^3 + 5) - (2x^3 + 5)}{x+h-h}$	B1	1.1b
		M1	2.1
	$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	B1	1.1b
	Gradient of chord = $\frac{(2(x^3 + 3x^2h + 3xh^2 + h^3) + 5) - (2x^3 + 5)}{1+h-1}$		
	= $\frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 + 5 - 2x^3 - 5}{1+h-1}$		
	= $\frac{6x^2h + 6xh^2 + 2h^3}{h}$		
	= $6x^2 + 6xh + 2h^2$	A1	1.1b
	$\frac{dy}{dx} = \lim_{h \rightarrow 0} (6x^2 + 6xh + 2h^2) = 6x^2$ and so at P, $\frac{dy}{dx} = 6(1)^2 = 6$	A1	2.2a
	(5)		
9 Alt 1	Let a point Q have x coordinate $1+h$, so $y_Q = 2(1+h)^3 + 5$	B1	1.1b
	$\{P(1, 7), Q(1+h, 2(1+h)^3 + 5)\} \Rightarrow$		
	Gradient PQ = $\frac{2(1+h)^3 + 5 - 7}{1+h-1}$	M1	2.1
	$(1+h)^3 = 1 + 3h + 3h^2 + h^3$	B1	1.1b
	Gradient PQ = $\frac{2(1 + 3h + 3h^2 + h^3) + 5 - 7}{1+h-1}$		
	= $\frac{2 + 6h + 6h^2 + 2h^3 + 5 - 7}{1+h-1}$		
	= $\frac{6h + 6h^2 + 2h^3}{h}$		
	= $6 + 6h + 2h^2$	A1	1.1b
	$\frac{dy}{dx} = \lim_{h \rightarrow 0} (6 + 6h + 2h^2) = 6$	A1	2.2a
	(5)		
(5 marks)			

Question 9 Notes:	
B1:	$2(x + h)^3 + 5$, seen or implied
M1:	Begins the proof by attempting to write the gradient of the chord in terms of x and h
B1:	$(x + h)^3 \rightarrow x^3 + 3x^2h + 3xh^2 + h^3$, by expanding brackets or by using a correct binomial expansion
M1:	Correct process to obtain the gradient of the chord as $\alpha x^2 + \beta xh + \gamma h^2$, $\alpha, \beta, \gamma \neq 0$
A1:	Correctly shows that the gradient of the chord is $6x^2 + 6xh + 2h^2$ and applies a limiting argument to deduce when $y = 2x^3 + 5$, $\frac{dy}{dx} = 6x^2$. E.g. $\lim_{h \rightarrow 0} (6x^2 + 6xh + 2h^2) = 6x^2$. Finally, deduces that at the point P , $\frac{dy}{dx} = 6$.
	Note: δx can be used in place of h
Alt 1	
B1:	Writes down the y coordinate of a point close to P . E.g. For a point Q with $x = 1 + h$, $\{y_Q\} = 2(1 + h)^3 + 5$
M1:	Begins the proof by attempting to write the gradient of the chord PQ in terms of h
B1:	$(1 + h)^3 \rightarrow 1 + 3h + 3h^2 + h^3$, by expanding brackets or by using a correct binomial expansion
M1:	Correct process to obtain the gradient of the chord PQ as $\alpha + \beta h + \gamma h^2$, $\alpha, \beta, \gamma \neq 0$
A1:	Correctly shows that the gradient of PQ is $6 + 6h + 2h^2$ and applies a limiting argument to deduce that at the point P on $y = 2x^3 + 5$, $\frac{dy}{dx} = 6$. E.g. $\lim_{h \rightarrow 0} (6 + 6h + 2h^2) = 6$
	Note: For Alt 1, δx can be used in place of h

