

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel  
Level 3 GCE**

Centre Number

--	--	--	--	--

Candidate Number

--	--	--	--	--

**Monday 13 May 2019**

Afternoon (Time: 1 hour 40 minutes)

Paper Reference **8FM0-01**

**Further Mathematics**

**Advanced Subsidiary**

**Paper 1: Core Pure Mathematics**

**You must have:**

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

P58333A

©2019 Pearson Education Ltd.

1/1/1/1



Pearson









3. Prove by mathematical induction that, for  $n \in \mathbb{N}$

$$\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$$

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA









5.

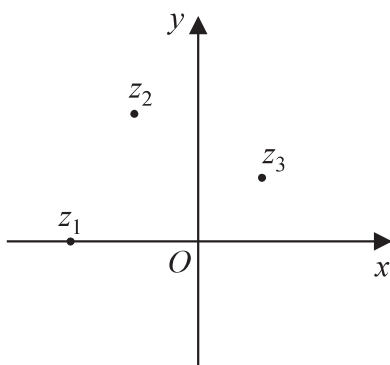


Figure 1

The complex numbers  $z_1 = -2$ ,  $z_2 = -1 + 2i$  and  $z_3 = 1 + i$  are plotted in Figure 1, on an Argand diagram for the complex plane with  $z = x + iy$

- (a) Explain why  $z_1$ ,  $z_2$  and  $z_3$  cannot all be roots of a quartic polynomial equation with real coefficients.

(2)

(b) Show that  $\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \frac{\pi}{4}$

(3)

(c) Hence show that  $\arctan(2) - \arctan\left(\frac{1}{3}\right) = \frac{\pi}{4}$

(2)

A copy of Figure 1, labelled Diagram 1, is given on page 12.

- (d) Shade, on Diagram 1, the set of points of the complex plane that satisfy the inequality

$$|z + 2| \leq |z - 1 - i|$$

(2)





Question 5 continued

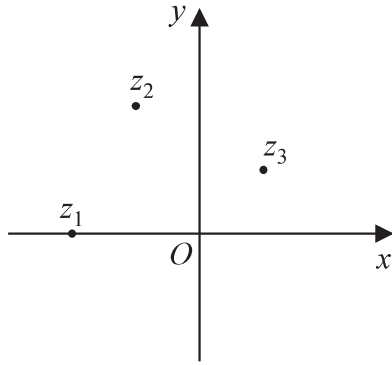


Diagram 1

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

























9.  $f(x) = 2x^{\frac{1}{3}} + x^{-\frac{2}{3}} \quad x > 0$

The finite region bounded by the curve  $y = f(x)$ , the line  $x = \frac{1}{8}$ , the  $x$ -axis and the line  $x = 8$  is rotated through  $\theta$  radians about the  $x$ -axis to form a solid of revolution.

Given that the volume of the solid formed is  $\frac{461}{2}$  units cubed, use algebraic integration to find the angle  $\theta$  through which the region is rotated.

(8)









10. The population of chimpanzees in a particular country consists of juveniles and adults. Juvenile chimpanzees do not reproduce.

In a study, the numbers of juvenile and adult chimpanzees were estimated at the start of each year. A model for the population satisfies the matrix system

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix} \begin{pmatrix} J_n \\ A_n \end{pmatrix} \quad n = 0, 1, 2, \dots$$

where  $a$  is a constant, and  $J_n$  and  $A_n$  are the respective numbers of juvenile and adult chimpanzees  $n$  years after the start of the study.

- (a) Interpret the meaning of the constant  $a$  in the context of the model. (1)

At the start of the study, the total number of chimpanzees in the country was estimated to be 64 000

According to the model, after one year the number of juvenile chimpanzees is 15 360 and the number of adult chimpanzees is 43 008

- (b) (i) Find, in terms of  $a$

$$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} \quad (3)$$

- (ii) Hence, or otherwise, find the value of  $a$ . (3)

- (iii) Calculate the change in the number of juvenile chimpanzees in the first year of the study, according to this model. (2)

Given that the number of juvenile chimpanzees is known to be in decline in the country,

- (c) comment on the short-term suitability of this model. (1)

A study of the population revealed that adult chimpanzees stop reproducing at the age of 40 years.

- (d) Refine the matrix system for the model to reflect this information, giving a reason for your answer.

*(There is no need to estimate any unknown values for the refined model, but any known values should be made clear.)*

(2)









