Practice examination questions for matrices

1. June 2005

1 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

(a) Calculate the matrices:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $\mathbf{AB}$. *(2 marks)*

(b) Show that $\mathbf{A} + \mathbf{B} - \mathbf{AB} = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

2. Jan 2006

7 (a) The transformation T is defined by the matrix **A**, where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(i) Describe the transformation T geometrically. *(2 marks)*

(ii) Calculate the matrix product $\mathbf{A}^2$. *(2 marks)*

(iii) Explain briefly why the transformation T followed by T is the identity transformation. *(1 mark)*

(b) The matrix **B** is defined by

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(i) Calculate $\mathbf{B}^2 - \mathbf{A}^2$. *(3 marks)*

(ii) Calculate $(\mathbf{B} + \mathbf{A})(\mathbf{B} - \mathbf{A})$. *(3 marks)*

3. June 2006

5 The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(a) Find the matrix:

(i) $\mathbf{M}^2$; *(3 marks)*

(ii) $\mathbf{M}^4$. *(1 mark)*

(b) Describe fully the geometrical transformation represented by **M**. *(2 marks)*

(c) Find the matrix $\mathbf{M}^{2006}$. *(3 marks)*

4. Jan 2007

2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $\mathbf{BA}$. *(3 marks)*

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\mathbf{A}$; *(2 marks)*

(ii) $\mathbf{B}$; *(2 marks)*

(iii) $\mathbf{BA}$. *(2 marks)*

5. June 2007

1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The matrix $\mathbf{M} = \mathbf{A} - 2\mathbf{B}$.

(a) Show that $\mathbf{M} = n\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, where n is a positive integer. *(2 marks)*

(b) The matrix **M** represents a combination of an enlargement of scale factor p and a reflection in a line L. State the value of p and write down the equation of L. *(2 marks)*

(c) Show that

$$\mathbf{M}^2 = q\mathbf{I}$$

where q is an integer and **I** is the 2×2 identity matrix. *(2 marks)*

6. Jan 2008

6 The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} \sqrt{3} & 3 \\ 3 & -\sqrt{3} \end{bmatrix}$$

(a) (i) Show that

$$\mathbf{M}^2 = p\mathbf{I}$$

where p is an integer and **I** is the 2×2 identity matrix. *(3 marks)*

(ii) Show that the matrix **M** can be written in the form

$$q\begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ \sin 60^\circ & -\cos 60^\circ \end{bmatrix}$$

where q is a real number. Give the value of q in surd form. *(3 marks)*

(b) The matrix **M** represents a combination of an enlargement and a reflection.

Find:

(i) the scale factor of the enlargement; *(1 mark)*

(ii) the equation of the mirror line of the reflection. *(1 mark)*

(c) Describe fully the geometrical transformation represented by $\mathbf{M}^4$. *(2 marks)*

7. June 2008

6 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

(a) Calculate the matrix **AB**. *(2 marks)*

(b) Show that $\mathbf{A}^2$ is of the form $k\mathbf{I}$, where k is an integer and **I** is the 2×2 identity matrix. *(2 marks)*

(c) Show that $(\mathbf{AB})^2 \neq \mathbf{A}^2\mathbf{B}^2$. *(3 marks)*

8. Jan 2009

5 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} k & k \\ k & -k \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -k & k \\ k & k \end{bmatrix}$$

where k is a constant.

(a) Find, in terms of k:

(i) $\mathbf{A} + \mathbf{B}$; *(1 mark)*

(ii) $\mathbf{A}^2$. *(2 marks)*

(b) Show that $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + \mathbf{B}^2$. *(4 marks)*

(c) It is now given that $k = 1$.

(i) Describe the geometrical transformation represented by the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) The matrix **A** represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(3 marks)*

9. Jan 2010

4 It is given that

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}$$

and that **I** is the 2×2 identity matrix.