Question	Scheme	Marks	AOs
10 (a)	$y = \frac{3x-5}{x+1} \Rightarrow y(x+1) = 3x-5 \Rightarrow xy + y = 3x-5 \Rightarrow y+5 = 3x-xy$	M1	1.1b
	$\Rightarrow y+5 = x(3-y) \Rightarrow \frac{y+5}{3-y} = x$	M1	2.1
	Hence $f^{-1}(x) = \frac{x+5}{3-x}, \quad x \in \mathbb{R}, x \neq 3$	A1	2.5
		(3)	
(b)	$ff(x) = \frac{3\left(\frac{3x-5}{x+1}\right) - 5}{\left(\frac{3x-5}{x+1}\right) + 1}$	M1	1.1a
	$\frac{3(3x-5) - 5(x+1)}{x+1}$	M1	1.1b
	$= \frac{(3x-5) + (x+1)}{x+1}$	A1	1.1b
	$= \frac{9x-15-5x-5}{3x-5+x+1} = \frac{4x-20}{4x-4} = \frac{x-5}{x-1}$ (note that $a = -5$)	A1	2.1
		(4)	
(c)	$fg(2) = f(4-6) = f(-2) = \frac{3(-2)-5}{-2+1} = 11$	M1	1.1b
		A1	1.1b
		(2)	
(d)	$g(x) = x^2 - 3x = (x-1.5)^2 - 2.25$. Hence $g_{\min} = -2.25$	M1	2.1
	Either $g_{\min} = -2.25$ or $g(x) \geq -2.25$ or $g(5) = 25 - 15 = 10$	B1	1.1b
	$-2.25 \leq g(x) \leq 10$ or $-2.25 \leq y \leq 10$	A1	1.1b
		(3)	
(e)	E.g. <ul style="list-style-type: none"> the function g is many-one the function g is not one-one the inverse is one-many $g(0) = g(3) = 0$ 	B1	2.4
		(1)	
(13 marks)			

Question 10 Notes:	
(a)	
M1:	Attempts to find the inverse by cross-multiplying and an attempt to collect all the x -terms (or swapped y -terms) onto one side
M1:	A fully correct method to find the inverse
A1:	A correct $f^{-1}(x) = \frac{x+5}{3-x}$, $x \in \mathbb{R}$, $x \neq 3$, expressed fully in function notation (including the domain)
(b)	
M1:	Attempts to substitute $f(x) = \frac{3x-5}{x+1}$ into $\frac{3f(x)-5}{f(x)+1}$
M1:	Applies a method of “rationalising the denominator” for both their numerator and their denominator.
A1:	$\frac{3(3x-5) - 5(x+1)}{\frac{x+1}{(3x-5) + (x+1)}} \text{ which can be simplified or un-simplified}$
A1:	Shows $ff(x) = \frac{x+a}{x-1}$ where $a = -5$ or $ff(x) = \frac{x-5}{x-1}$, with no errors seen.
(c)	
M1:	Attempts to substitute the result of $g(2)$ into f
A1:	Correctly obtains $fg(2) = 11$
(d)	
M1:	Full method to establish the minimum of g . E.g.
	<ul style="list-style-type: none"> • $(x \pm a)^2 + \beta$ leading to $g_{\min} = \beta$ • Finds the value of x for which $g'(x) = 0$ and inserts this value of x back into $g(x)$ in order to find to g_{\min}
B1:	For either <ul style="list-style-type: none"> • finding the correct minimum value of g (Can be implied by $g(x) \geq -2.25$ or $g(x) > -2.25$) • stating $g(5) = 25 - 15 = 10$
A1:	States the correct range for g . E.g. $-2.25 \leq g(x) \leq 10$ or $-2.25 \leq y \leq 10$
(e)	
B1:	See scheme

