Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Sample Assessment Material

(Time: 1 hour 30 minutes) | Paper Reference **9FM0/02**

Further Mathematics

Advanced

Paper 2: Core Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Find

$$\int \frac{1}{x^2 + 6x + 25} \mathrm{d}x$$

(3)

(b) Hence find the exact value of

$$\int_{-3}^{1} \left(1 - \frac{25}{x^2 + 6x + 25}\right) \mathrm{d}x$$

giving the answer in simplest form.

(3)

A student claims that the magnitude of the answer to part (b) gives the total area bounded by the curve $y = 1 - \dfrac{25}{x^2 + 6x + 25}$ and the x-axis between the line $x = -3$ and the line $x = 1$

(c) State, with a reason, whether or not the student is correct.

(1)

Question 1 continued

(Total for Question 1 is 7 marks)

2. A company operating a coal mine is concerned about the mine running out of coal. It is estimated that 2.5 million tonnes of coal are left in the mine. The company wishes to mine all of this coal in 20 years.

In order to mine the coal in a regulated manner, the company models the amount of coal to be mined in the coming years by the formula

$$M_r = \frac{10}{r^2 + 8r + 15}$$

where M_r is the amount of coal, in millions of tonnes, mined in year r, with the first year being year 1

(a) Show that, according to the model, the total amount of coal, in millions of tonnes, mined in the first n years is given by

$$T_n = \frac{9n^2 + 41n}{k(n+4)(n+5)}$$

where k is a constant to be determined.

(6)

(b) Explain why, according to this model, the mine will never run out of coal.

(2)

The company decides to mine an extra fixed amount each year so that all the coal will be mined in exactly 20 years.

(c) Refine the formula for M_r so that 2.5 million tonnes of coal will be exhausted in exactly 20 years of mining.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3.

$$\mathbf{P} = \begin{pmatrix} 3 & 3 \\ 4 & 7 \end{pmatrix}$$

The matrix **P** represents a linear transformation, *T*, of the plane.

(a) Describe the invariant points of the transformation *T*.

(3)

(b) Describe the invariant lines of the transformation *T*.

(6)

Question 3 continued

(Total for Question 3 is 9 marks)

4. (a) Using the identity $zz^* = |z|^2$, or otherwise, show that if w is any root of unity then

$$|w - 2|^2 = 5 - 2(w + w^*)$$

(3)

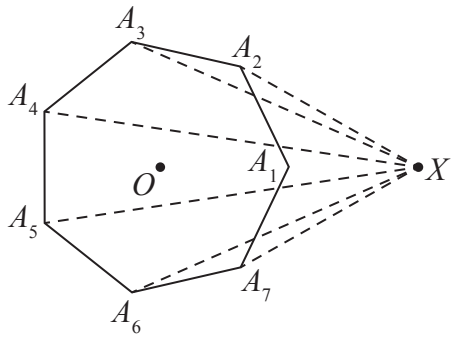

Figure 1

Figure 1 shows a regular heptagon $A_1A_2A_3A_4A_5A_6A_7$ whose vertices all lie on the unit circle with centre at the origin O and A_1 at (1, 0). The point X lies in the same plane as the heptagon and has coordinates (2, 0).

Using the result given in part (a),

(b) find $\sum_{i=1}^{7} \left(XA_i\right)^2$

(4)

Question 4 continued

(Total for Question 4 is 7 marks)

5.

$$y = \arctan(\sinh(x))$$

(a) Show that $\frac{d^3y}{dx^3} = \frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^3$

(7)

(b) Hence find $\frac{d^5y}{dx^5}$ in terms of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$

(4)

(c) Find the Maclaurin series for y, in ascending powers of x, up to and including the term in x^5

(3)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 14 marks)

6. A damped spring is part of a car suspension system. In tests for the system, a mass is attached to the damped spring and is made to move upwards in a vertical line.

The motion of the system is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 2e^{-3t}$$

where x cm is the vertical displacement of the mass above its equilibrium position and t is the time, in seconds, after motion begins.

In one particular test, the mass is moved to a position 20 cm above its equilibrium position and given an initial velocity of 1 ms^{-1} upwards. For this test, use the model to

(a) find an equation for x in terms of t,

(9)

(b) find, to the nearest mm, the maximum displacement of the mass from its equilibrium position.

(3)

In this test, the time taken for the mass to return to its equilibrium position was measured as 2.86 seconds.

(c) State, with justification, whether or not this supports the model.

(1)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 13 marks)

7.

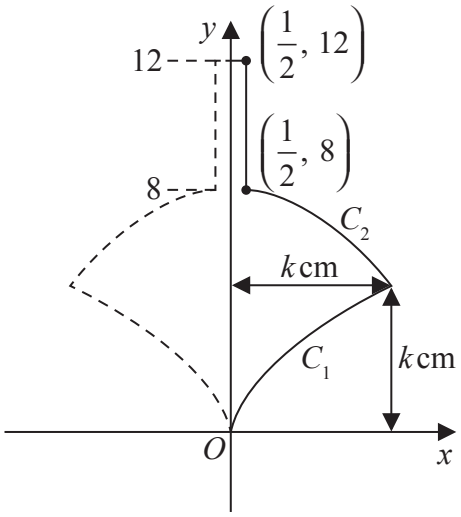

Diagram not to scale

Figure 2

Figure 2 shows a sketch of the cross-section of a design for a child's spinning top. The top is formed by rotating the region bounded by the y-axis, the curve C_1, the curve C_2, the line with equation $x = \frac{1}{2}$ and the line with equation $y = 12$, through 360° about the y-axis.

The curve C_1 has equation

$$y = k^{\frac{2}{3}}x^{\frac{1}{3}} \qquad 0 \leqslant x \leqslant k$$

and the curve C_2 has equation

$$y = \frac{32k^2 - k - (32 - 4k)x^2}{4k^2 - 1} \qquad \frac{1}{2} \leqslant x \leqslant k$$

(a) Show that $\int_k^8 \left((4k^2 - 1)y - (32k^2 - k)\right)\mathrm{d}y = \frac{1}{2}(8 - k)\left(4k^3 - 32k^2 + k - 8\right)$

(3)

Hence find

(b) the value of k that gives the maximum value for the volume of the spinning top,

(9)

(c) the maximum volume of the spinning top.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS