

2. [With respect to the **right-hand rule**, a rotation through θ° anticlockwise about the z -axis is represented by the matrix
- $$\begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Given that the matrix \mathbf{M} , where

$$\mathbf{M} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

represents a rotation through α° anticlockwise about the z -axis with respect to the **right-hand rule**,

- (a) determine the value of α . (1)

- (b) Hence determine the smallest possible positive integer value of k for which $\mathbf{M}^k = \mathbf{I}$ (2)

The 3×3 matrix \mathbf{N} represents a reflection in the plane with equation $y = 0$

- (c) Write down the matrix \mathbf{N} . (1)

The point A has coordinates $(-2, 4, 3)$

The point B is the image of the point A under the transformation represented by matrix \mathbf{M} followed by the transformation represented by matrix \mathbf{N} .

- (d) Show that the coordinates of B are $(2 + \sqrt{3}, 2\sqrt{3} - 1, 3)$ (2)

Given that O is the origin,

- (e) show that, to 3 significant figures, the size of angle AOB is 66.9° (2)

- (f) Hence determine the area of triangle AOB , giving your answer to 3 significant figures. (2)



8.

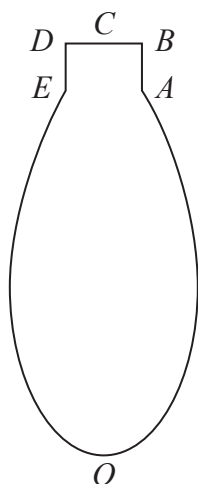


Figure 1

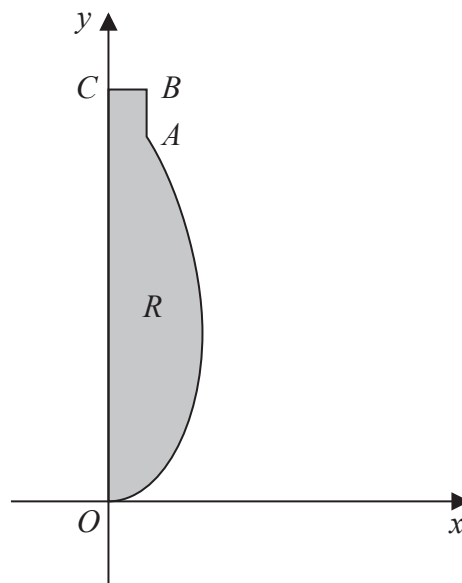


Figure 2

Figure 1 shows the central vertical cross-section, $OABCDEO$, of the design for a solid glass ornament.

Figure 2 shows the finite region, R , which is bounded by the y -axis, the horizontal line CB , the vertical line BA , and the curve AO .

The ornament is formed by rotating the region R through 360° about the y -axis.

The curve AO is modelled by the equation

$$x = ky^2 + \sqrt{y} \quad 0 \leq y \leq 4$$

where k is a constant.

The point A has coordinates $(0.4, 4)$ and the point B has coordinates $(0.4, 4.5)$

The units are centimetres.

- (a) Determine the value of k according to this model. (2)
- (b) Use algebraic integration to determine the exact volume of glass that would be required to make the ornament, according to the model. (7)
- (c) State a limitation of the model. (1)

When the ornament was manufactured, 9 cm^3 of glass was required.

- (d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning. (1)