Question	Scheme	Marks	AOs
11 (a)	$f'(x) = k - 4x - 3x^2$		
	$f''(x) = -4 - 6x = 0$	M1	1.1b
	Criteria 1 Either $f''(x) = -4 - 6x = 0 \Rightarrow x = \frac{4}{-6} \Rightarrow x = -\frac{2}{3}$ or $f''\left(-\frac{2}{3}\right) = -4 - 6\left(-\frac{2}{3}\right) = 0$		
	Criteria 2 Either • $f''(-0.7) = -4 - 6(-0.7) = 0.2 > 0$ $f''(-0.6) = -4 - 6(-0.6) = -0.4 < 0$ or • $f'''\left(-\frac{2}{3}\right) = -6 \neq 0$		
	At least one of Criteria 1 or Criteria 2	B1	2.4
	Both Criteria 1 and Criteria 2 and concludes C has a point of inflection at $x = -\frac{2}{3}$	A1	2.1
		(3)	
(b)	$f'(x) = k - 4x - 3x^2, AB = 4\sqrt{2}$		
	$f(x) = kx - 2x^2 - x^3 \{+c\}$	M1	1.1b
		A1	1.1b
	$f(0) = 0$ or $(0, 0) \Rightarrow c = 0 \Rightarrow f(x) = kx - 2x^2 - x^3$ $\{f(x) = 0 \Rightarrow\} f(x) = x(k - 2x - x^2) = 0 \Rightarrow \{x = 0,\} k - 2x - x^2 = 0$	A1	2.2a
	$\{x^2 + 2x - k = 0\} \Rightarrow (x+1)^2 - 1 - k = 0, x = \dots$	M1	2.1
	$\Rightarrow x = -1 \pm \sqrt{k+1}$	A1	1.1b
	$AB = \left(-1 + \sqrt{k+1}\right) - \left(-1 - \sqrt{k+1}\right) = 4\sqrt{2} \Rightarrow k = \dots$	M1	2.1
	So, $2\sqrt{k+1} = 4\sqrt{2} \Rightarrow k = 7$	A1	1.1b
	(7)		
(10 marks)			

Question 11 Notes:**(a)****M1:**

E.g.

- attempts to find $f''\left(-\frac{2}{3}\right)$
- finds $f''(x)$ and sets the result equal to 0

B1:

See scheme

A1:

See scheme

(b)**M1:**Integrates $f'(x)$ to give $f(x) = \pm kx \pm \alpha x^2 \pm \beta x^3$, $\alpha, \beta \neq 0$ with or without the constant of integration**A1:** $f(x) = kx - 2x^2 - x^3$, with or without the constant of integration**A1:**Finds $f(x) = kx - 2x^2 - x^3 + c$, and makes some reference to $y = f(x)$ passing through the origin to deduce $c = 0$. Proceeds to produce the result $k - 2x - x^2 = 0$ or $x^2 + 2x - k = 0$ **M1:**Uses a valid method to solve the quadratic equation to give x in terms of k **A1**Correct roots for x in terms of k . i.e. $x = -1 \pm \sqrt{k+1}$ **M1:**Applies $AB = 4\sqrt{2}$ on $x = -1 \pm \sqrt{k+1}$ in a complete method to find $k = \dots$ **A1:**Finds $k = 7$ from correct solution only

Question	Scheme	Marks	AOs
12	$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1 + \cos \theta} d\theta$		
	Attempts this question by applying the substitution $u = 1 + \cos \theta$ and progresses as far as achieving $\int \dots \frac{(u-1)}{u} \dots$	M1	3.1a
	$u = 1 + \cos \theta \Rightarrow \frac{du}{d\theta} = -\sin \theta$ and $\sin 2\theta = 2\sin \theta \cos \theta$	M1	1.1b
	$\left\{ \int \frac{\sin 2\theta}{1 + \cos \theta} d\theta = \right\} \int \frac{2\sin \theta \cos \theta}{1 + \cos \theta} d\theta = \int \frac{-2(u-1)}{u} du$	A1	2.1
	$-2 \int \left(1 - \frac{1}{u} \right) du = -2(u - \ln u)$	M1	1.1b
		M1	1.1b
	$\left\{ \int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1 + \cos \theta} d\theta = \right\} = -2[u - \ln u]_2^1 = -2((1 - \ln 1) - (2 - \ln 2))$	M1	1.1b
	$= -2(-1 + \ln 2) = 2 - 2\ln 2 *$	A1*	2.1
	(7)		
12 Alt 1	Attempts this question by applying the substitution $u = \cos \theta$ and progresses as far as achieving $\int \dots \frac{u}{u+1} \dots$	M1	3.1a
	$u = \cos \theta \Rightarrow \frac{du}{d\theta} = -\sin \theta$ and $\sin 2\theta = 2\sin \theta \cos \theta$	M1	1.1b
	$\left\{ \int \frac{\sin 2\theta}{1 + \cos \theta} d\theta = \right\} \int \frac{2\sin \theta \cos \theta}{1 + \cos \theta} d\theta = \int \frac{-2u}{u+1} du$	A1	2.1
	$\left\{ = -2 \int \frac{(u+1) - 1}{u+1} du = -2 \int 1 - \frac{1}{u+1} du \right\} = -2(u - \ln(u+1))$	M1	1.1b
		M1	1.1b
	$\left\{ \int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1 + \cos \theta} d\theta = \right\} = -2[u - \ln(u+1)]_1^0 = -2((0 - \ln 1) - (1 - \ln 2))$	M1	1.1b
	$= -2(-1 + \ln 2) = 2 - 2\ln 2 *$	A1*	2.1
		(7)	
(7 marks)			

