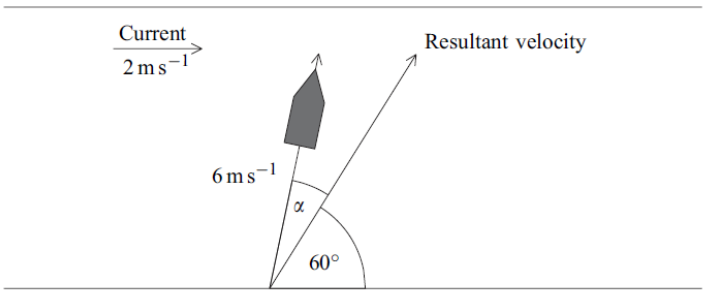
Practice examination questions for vectors

1. Jan 2005

6 A motor boat can travel at a speed of $6\,\mathrm{m\,s^{-1}}$ relative to the water. It is used to cross a river in which the current flows at $2\,\mathrm{m\,s^{-1}}$. The resultant velocity of the boat makes an angle of 60° to the river bank, as shown in the diagram.



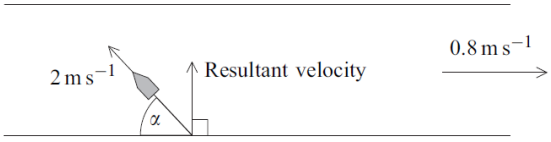
The angle between the direction in which the boat is travelling relative to the water and the resultant velocity is α.

(a) Show that $\alpha = 16.8°$, correct to three significant figures. *(4 marks)*

(b) Find the magnitude of the resultant velocity. *(3 marks)*

2.June2005

3 A boat can travel at a speed of $2\,\mathrm{m\,s^{-1}}$ in still water. The boat is to cross a river in which a current flows at a speed of $0.8\,\mathrm{m\,s^{-1}}$. The angle between the direction in which the boat is pointing and the bank is α. The boat travels so that the resultant velocity of the boat is perpendicular to the bank.



(a) Show that $\alpha = 66.4°$ correct to three significant figures. *(3 marks)*

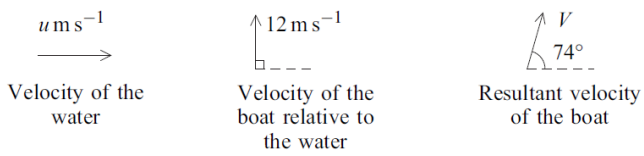
(b) (i) Find the magnitude of the resultant velocity of the boat. *(2 marks)*

(ii) The width of the river is 14 metres. Find the time that it takes for the boat to cross the river. *(2 marks)*

3.Jan2006

4 Water flows in a constant direction at a constant speed of $u\,\mathrm{m\,s^{-1}}$. A boat travels in the water at a speed of $12\,\mathrm{m\,s^{-1}}$ relative to the water.

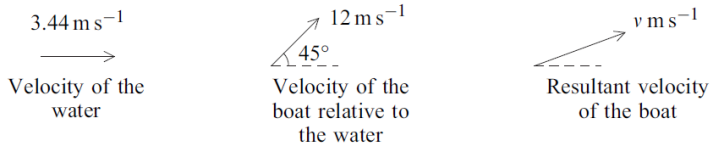
(a) The direction in which the boat travels relative to the water is perpendicular to the direction of motion of the water. The resultant velocity of the boat is $V\,\mathrm{m\,s^{-1}}$ at an angle of 74° to the direction of motion of the water, as shown in the diagram.



(i) Find V. *(2 marks)*

(ii) Show that $u = 3.44$, correct to three significant figures. *(3 marks)*

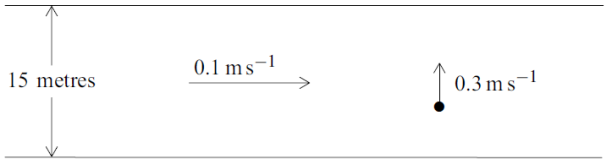
(b) The boat changes course so that it travels relative to the water at an angle of 45° to the direction of motion of the water. The resultant velocity of the boat is now of magnitude v m s^{-1}. The velocity of the water is unchanged, as shown in the diagram below.



Find the value of v. *(4 marks)*

4.Jan2007

5 A girl in a boat is rowing across a river, in which the water is flowing at 0.1 m s^{-1}. The velocity of the boat relative to the water is 0.3 m s^{-1} and is perpendicular to the bank, as shown in the diagram.



