Simpson’s Rule – From tree-trunks to spheres

Simpson’s Rule for approximating volume:

This rule was quite tricky to develop, but it can be applied – with varying degrees of accuracy – to any three-dimensional solid.

\[ V = \frac{h(A_b + 4A_m + A_t)}{6} \]

where:
- \( V \) = volume
- \( h \) = height
- \( A_b \) = cross-sectional area of the base
- \( A_m \) = cross-sectional area of the middle
- \( A_t \) = cross-sectional area of the top

There are a number of shapes where Simpson’s rule gives the exact volume.

1) Show, by substituting into the formula the appropriate values, that Simpson’s rule gives the exact volume of a cylinder.

Hint:
The volume of a cylinder is given by: \( V = \pi r^2 h \) where \( r \) is the radius of the base and \( h \) is the height.

2) The volume of a cone is given by: \( V = \frac{1}{3} \pi r^2 h \). Show that Simpson’s rule also works for a cone.

Hint:
The radius of the circle produced by cutting midway down a cone will be half that of the circle at the base.

3) Does Simpson’s rule work for a sphere? The volume of a sphere is given by: \( V = \frac{4}{3} \pi r^3 \). Make a guess, then attempt to calculate the Simpson’s rule approximation to prove or disprove.

Hint:
The cross-sectional area of the ‘end’ of a sphere, like the pointed end of the cone, will be 0.