Question 12 Notes:	
M1:	See scheme
M1:	Attempts to differentiate $u = 1 + \cos \theta$ to give $\frac{du}{d\theta} = \dots$ and applies $\sin 2\theta = 2\sin \theta \cos \theta$
A1:	Applies $u = 1 + \cos \theta$ to show that the integral becomes $\int \frac{-2(u-1)}{u} du$
M1:	Achieves an expression in u that can be directly integrated (e.g. dividing each term by u or applying partial fractions) and integrates to give an expression in u of the form $\pm \lambda u \pm \mu \ln u, \lambda, \mu \neq 0$
M1:	For integration in u of the form $\pm 2(u - \ln u)$
M1:	Applies u -limits of 1 and 2 to an expression of the form $\pm \lambda u \pm \mu \ln u, \lambda, \mu \neq 0$ and subtracts either way round
A1*:	Applies u -limits the right way round, i.e. <ul style="list-style-type: none"> • $\int_2^1 \frac{-2(u-1)}{u} du = -2 \int_2^1 \left(1 - \frac{1}{u}\right) du = -2[u - \ln u]_2^1 = -2((1 - \ln 1) - (2 - \ln 2))$ • $\int_2^1 \frac{-2(u-1)}{u} du = 2 \int_1^2 \left(1 - \frac{1}{u}\right) du = 2[u - \ln u]_1^2 = 2((2 - \ln 2) - (1 - \ln 1))$ and correctly proves $\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1 + \cos \theta} d\theta = 2 - 2\ln 2$, with no errors seen
Alt 1	
M1:	See scheme
M1:	Attempts to differentiate $u = \cos \theta$ to give $\frac{du}{d\theta} = \dots$ and applies $\sin 2\theta = 2\sin \theta \cos \theta$
A1:	Applies $u = \cos \theta$ to show that the integral becomes $\int \frac{-2u}{u+1} du$
M1:	Achieves an expression in u that can be directly integrated (e.g. by applying partial fractions or a substitution $v = u+1$) and integrates to give an expression in u of the form $\pm \lambda u \pm \mu \ln(u+1), \lambda, \mu \neq 0$ or $\pm \lambda v \pm \mu \ln v, \lambda, \mu \neq 0$, where $v = u+1$
M1:	For integration in u in the form $\pm 2(u - \ln(u+1))$
M1:	Either <ul style="list-style-type: none"> • Applies u-limits of 0 and 1 to an expression of the form $\pm \lambda u \pm \mu \ln(u+1), \lambda, \mu \neq 0$ and subtracts either way round • Applies v-limits of 1 and 2 to an expression of the form $\pm \lambda v \pm \mu \ln v, \lambda, \mu \neq 0$, where $v = u+1$ and subtracts either way round
A1*:	Applies u -limits the right way round, (o.e. in v) i.e. <ul style="list-style-type: none"> • $\int_1^0 \frac{-2u}{u+1} du = -2 \int_1^0 \left(1 - \frac{1}{u+1}\right) du = -2[u - \ln(u+1)]_1^0 = -2((0 - \ln 1) - (1 - \ln 2))$ • $\int_1^0 \frac{-2u}{u+1} du = 2 \int_0^1 \left(1 - \frac{1}{u+1}\right) du = 2[u - \ln(u+1)]_0^1 = 2((1 - \ln 2) - (0 - \ln 1))$ and correctly proves $\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1 + \cos \theta} d\theta = 2 - 2\ln 2$, with no errors seen