(a) Find the magnitude of the resultant velocity of the boat. *(2 marks)*

(b) Find the acute angle between the resultant velocity and the bank. *(3 marks)*

(c) The width of the river is 15 metres.

(i) Find the time that it takes the boat to cross the river. *(2 marks)*

(ii) Find the total distance travelled by the boat as it crosses the river. *(2 marks)*

5.June2007

5 An aeroplane flies in air that is moving due east at a speed of V m s^{-1}. The velocity of the aeroplane relative to the air is 150 m s^{-1} due north. The aeroplane actually travels on a bearing of 030°.

(a) Show that $V = 86.6$ m s^{-1}, correct to three significant figures. *(2 marks)*

(b) Find the magnitude of the resultant velocity of the aeroplane. *(3 marks)*

6.Jan2008

2 The velocity of a ship, relative to the water in which it is moving, is $8\,\mathrm{m\,s^{-1}}$ due north. The water is moving due east with a speed of $U\,\mathrm{m\,s^{-1}}$. The resultant velocity of the ship has magnitude $10\,\mathrm{m\,s^{-1}}$.

(a) Find U. *(2 marks)*

(b) Find the direction of the resultant velocity of the ship. Give your answer as a bearing to the nearest degree. *(2 marks)*

7.June2008

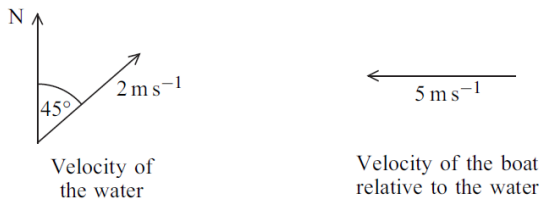
4 An aeroplane is travelling due north at $180\,\mathrm{m\,s^{-1}}$ relative to the air. The air is moving north-west at $50\,\mathrm{m\,s^{-1}}$.

(a) Find the magnitude of the resultant velocity of the aeroplane. *(4 marks)*

(b) Find the direction of the resultant velocity, giving your answer as a three-figure bearing to the nearest degree. *(4 marks)*

8.Jan2009

7 A boat is travelling in water that is moving north-east at a speed of $2\,\mathrm{m\,s^{-1}}$. The velocity of the boat relative to the water is $5\,\mathrm{m\,s^{-1}}$ due west.



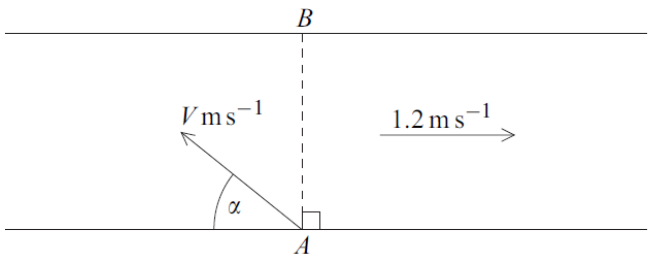
(a) Show that the magnitude of the resultant velocity of the boat is $3.85\,\mathrm{m\,s^{-1}}$, correct to three significant figures. *(4 marks)*

(b) Find the bearing on which the boat is travelling, giving your answer to the nearest degree. *(4 marks)*

9.June2009

4 A river has parallel banks which are 16 metres apart. The water in the river flows at $1.2\,\mathrm{m\,s^{-1}}$ parallel to the banks. A boat sets off from one bank at the point A and travels perpendicular to the bank so that it reaches the point B, which is directly opposite the point A. It takes the boat 10 seconds to cross the river.

The velocity of the boat relative to the water has magnitude $V\,\mathrm{m\,s^{-1}}$ and is at an angle α to the bank, as shown in the diagram.