(a) Show that $(\mathbf{A} - \mathbf{I})^2 = k\mathbf{I}$ for some integer k. *(3 marks)*

(b) Given further that

$$\mathbf{B} = \begin{bmatrix} 1 & 3 \\ p & 1 \end{bmatrix}$$

find the integer p such that

$$(\mathbf{A} - \mathbf{B})^2 = (\mathbf{A} - \mathbf{I})^2$$ *(4 marks)*

10.Jan 2011

3 (a) Write down the 2×2 matrix corresponding to each of the following transformations:

(i) a rotation about the origin through 90° clockwise; *(1 mark)*

(ii) a rotation about the origin through 180°. *(1 mark)*

(b) The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}$$

(i) Calculate the matrix $\mathbf{AB}$. *(2 marks)*

(ii) Show that $(\mathbf{A} + \mathbf{B})^2 = k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, for some integer k. *(3 marks)*

(c) Describe the single geometrical transformation, or combination of two geometrical transformations, represented by each of the following matrices:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $(\mathbf{A} + \mathbf{B})^2$; *(2 marks)*

(iii) $(\mathbf{A} + \mathbf{B})^4$. *(2 marks)*

Answers

1. June 2005

1(a)(i)	$\mathbf{A}+\mathbf{B}=\begin{bmatrix}3 & 6\\ 6 & 3\end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct
(ii)	$\mathbf{AB}=\begin{bmatrix}8 & 6\\ 6 & 8\end{bmatrix}$	M1A1	2	Ditto
(b)	$\mathbf{A}+\mathbf{B}-\mathbf{AB}=\begin{bmatrix}-5 & 0\\ 0 & -5\end{bmatrix}$	B1F		ft wrong answers in (a)
	$\ldots=-5\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	B1	2	
	Total		**6**	

2. Jan 2006

7(a)(i)	Reflection ...	M1		
	... in $y=-x$	A1	2	OE
(ii)	$\mathbf{A}^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	M1A1	2	M1A0 for three correct entries
(iii)	$\mathbf{A}^2=\mathbf{I}$ or geometrical reasoning	E1	1	
(b)(i)	$\mathbf{B}^2=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$	M1A1		M1A0 for three correct entries
	$\mathbf{B}^2-\mathbf{A}^2=\begin{bmatrix}0 & 2\\ 0 & 0\end{bmatrix}$	A1√	3	ft errors, dependent on both M marks
(ii)	$(\mathbf{B}+\mathbf{A})(\mathbf{B}-\mathbf{A})=\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}$	B1		
	$\ldots=\begin{bmatrix}1 & 2\\ 0 & -1\end{bmatrix}$	M1 A1√	3	ft one error; M1A0 for three correct (ft) entries
	Total		**11**	

3. June 2006

5(a)(i)	$\mathbf{M}^2=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$	M1 A2,1	3	M1 if 2 entries correct M1A1 if 3 entries correct
(ii)	$\mathbf{M}^4=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$	B1√	1	ft error in $\mathbf{M}^2$ provided no surds in $\mathbf{M}^2$
(b)	Rotation (about the origin)	M1		
	... through 45° clockwise	A1	2	
(c)	Awareness of $\mathbf{M}^8=\mathbf{I}$	M1		OE; NMS 2/3
	$\mathbf{M}^{2006}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$	m1 A1√	3	complete valid method ft error in $\mathbf{M}^2$ as above
	Total		**9**	

4. Jan 2007

2(a)(i)	$\mathbf{A}+\mathbf{B}=\begin{bmatrix}\sqrt{3} & 0\\ 1 & 0\end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct; Condone $\frac{2\sqrt{3}}{2}$ for $\sqrt{3}$
(ii)	$\mathbf{BA}=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$	B3,2,1	3	Deduct one for each error; **SC** B2,1 for **AB**
(b)(i)	Rotation 30° anticlockwise (abt O)	M1A1	2	M1 for rotation
(ii)	Reflection in $y=(\tan 15°)x$	M1A1	2	M1 for reflection
(iii)	Reflection in x-axis	B2F	2	1/2 for reflection in y-axis ft (M1A1) only for the **SC**
	Alt: Answer to (i) followed by answer to (ii)	M1A1F	(2)	M1A0 if in wrong order or if order not made clear
	Total		**11**	

5. June 2007

1(a)	$\mathbf{M}=\begin{bmatrix}0 & -3\\ -3 & 0\end{bmatrix}$	B2,1	2	B1 if subtracted the wrong way round
(b)	$p=3$	B1F		ft after B1 in (a)
	L is $y=-x$	B1	2	Allow $p=-3$, L is $y=x$
(c)	$\mathbf{M}^2=\begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix}$	B1F		Or by geometrical reasoning; ft as before
	$\ldots=9\mathbf{I}$	B1F	2	ft as before
	Total		**6**	

