

Trial and Improvement

Method:

1) If necessary, rearrange the equation to give just a number on the right.

Eg, if you have $2x^2 = 5x - 4$, write it as $2x^2 - 5x = -4$.

2) Make a table for your 'Guess for x ', the 'Value of ...' and 'Too high / Too low'.

3) Pick a number to try.

Even if you choose something that's way too big or small, it doesn't matter much.

Sometimes the question will tell you that x lies between, say, 2 and 3, so use those first.

4) Work out the value of the left side of the equation, then write 'Too high' or 'Too low'.

5) Pick another number to try.

If you already know that 4 is too high but 2 is too low, pick something in between.

6) Continue until you know enough about the number, and write what you know.

For instance, "3.55 is too high, and 3.5 is too low."

7) Give your final answer, to the level of accuracy required.

Remember you can't be sure the answer is 2.7 to 1 d.p. unless you know for certain that the answer is between 2.65 and 2.75.

Example:

Find the solution to $x^3 = 5x + 200$ correct to 1 decimal place.

My thinking...

I need just a number on the right

$$x^3 = 5x + 200$$

$$x^3 - 5x = 200$$

So I need 200 as the answer

Try an easy number first

10 must be too big, so try 1

Between 1 and 10, so try 5

Between 5 and 10, so try 7

Between 5 and 7, so try 6

Between 6 and 7, so try 6.5

Between 6 and 6.5, so try 6.3

Between 6 and 6.3, so try 6.2

Between 6 and 6.2, so try 6.1

Between 6.1 and 6.2, so try 6.15

Guess for x	Value of $x^3 - 5x$	Too high / Too low
10	$10^3 - 5 \times 10 = 950$	Too high
1	$1^3 - 5 \times 1 = -4$	Too low
5	$5^3 - 5 \times 5 = 100$	Too low
7	$7^3 - 5 \times 7 = 308$	Too high
6	$6^3 - 5 \times 6 = 186$	Too low
6.5	$6.5^3 - 5 \times 6.5 = 242.125$	Too high
6.3	$6.3^3 - 5 \times 6.3 = 218.547$	Too high
6.2	$6.2^3 - 5 \times 6.2 = 207.328$	Too high
6.1	$6.1^3 - 5 \times 6.1 = 196.481$	Too low
6.15	$6.15^3 - 5 \times 6.15 = 201.8 \dots$	Too high

6.1 is too low, and 6.15 is too high,
so x must be between 6.1 and 6.15.

Therefore $x = 6.1$ to 1 decimal place.

Trial and Improvement Questions Continued

Use the trial and improvement method to answer the following questions.

Remember that these questions are a way of **proving** what the answer must be, so the marks are **all** for a correct method. An answer alone, or without proper working, scores no marks.

3.
Show that the equation $y^3 = 4y^2 + 20$ has a solution between 4 and 5.

Use a trial and improvement method to find this solution.

Give your answer correct to **two** decimal place.

You must show **ALL** your working.



4.
Use a trial and improvement method to find the square root of 1000.

Give your solution correct to the **nearest whole number**.

You must show **ALL** your working.



Trial and Improvement SOLUTIONS

Note: the method is the most important part of these questions, but since the numbers chosen by each student will be different, methods are **not** shown in full below. For a model solution, see the example given in the notes on the first page.

1.

The equation $x^3 - x = 20$ has a solution between 2 and 3.

Use a trial and improvement method to find this solution.

Give your answer correct to one decimal place.

You must show **ALL** your working.

(GCSE question, November 2005)

$$2^3 - 2 = 6 \text{ which is too low}$$

$$3^3 - 3 = 24 \text{ which is too high}$$

$$2.5^3 - 2.5 = 13.125 \text{ which is too low}$$

...

$$x = \mathbf{2.8} \text{ to 1 decimal place}$$

2.

The equation $x^3 + 2x = 65$ has a solution between 3 and 4.

Use a trial and improvement method to find this solution.

Give your solution correct to one decimal place.

You must show **ALL** your working.

(GCSE question, November 2005)

$$3^3 + 2 \times 3 = 33 \text{ which is too low}$$

$$4^3 + 2 \times 4 = 72 \text{ which is too high}$$

$$3.5^3 + 2 \times 3.5 = 49.875 \text{ which is too low}$$

...

$$x = \mathbf{3.9} \text{ to 1 decimal place}$$

3.

Show that the equation $y^3 = 4y^2 + 20$ has a solution between 4 and 5.

Use a trial and improvement method to find this solution.

Give your answer correct to **two** decimal place.

You must show **ALL** your working.

$$y^3 - 4y^2 = 20$$

$$4^3 - 4 \times 4^2 = 0 \text{ which is too low}$$

$$5^3 - 4 \times 5^2 = 25 \text{ which is too high}$$

Therefore there is a solution between 4 and 5

$$4.5^3 - 4 \times 4.5^2 = 10.125 \text{ which is too low}$$

...

$$x = \mathbf{4.85} \text{ to 2 decimal places}$$

4.

Use a trial and improvement method to find the square root of 1000.

Give your solution correct to the **nearest whole number**.

You must show **ALL** your working.

$$x^2 = 1000$$

$$10^2 = 100 \text{ which is too low}$$

$$50^2 = 2500 \text{ which is too high}$$

...

$$x = \mathbf{32} \text{ to the nearest whole number}$$