(a) Show that the magnitude of the resultant velocity of the boat is $1.6\,\mathrm{m\,s^{-1}}$. *(1 mark)*

(b) Find V. *(3 marks)*

(c) Find α. *(2 marks)*

(d) State one modelling assumption that you needed to make about the boat. *(1 mark)*

10.June2010

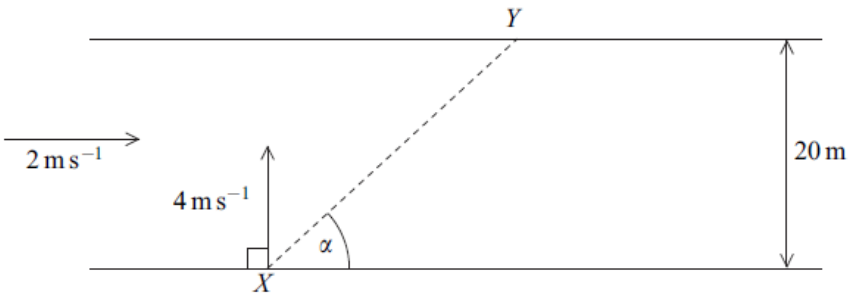
5 An aeroplane is travelling along a straight line between two points, A and B, which are at the same height. The air is moving due east at a speed of $30\,\mathrm{m\,s^{-1}}$. Relative to the air, the aeroplane travels due north at a speed of $100\,\mathrm{m\,s^{-1}}$.

(a) Find the magnitude of the resultant velocity of the aeroplane. *(3 marks)*

(b) Find the bearing on which the aeroplane is travelling, giving your answer to the nearest degree. *(2 marks)*

11.Jan2011

4 A canoe is paddled across a river which has a width of 20 metres. The canoe moves from the point X on one bank of the river to the point Y on the other bank, so that its path is a straight line at an angle α to the banks. The velocity of the canoe relative to the water is $4\,\mathrm{m\,s^{-1}}$ perpendicular to the banks. The water flows at $2\,\mathrm{m\,s^{-1}}$ parallel to the banks.



Model the canoe as a particle.

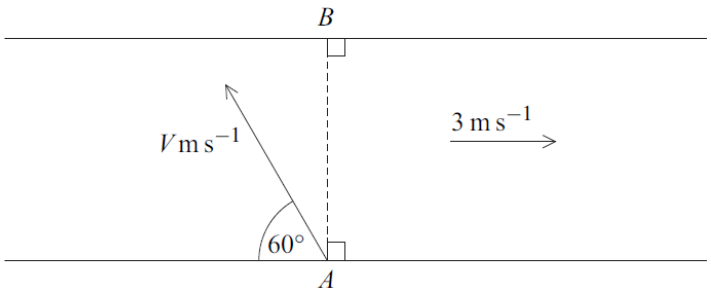
(a) Find the magnitude of the resultant velocity of the canoe. *(2 marks)*

(b) Find the angle α. *(2 marks)*

(c) Find the time that it takes for the canoe to travel from X to Y. *(2 marks)*

12. Jan2012

4 A small ferry is used to cross a river which has straight parallel banks that are 200 metres apart. The water in the river moves at a constant speed of $3\,\mathrm{m\,s^{-1}}$. The ferry travels from a point A on one bank to a point B directly opposite A on the other bank. The velocity of the ferry relative to the water is $V\,\mathrm{m\,s^{-1}}$ at an angle of $60°$ to the upstream bank, as shown in the diagram.



(a) Find V. *(3 marks)*

(b) Find the time that it takes for the ferry to cross the river, giving your answer to the nearest second. *(3 marks)*

13. June2012

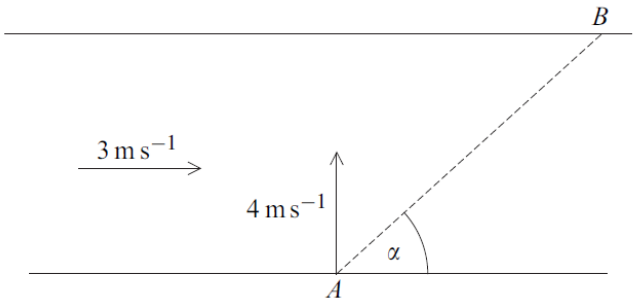
1 As a boat moves, it travels at $5\,\mathrm{m\,s^{-1}}$ due north, relative to the water. The water is moving due west at $2\,\mathrm{m\,s^{-1}}$.

(a) Find the magnitude of the resultant velocity of the boat. *(2 marks)*

(b) Find the bearing of the resultant velocity of the boat. *(3 marks)*

14. Jan2013

6 A river has straight parallel banks. The water in the river is flowing at a constant velocity of $3\,\mathrm{m\,s^{-1}}$ parallel to the banks. A boat crosses the river, from the point A to the point B, so that its path is at an angle α to the bank. The velocity of the boat relative to the water is $4\,\mathrm{m\,s^{-1}}$ perpendicular to the bank. The diagram shows these velocities and the path of the boat.