Question	Scheme	Marks	AOs
13 (a)	$R = 2.5$	B1	1.1b
	$\tan \alpha = \frac{1.5}{2}$ o.e.	M1	1.1b
	$\alpha = 0.6435$, so $2.5 \sin(\theta - 0.6435)$	A1	1.1b
		(3)	
(b)	e.g. $D = 6 + 2 \sin\left(\frac{4\pi(0)}{25}\right) - 1.5 \cos\left(\frac{4\pi(0)}{25}\right) = 4.5\text{m}$	B1	3.4
	or $D = 6 + 2.5 \sin\left(\frac{4\pi(0)}{25} - 0.6435\right) = 4.5\text{m}$		
		(1)	
(c)	$D_{\max} = 6 + 2.5 = 8.5 \text{ m}$	B1ft	3.4
		(1)	
(d)	Sets $\frac{4\pi t}{25} - "0.6435" = \frac{5\pi}{2}$ or $\frac{\pi}{2}$	M1	1.1b
	Afternoon solution $\Rightarrow \frac{4\pi t}{25} - "0.6435" = \frac{5\pi}{2} \Rightarrow t = \frac{25}{4\pi} \left(\frac{5\pi}{2} + "0.6435"$	M1	3.1b
	$\Rightarrow t = 16.9052... \Rightarrow \text{Time} = 16:54$ or 4:54 pm	A1	3.2a
		(3)	
(e)(i)	<ul style="list-style-type: none"> An attempt to find the depth of water at 00:00 on 19th October 2017 for at least one of either Tom's model or Jolene's model. 	M1	3.4
	<ul style="list-style-type: none"> At 00:00 on 19th October 2017, Tom: $D = 3.72... \text{ m}$ and Jolene: $H = 4.5 \text{ m}$ and e.g. <ul style="list-style-type: none"> As $4.5 \neq 3.72$ then Jolene's model is not true Jolene's model is not continuous at 00:00 on 19th October 2017 Jolene's model does not continue on from where Tom's model has ended 	A1	3.5a
(ii)	To make the model continuous, e.g.		
	<ul style="list-style-type: none"> $H = 5.22 + 2 \sin\left(\frac{4\pi x}{25}\right) - 1.5 \cos\left(\frac{4\pi x}{25}\right), \quad 0 \leq x < 24$ $H = 6 + 2 \sin\left(\frac{4\pi(x+24)}{25}\right) - 1.5 \cos\left(\frac{4\pi(x+24)}{25}\right), \quad 0 \leq x < 24$ 	B1	3.3
		(3)	
(11 marks)			

Question	Scheme	Marks	AOs
13 (d) Alt 1	Sets $\frac{4\pi t}{25} - "0.6435" = \frac{\pi}{2}$	M1	1.1b
	Period = $2\pi \div \left(\frac{4\pi}{25}\right) = 12.5$ Afternoon solution $\Rightarrow t = 12.5 + \frac{25}{4\pi} \left(\frac{\pi}{2} + "0.6435"\right)$	M1	3.1b
	$\Rightarrow t = 16.9052... \Rightarrow$ Time = 16:54 or 4:54 pm	A1	3.2a
		(3)	

Question 13 Notes:

