Up The Wall (1)

A popular feat among stunt drivers which relies on the mechanics of horizontal circular motion is the Wall of Death.

This usually comprises a near-vertical slope which vehicles traverse in a fairly tight circle, relying on sufficient speed to ensure they don't 'fall' down the slope.

A typical wall of death might be 10 metres in diameter. Assuming an angle of 80 degrees from the horizontal and ignoring friction, draw a force diagram for the car above.

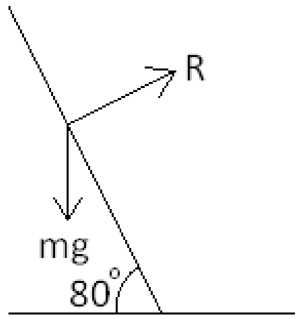
(a)

Find an expression for the normal reaction force, and hence find the speed of the car required for it to describe horizontal circles.

(b)

Up The Wall (1) Solutions

a)



b)

$$Resolving\ vertically: \quad R\cos 80 = mg \quad \Longrightarrow \quad \boldsymbol{R = \frac{mg}{\cos 80}}$$

$$Resolving\ radially\ and\ using\ circular\ motion\ formula\ F = \frac{mv^2}{r}:$$

$$R\sin 80 = ma = \frac{mv^2}{r} = \frac{mv^2}{5} \quad \Longrightarrow \quad \frac{mg\sin 80}{\cos 80} = \frac{mv^2}{5} \quad \Longrightarrow \quad v = \sqrt{5g\tan 80} = \boldsymbol{16.7ms^{-1}\ to\ 3\ s.f.}$$

Up The Wall (2)

The sport isn't limited to cars – motorbikes are a popular choice of vehicle, but ordinary bicycles can work just as well, provided you get the maths right.

Since friction does, in actual fact, make quite a large difference to the outcome of these equations, for the following situation we will take it into account. The coefficient of friction between the bicycle tyres and the wall is 0.75.

Draw a force diagram for a bicycle in the same ring as the car, including friction, and use it to calculate the lowest possible speed the cyclist must ride at to maintain circular motion.

(a)

Draw a modified force diagram and use it to explain why there is no upper limit to the speed the cyclist can travel at. That is, there is no speed great enough that the cyclist would move up the slope rather than remain travelling in a horizontal circle.

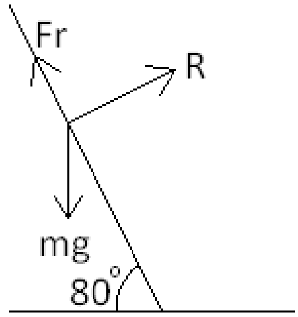
(b)

Once we take friction into account, we should be able to see why motion ought to be possible even around a completely vertical track. Calculate the speed required for a motorbike to traverse a 100m diameter vertical wall of death given a coefficient of friction of 1.

(c)

Up The Wall (2) Solutions

a)



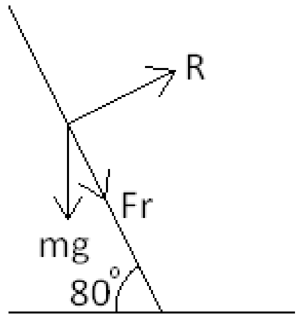
$$Resolving\ vertically: \quad Fr\sin 80 + R\cos 80 = mg \quad \Longrightarrow \quad 0.75R\sin 80 + R\cos 80 = mg$$

$$R = \frac{mg}{0.75\sin 80 + \cos 80}$$

$$R\sin 80 - Fr\cos 80 = ma = \frac{mv^2}{r} = \frac{mv^2}{5} \quad \Longrightarrow \quad R\sin 80 - 0.75R\cos 80 = \frac{mv^2}{5}$$

$$\Longrightarrow \quad \frac{5mg(\sin 80 - 0.75\cos 80)}{0.75\sin 80 + \cos 80} = mv^2 \quad \Longrightarrow \quad v = \sqrt{\frac{5g(\sin 80 - 0.75\cos 80)}{0.75\sin 80 + \cos 80}} = \mathbf{6.78ms^{-1}\ to\ 3\ s.f.}$$

b)

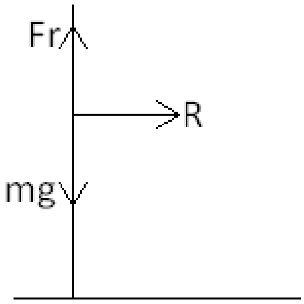


$$Upward\ forces: R\cos 80 \quad Downward\ forces: Fr\sin 80 + mg$$
$$\boldsymbol{Max\ F_r} = 0.75R\sin 80 + mg$$

$$\cos 80 = 0.173\ldots \quad and\ 0.75\sin 80 = 0.738\ldots$$
$$So\ \mathbf{0.173R}\ acts\ upwards, and\ \mathbf{up\ to\ 0.738R + mg}\ acts\ downwards$$

Since Friction is directly proportional to the normal reaction, at this angle, and with this coefficient of friction, the maximum possible downward force will never be exceeded by the upward force.

c)



$$\mu = 1 \quad so \quad Fr = R$$

$$but\ resolving\ vertically\ gives\ Fr = mg \quad so \quad R = mg$$

$$mg = ma = \frac{mv^2}{r} \quad \Longrightarrow \quad v = \sqrt{gr} = \sqrt{9.8 \times 50}$$

$$= \mathbf{22.1ms^{-1}\ to\ 3\ s.f.}$$