(a)	Show that $\alpha = 53.1°$, correct to three significant figures.	*(2 marks)*
(b)	The boat returns along the same straight path from B to A. Given that the speed of the boat relative to the water is still $4\,\text{m}\,\text{s}^{-1}$, find the magnitude of the resultant velocity of the boat on the return journey.	*(6 marks)*

Answers

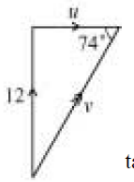
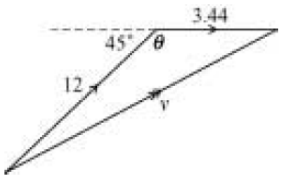
1. Jan 2005

6(a)	$\frac{\sin 60°}{6} = \frac{\sin \alpha}{2}$ $\alpha = 16.8°$	M1 A1 A1 A1	 4	Use of sine rule Correct LHS Correct RHS AG; correct α from correct working
(b)	$\frac{v}{\sin(180-60-16.8)} = \frac{6}{\sin 60°}$ $v = 6.74$ or $6.75\ \text{ms}^{-1}$	M1 A1 A1	 3	use of sine rule to find v Correct equation Correct v
	Total		7	

2.June2005

3(a)	$2\cos\alpha = 0.8$	M1		M1: Use of cos or sin to find α with 2 and 0.8
	$\cos\alpha = \frac{0.8}{2}$ **AG**	A1		A1: Correct equation
	$\alpha = \cos^{-1}\left(\frac{0.8}{2}\right) = 66.4°$	A1	3	A1: Correct α from correct working
(b)(i)	$v = \sqrt{2^2 - 0.8^2} = 1.83\ \text{ms}^{-1}$ or	M1		M1: Use of Pythagoras with 2 and 0.8 or trigonometry with angle from above
	$v = 2\sin 66.4° = 1.83\ \text{ms}^{-1}$	A1	2	A1: Correct velocity
(ii)	$t = \frac{14}{1.83} = 7.64$ s	M1		M1: Use of distance over speed from previous
	Allow 7.65 s	A1	2	A1: Correct time
	Total		**7**	

3.Jan2006

4(a)(i)	$v = 12.5$ (12.48)	M1		$\frac{12}{\sin \text{ or } \cos \text{ of } 74° \text{ or } 16°}$
		A1	2	or Pythagoras with 3.44 **SC** if Pythagoras used in circular solution M1 (1st use) A1 A1 each answer (3 max)
(ii)				
	$\tan 74° = \frac{12}{u}$	M1A1F		√incorrect v if used
	$u = 3.44$	A1	3	cao
(b)				Alt: 12 cos or sin 45° B1 Full method $v^2 = (12\sin 45)^2 + (3.44 + 12\sin 45)^2$ M1 (8.485) (11.925) A1 14.6 A1
	$\theta = 135°$	B1		
	$v^2 = 3.44^2 + 12^2 - 2\times 12\times 3.44\cos 135°$	M1 A1√		subs all correct
	$v = 14.6$	A1√	4	√ incorrect subtraction $\rightarrow 135°$
	Total		**9**	

4.Jan2007

5(a)	$v = \sqrt{0.3^2 + 0.1^2} = \sqrt{0.1} = 0.316\ \text{ms}^{-1}$	M1A1	2	Use of Pythagoras to find v. Correct v
(b)	$\alpha = \tan^{-1}\left(\frac{0.3}{0.1}\right) = 71.6°$	M1A1		Use of trigonometry with reasonable choice of sides to find α. Correct expression
		A1	3	Correct angle **CAO**
(i)	$t = \frac{15}{0.3} = 50\text{s}$	M1 A1	2	Use of s/v to find t with s and t consistent Correct t
(ii)	$s = 50\times\sqrt{0.1} = 15.8\text{m}$	M1A1	2	Use of their t in $t\times v$ to find s or the use of trigonometry. Correct distance **CAO**
	Total		**9**	