6. Jan 2008

6(a)(i)	$\mathbf{M}^2=\begin{bmatrix}12 & 0\\ 0 & 12\end{bmatrix}$	M1A1		M1 if zeroes appear in the right places
	$=12\mathbf{I}$	A1F	3	ft provided of right form
(ii)	$q\cos 60°=\frac{1}{2}q=\sqrt{3}\Rightarrow q=2\sqrt{3}$	M1A1		OE **SC** $q=2\sqrt{3}$ NMS 1/3
	Other entries verified	E1	3	surd for sin 60° needed
(b)(i)	SF $=q=2\sqrt{3}$	B1F	1	ft wrong value for q
(ii)	Equation is $y=x\tan 30°$	B1	1	
(c)	$\mathbf{M}^4=144\mathbf{I}$	B1F		PI; ft wrong value in (a)(i)
	$\mathbf{M}^4$ gives enlargement SF 144	B1F	2	ft if c's $\mathbf{M}^4=k\mathbf{I}$
	Total		**10**	

7. June 2008

6(a)	$\mathbf{AB}=\begin{bmatrix}0 & -4\\ 4 & 0\end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct
(b)	$\mathbf{A}^2=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$	B1		
	$\ldots=4\mathbf{I}$	B1	2	

(c)	$(\mathbf{AB})^2 = -16\mathbf{I}$ $\mathbf{B}^2 = 4\mathbf{I}$ so $\mathbf{A}^2\,\mathbf{B}^2 = 16\mathbf{I}$ (hence result)	B1 B1 B1	 3	 PI Condone absence of conclusion
	Total		**7**	

8. Jan 2009

5(a)(i)	$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 0 & 2k \\ 2k & 0 \end{bmatrix}$	B1	1	
(ii)	$\mathbf{A}^2 = \begin{bmatrix} 2k^2 & 0 \\ 0 & 2k^2 \end{bmatrix}$	B2,1	2	B1 if three entries correct
(b)	$(\mathbf{A} + \mathbf{B})^2 = = \begin{bmatrix} 4k^2 & 0 \\ 0 & 4k^2 \end{bmatrix}$ $\mathbf{B}^2 = \mathbf{A}^2$, hence result	B2,1 B1B1	 4	B1 if three entries correct
(c)(i)	$\mathbf{A}^2$ is an enlargement (centre O) with SF 2	M1 A1	 2	 Condone $2k^2$
(ii)	Scale factor is now $\sqrt{2}$ Mirror line is $y = x\tan 22\frac{1}{2}°$	B1 M1A1	 3	Condone $\sqrt{2}k$
	Total		**12**	

9. Jan 2010

4(a)	$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	B1		stated or used at any stage
	Attempt at $(\mathbf{A} - \mathbf{I})^2$	M1		with at most one numerical error
	$(\mathbf{A} - \mathbf{I})^2 = \begin{bmatrix} 0 & 4 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 3 & 0 \end{bmatrix} = 12\mathbf{I}$	A1	3	
(b)	$\mathbf{A} - \mathbf{B} = \begin{bmatrix} 0 & 1 \\ 3-p & 0 \end{bmatrix}$	B1		
	$(\mathbf{A} - \mathbf{B})^2 = \begin{bmatrix} 3-p & 0 \\ 0 & 3-p \end{bmatrix}$	M1A1		M1 A0 if 3 entries correct
	$\ldots = (\mathbf{A} - \mathbf{I})^2$ for $p = -9$	A1F	4	ft wrong value of k
	Total		**7**	

10.Jan 2011

3(a)(i)	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	**B1**	**1**	
(ii)	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	**B1**	**1**	
(b)(i)	$\mathbf{AB} = \begin{bmatrix} -20 & 14 \\ 14 & -10 \end{bmatrix}$	**M1A1**	**2**	**M1A0 if 3 entries correct**

(ii)	$\mathbf{A}+\mathbf{B}=\begin{bmatrix}0 & 5\\ -5 & 0\end{bmatrix}$	B1		
	$(\mathbf{A}+\mathbf{B})^2=\begin{bmatrix}-25 & 0\\ 0 & -25\end{bmatrix}$	B1		
	$\ldots=-25\mathbf{I}$	B1F	3	ft if c's $(\mathbf{A}+\mathbf{B})^2$ is of the form $k\mathbf{I}$
(c)(i)	Rot'n 90° clockwise, enlargem't SF 5	B2, 1	2	OE
(ii)	Rotation 180°, enlargement SF 25	B2, 1F	2	Accept 'enlargement SF −25'; ft wrong value of k
(iii)	Enlargement SF 625	B2, 1F	2	B1 for pure enlargement; ft ditto
	Total		**13**	