(a)	
B1:	$R = 2.5$ Condone $R = \sqrt{6.25}$
M1:	For either $\tan \alpha = \frac{1.5}{2}$ or $\tan \alpha = -\frac{1.5}{2}$ or $\tan \alpha = \frac{2}{1.5}$ or $\tan \alpha = -\frac{2}{1.5}$
A1:	$\alpha = \text{awrt } 0.6435$
(b)	
B1:	Uses Tom's model to find $D = 4.5$ (m) at 00:00 on 18th October 2017
(c)	
B1ft:	Either 8.5 or follow through "6 + their R" (by using their R found in part (a))
(d)	
M1:	Realises that $D = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right) = 6 + "2.5"\sin\left(\frac{4\pi t}{25} - "0.6435"\right)$ and so maximum depth occurs when $\sin\left(\frac{4\pi t}{25} - "0.6435"\right) = 1 \Rightarrow \frac{4\pi t}{25} - "0.6435" = \frac{\pi}{2}$ or $\frac{5\pi}{2}$
M1:	Uses the model to deduce that a p.m. solution occurs when $\frac{4\pi t}{25} - "0.6435" = \frac{5\pi}{2}$ and rearranges this equation to make $t = \dots$
A1:	Finds that maximum depth occurs in the afternoon at 16:54 or 4:54 pm
(d)	
Alt 1	
M1:	Maximum depth occurs when $\sin\left(\frac{4\pi t}{25} - "0.6435"\right) = 1 \Rightarrow \frac{4\pi t}{25} - "0.6435" = \frac{\pi}{2}$
M1:	Rearranges to make $t = \dots$ and adds on the period, where period = $2\pi \div \left(\frac{4\pi}{25}\right) \{= 12.5\}$
A1:	Finds that maximum depth occurs in the afternoon at 16:54 or 4:54 pm

Question 13 Notes Continued:	
(e)(i)	
M1:	See scheme
A1:	See scheme
	Note: Allow Special Case M1 for a candidate who just states that Jolene's model is not continuous at 00:00 on 19th October 2017 o.e.
(e)(ii)	
B1:	Uses the information to set up a new model for H . (See scheme)

Question	Scheme	Marks	AOs
14	$x = 4\cos\left(t + \frac{\pi}{6}\right), y = 2\sin t$		
	$x + y = 4\left[\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right] + 2\sin t$	M1	3.1a
		M1	1.1b
	$x + y = 2\sqrt{3}\cos t$	A1	1.1b
	$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	M1	3.1a
	$\frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$		
	$(x+y)^2 + 3y^2 = 12$	A1	2.1
	(5)		
14 Alt 1	$(x+y)^2 = \left(4\cos\left(t + \frac{\pi}{6}\right) + 2\sin t\right)^2$		
	$= \left(4\left[\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right] + 2\sin t\right)^2$	M1	3.1a
		M1	1.1b
	$= \left(2\sqrt{3}\cos t\right)^2$ or $12\cos^2 t$	A1	1.1b
	So, $(x+y)^2 = 12(1 - \sin^2 t) = 12 - 12\sin^2 t = 12 - 12\left(\frac{y}{2}\right)^2$	M1	3.1a
	$(x+y)^2 + 3y^2 = 12$	A1	2.1
	(5)		
(5 marks)			
Question 14 Notes:			
M1:	Looks ahead to the final result and uses the compound angle formula in a full attempt to write down an expression for $x + y$ which is in terms of t only.		
M1:	Applies the compound angle formula on their term in x . E.g. $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$		
A1:	Uses correct algebra to find $x + y = 2\sqrt{3}\cos t$		
M1:	Complete strategy of applying $\cos^2 t + \sin^2 t = 1$ on a rearranged $x + y = "2\sqrt{3}\cos t"$, $y = 2\sin t$ to achieve an equation in x and y only		
A1:	Correctly proves $(x + y)^2 + ay^2 = b$ with both $a = 3$, $b = 12$, and no errors seen		

Question 14 Notes Continued:**Alt 1****M1:** Apply in the same way as in the main scheme**M1:** Apply in the same way as in the main scheme**A1:** Uses correct algebra to find $(x + y)^2 = (2\sqrt{3}\cos t)^2$ or $(x + y)^2 = 12\cos^2 t$ **M1:** Complete strategy of applying $\cos^2 t + \sin^2 t = 1$ on $(x + y)^2 = (2\sqrt{3}\cos t)^2$ to achieve an equation in x and y only**A1:** Correctly proves $(x + y)^2 + ay^2 = b$ with both $a = 3$, $b = 12$, and no errors seen