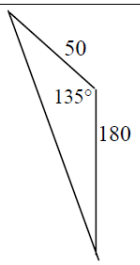
5.June2007

5(a)	$V = 150\tan 30°$	M1		Using trigonometry (usually tan or sine rule) to find V
	$= 86.6\ \text{ms}^{-1}$	A1	2	AG Correct answer from correct working (Division by 2 only acceptable if sin30° or cos60° seen)
	OR $\frac{V}{\sin 30°} = \frac{150}{\sin 60°}$ AG $V = 86.6\ \text{ms}^{-1}$			
(b)	$\frac{150}{v} = \cos 30°$	M1		Using trigonometry or Pythagoras to find v
		A1		Correct expression
	$v = \frac{150}{\cos 30°} = 173\ \text{ms}^{-1}$ (to 3sf)	A1	3	Correct answer
	Total		**5**	

6.Jan2008

2.(a)	$U = \sqrt{10^2 - 8^2} = 6$	M1		Expression/equation for U based on a right angled triangle.
		A1	2	Correct U. Note $10^2 + 8^2$ gives M1A0
(b)	$\cos\theta = \frac{8}{10}$	M1		Use of trigonometry to find angle. Allow $\begin{cases} \tan\theta = \frac{8}{6} \text{ or } \frac{6}{8} \\ \sin/\cos\theta = \frac{8}{10} \text{ or } \frac{6}{10} \end{cases}$
	$\theta = 037°$	A1	2	Correct angle. Accept 36.9° etc. Note 143° gives M1A0
	Total		**4**	

7.June2008

4(a)		B1		Diagram (may be implied) The shape is sufficient, but 50 and 180 must be seen. The 135° may be replaced by 45° or be absent.
	$v^2 = 50^2 + 180^2 - 2\times 50\times 180\cos 135°$	M1		Use of cosine rule with 50, 180 and either 135° or 45°
		A1		Correct equation
	$v = 218\ \text{ms}^{-1}$	A1	4	Correct result for v

	ALTERNATIVE SOLUTION $180+50\cos 45° = 215.36$ $50\sin 45° = 35.36$ $v=\sqrt{215.36^2+35.36^2} = 218 \text{ ms}^{-1}$	**OR** (M1) (A1) (B1) (A1)		Calculation of northerly component with 180, 50 and 45° Correct component Correct westerly component Correct result for v Accept AWRT 218
(b)	$\frac{\sin\alpha}{50}=\frac{\sin 135°}{218.24}$ $\alpha = 9.3°$ Bearing is 351°	M1 A1F A1 A1	4	Use of the sine rule with 50, 135° or 45° and AWRT 218 or candidate's answer to part (a) to at least 3SF. Correct equation (must have 135° not 45°). Correct angle Three figure bearing
	ALTERNATIVE SOLUTION $\tan\alpha = \frac{35.36}{215.36}$ $\alpha = 9.3°$ Bearing is 351°	(M1) (A1) (A1) (A1)		as for sine rule. Use of trig to find angle Correct equation Correct angle Three figure bearing
	Total		**8**	

8.Jan2009

7(b)	$\frac{\sin\theta}{2}=\frac{\sin 45°}{3.855}$ $\theta = 21.5°$ Bearing = 270 +21.5 = 292° **OR** $\frac{\sin\theta}{5}=\frac{\sin 45°}{3.855}$ $\theta = 113°$ Bearing = 360 – (113.3 – 45) = 292°	M1 A1 A1 A1 (M1) (A1) (A1) (A1)		M1: Use of sine rule, with 2, 3.855 or 3.85 or awrt 3.85 and any angle. A1: Correct expression A1: Correct angle. Awrt 21° or 22° A1: Correct bearing. Do not penalise candidates who include decimals. Accept 291° M1: Use of sine rule, with 5, 3.855 or 3.85 or awrt 3.85 and any angle. A1: Correct expression A1: Correct angle. Allow awrt 113° or 114°. Also allow awrt 66° or 67°. A1: Correct bearing. Do not penalise candidates who include decimals. Accept 291°
	OR $\tan\theta = \frac{2\cos 45°}{5-2\cos 45°}$ $\theta = 21.5°$ Bearing = 270 +21.5 = 292°	(M1) (A1) (A1) (A1)		M1: Consideration of perpendicular components using values from part (a). A1: Correct expression A1: Correct positive angle. Awrt 21° or 22° Also allow method leading to awrt 68° or 69° A1: Correct bearing. Do not penalise candidates who include decimals. Accept 291°
	OR $\cos\theta = \frac{3.855^2+5^2-2^2}{2\times 5\times 3.855}$ $\theta = 21.5°$ Bearing = 270 +21.5 = 292°	(M1) (A1) (A1) (A1)	4	M1: Use of cosine rule, with 2, 3.855 or 3.85 or awrt 3.85 and 5. A1: Correct expression A1: Correct angle. Awrt 21° or 22° A1: Correct bearing. Do not penalise candidates who include decimals. Accept 291°
	Total		**8**	

