

## Trial and Improvement

### Method:

1) If necessary, rearrange the equation to give just a number on the right.

*Eg, if you have  $2x^2 = 5x - 4$ , write it as  $2x^2 - 5x = -4$ .*

2) Make a table for your 'Guess for  $x$ ', the 'Value of ...' and 'Too high / Too low'.

3) Pick a number to try.

*Even if you choose something that's way too big or small, it doesn't matter much.*

*Sometimes the question will tell you that  $x$  lies between, say, 2 and 3, so use those first.*

4) Work out the value of the left side of the equation, then write 'Too high' or 'Too low'.

5) Pick another number to try.

*If you already know that 4 is too high but 2 is too low, pick something in between.*

6) Continue until you know enough about the number, and write what you know.

*For instance, "3.55 is too high, and 3.5 is too low."*

7) Give your final answer, to the level of accuracy required.

*Remember you can't be sure the answer is 2.7 to 1 d.p. unless you know for certain that the answer is between 2.65 and 2.75.*

### Example:

Find the solution to  $x^3 = 5x + 200$  correct to 1 decimal place.

*My thinking...*

*I need just a number on the right*

$$x^3 = 5x + 200$$

$$x^3 - 5x = 200$$

*So I need 200 as the answer*

*Try an easy number first*

*10 must be too big, so try 1*

*Between 1 and 10, so try 5*

*Between 5 and 10, so try 7*

*Between 5 and 7, so try 6*

*Between 6 and 7, so try 6.5*

*Between 6 and 6.5, so try 6.3*

*Between 6 and 6.3, so try 6.2*

*Between 6 and 6.2, so try 6.1*

*Between 6.1 and 6.2, so try 6.15*

Guess for $x$	Value of $x^3 - 5x$	Too high / Too low
10	$10^3 - 5 \times 10 = 950$	Too high
1	$1^3 - 5 \times 1 = -4$	Too low
5	$5^3 - 5 \times 5 = 100$	Too low
7	$7^3 - 5 \times 7 = 308$	Too high
6	$6^3 - 5 \times 6 = 186$	Too low
6.5	$6.5^3 - 5 \times 6.5 = 242.125$	Too high
6.3	$6.3^3 - 5 \times 6.3 = 218.547$	Too high
6.2	$6.2^3 - 5 \times 6.2 = 207.328$	Too high
6.1	$6.1^3 - 5 \times 6.1 = 196.481$	Too low
6.15	$6.15^3 - 5 \times 6.15 = 201.8 \dots$	Too high

6.1 is too low, and 6.15 is too high,  
so  $x$  must be between 6.1 and 6.15.

Therefore  $x = 6.1$  to 1 decimal place.





## Trial and Improvement SOLUTIONS

Note: the method is the most important part of these questions, but since the numbers chosen by each student will be different, methods are **not** shown in full below. For a model solution, see the example given in the notes on the first page.

1.

The equation  $x^3 - x = 20$  has a solution between 2 and 3.

Use a trial and improvement method to find this solution.

Give your answer correct to one decimal place.

You must show **ALL** your working.

(GCSE question, November 2005)

$$\begin{aligned}2^3 - 2 &= 6 \text{ which is too low} \\3^3 - 3 &= 24 \text{ which is too high} \\2.5^3 - 2.5 &= 13.125 \text{ which is too low} \\&\dots \\x &= \mathbf{2.8} \text{ to 1 decimal place}\end{aligned}$$

2.

The equation  $x^3 + 2x = 65$  has a solution between 3 and 4.

Use a trial and improvement method to find this solution.

Give your solution correct to one decimal place.

You must show **ALL** your working.

(GCSE question, November 2005)

$$\begin{aligned}3^3 + 2 \times 3 &= 33 \text{ which is too low} \\4^3 + 2 \times 4 &= 72 \text{ which is too high} \\3.5^3 + 2 \times 3.5 &= 49.875 \text{ which is too low} \\&\dots \\x &= \mathbf{3.9} \text{ to 1 decimal place}\end{aligned}$$

3.

**Show that** the equation  $y^3 = 4y^2 + 20$  has a solution between 4 and 5.

Use a trial and improvement method to find this solution.

Give your answer correct to **two** decimal place.

You must show **ALL** your working.

$$\begin{aligned}y^3 - 4y^2 &= 20 \\4^3 - 4 \times 4^2 &= 0 \text{ which is too low} \\5^3 - 4 \times 5^2 &= 25 \text{ which is too high} \\&\mathbf{\text{Therefore there is a solution between 4 and 5}} \\4.5^3 - 4 \times 4.5^2 &= 10.125 \text{ which is too low} \\&\dots \\x &= \mathbf{4.85} \text{ to 2 decimal places}\end{aligned}$$

4.

Use a trial and improvement method to find the square root of 1000.

Give your solution correct to the **nearest whole number**.

You must show **ALL** your working.

$$\begin{aligned}x^2 &= 1000 \\10^2 &= 100 \text{ which is too low} \\50^2 &= 2500 \text{ which is too high} \\&\dots \\x &= \mathbf{32} \text{ to the nearest whole number}\end{aligned}$$