9.June2009

4(a)	$v=\frac{16}{10}=1.6\ \text{ms}^{-1}$ AG	B1	1	B1: Printed result obtained from correct division. Must see 16 divided by 10.
(b)	$V^2=1.6^2+1.2^2$ $V=\sqrt{4}=2\ \text{ms}^{-1}$	M1A1 A1	3	M1: Equation to find V based on Pythagoras. Must involve addition of the squares of two components. A1: Correct equation A1: Correct V

10.June2010

Q	Solution	Marks	Total	Comments
5(a)	$(v=)\sqrt{30^2+100^2}$ $=104.4$ $=104\ \text{ms}^{-1}$ (to 3SF)	M1A1 A1	3	M1: Equation or expression to find v based on Pythagoras. Must be +. For example: 10 900 oe scores M1. A1: Correct equation or expression, with square root. A1: Correct v. Accept 104.4.
(b)	$\theta=\tan^{-1}\left(\frac{30}{100}\right)$ or $\tan^{-1}\left(\frac{100}{30}\right)$ $=017°$ **OR** $\theta=\sin^{-1}\left(\frac{30}{104.4}\right)$ or $\sin^{-1}\left(\frac{100}{104.4}\right)$ $=017°$ **OR** $\theta=\cos^{-1}\left(\frac{100}{104.4}\right)$ or $\cos^{-1}\left(\frac{30}{104.4}\right)$ $=017°$	M1 A1F (M1) (A1F) (M1) (A1F)	2	M1: Trigonometric equation to find α. A1F: Correct α. Follow through incorrect answer from (b). Note: Subtracting 17 etc from other values such as 360 or 90 can not be ignored and will score M1. Accept 16 or 17 or 16.6 or 16.7 or 16.8. Also accept all of these with a zero in front, eg 016.
	Total		**5**	

11.Jan2011

4(a)	$(V=)\sqrt{2^2+4^2}=\sqrt{20}$ $=2\sqrt{5}$ $=4.47\ \text{ms}^{-1}$	M1A1	2	M1: Equation or expression to find V based on Pythagoras. Must be +. A1: Correct velocity. Accept $\sqrt{20}$, $2\sqrt{5}$, 4.47 or more accurate answer from 4.472135…
(b)	$\tan\alpha=\frac{4}{2}$ $\alpha=63.4°$ **OR** $\sin\alpha=\frac{4}{2\sqrt{5}}$ or $\frac{4}{4.47}$ $\alpha=63.4°$ **OR** $\cos\alpha=\frac{2}{2\sqrt{5}}$ or $\frac{2}{4.47}$ $\alpha=63.4°$	M1 A1F (M1) (A1F) (M1) (A1F)	2	M1: Trigonometric equation to find angle. Can be any of those as shown. For tan, fraction can be inverted. For sin, 2 can be used instead of 4. For cos, 4 can be used instead of 2. Can use their V from part (a). A1F: Correct angle. Accept 63 or AWRT 63.4 or 63.5.

(c)	$t=\frac{20}{4}=5$ seconds **OR** $t=\frac{\sqrt{500}}{\sqrt{20}}=5$ seconds	M1 A1	2	M1: Division of distance by speed (for example, $\frac{10}{2}$ or $\frac{20}{4}$ or $\frac{\sqrt{500}}{\sqrt{20}}$ or $\frac{22.4}{4.47}$) Do not award M1 if distance and speed don't correspond (eg $\frac{10}{4}$ or $\frac{20}{2}$ or $\frac{20}{4.47}$) A1: Correct time CAO. Accept 5.00 or 5.0
	Total		**6**	

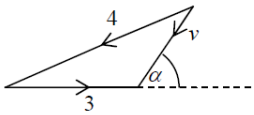
12. Jan2012

4(a)	$V\sin 30° = 3$ $V=\frac{3}{\sin 30°}=6$	M1A1 A1	3	M1: Resolving parallel to the bank. Accept $V\cos 30° = 3$. A1: Correct equation. A1: Correct V.
(b)	$t=\frac{200}{6\sin 60°}=38$ (seconds) OR $h=\frac{200}{\sin 60°}=230.94$ $t=\frac{230.94}{6}=38$ (seconds) OR resultant velocity $=\sqrt{27}$ $t=\frac{200}{\sqrt{27}}=38$ (seconds)	M1 A1F A1F (M1) (A1F) (A1F) (M1) (A1F) (A1F)	3	M1: 200 divided by $V\sin 60°$ or $V\sin 30°$ or equivalent with their value for V from (a). A1F: Correct expression for t. A1F: Correct value for t to nearest second. Follow through their answer to part (a) M1:Distance divided by corresponding velocity. A1F: Correct expression for t A1F: Correct value for t to nearest second. Follow through their answer to part (a) Do not accept 38.5
	Total		**6**	

13. June2012

1(a)	$(V^2=)5^2+2^2$ $(V=)5.39$ ms^{-1}	M1 A1	2	M1: Correct expression for V or V^2. A1: Correct speed. Accept 5.38 or $\sqrt{29}$ or AWRT 5.39 or 5.38. Do not accept 5.4
(b)	$\tan\theta=\frac{2}{5}$ $\theta=21.8°$ Bearing $=360-21.8=338°$ (to 3sf) Or $\tan\theta=\frac{5}{2}$ $\theta=68.2°$ Bearing $=270+68.2=338°$ (to 3sf)	M1 A1 A1	3	M1: Accept $\tan\theta=\frac{2}{5}$ or $\frac{5}{2}$ or $\sin\theta$ or $\cos\theta=\frac{2}{V}$ or $\frac{5}{V}$ with their V from part (a). Note: With use of sine or cosine rules, must get to $\sin\theta$ or $\cos\theta=\frac{2}{V}$ or $\frac{5}{V}$ OE A1: Correct angle. Accept AWRT 22° or 68° from correct working. A1: Correct bearing. Accept AWRT 338. Note that incorrect diagrams should not be penalised if "correct" working shown.
	Total		**5**	

14. Jan2013

6(a)	$\tan\alpha = \frac{4}{3}$ or $\cos\alpha = \frac{3}{5}$ or $\sin\alpha = \frac{4}{5}$ $\alpha = 53.1°$ AG	M1 A1	2	M1: Trig equation to find the angle with: cos with 3 or 4 in the numerator and 5 in denominator sin with 3 or 4 in the numerator and 5 in denominator tan with 3 and 4 in any position A1: Correct angle from correct working. (Allow 90 – 36.9 = 53.1°). Final answer must be 53.1 Note, for example, $\tan^{-1}\frac{4}{3} = 53.1$ scores M1A1
(b)	 $4^2 = 3^2 + v^2 - 2\times3\times v\times\cos(180-53.1...)$ $v^2 + 3.6v - 7 = 0$ $v = 1.40$ or $v = -5.00$ $v = 1.40$ m s^{-1} OR	B1 M1A1 A1 dM1 A1	6	(Note: diagram not needed for the award of marks) B1: For seeing 180 – 53.1 (= 126.9). M1: Using cosine rule with 3, 4, v and any angle. Must see v **and** v^2. A1: Correct equation. A1: Correct simplified quadratic. dM1: Solving the quadratic. A1: Selecting positive root. (Can be implied.) Accept 1.4 or 1.39
	$\frac{\sin(180-53.13)}{4} = \frac{\sin\theta}{3}$ $\theta = 36.87°$ $180 - 36.87 - 126.87 = 16.26°$ $\frac{v}{\sin 16.26°} = \frac{4}{\sin(180-53.13)}$ OR $\frac{3}{\sin 36.87°}$ $v = 1.40$ m s^{-1}	(B1) (M1A1) (A1) (dM1) (A1)	(6)	B1: For seeing 180 – 53.1 (= 126.9). M1: Using sine rule with 3, 4 and 126.9°. A1: Correct equation. A1: For finding 16.26. Accept 16.3 or 16.2 or 16.26... . dM1: Second application of sine rule with v and 3 or 4 with at least one correct angle. A1: Correct velocity. Accept 1.4 or 1.39. Note: the result below can be proved. $v = 4\sin\alpha - 3\cos\alpha$ SC4: seeing $4\sin\alpha - 3\cos\alpha$ with incorrect answer. SC6: seeing $4\sin\alpha - 3\cos\alpha$ with answer as 1.4 or 1.39.
	Total		**8**	