Practice examination questions for trigonometry

1. Jan 2005

6 The angle x radians satisfies the equation

$$\cos\left(2x+\frac{\pi}{6}\right)=\frac{1}{\sqrt{2}}$$

(a) Find the general solution of this equation, giving the roots as exact values in terms of π. *(6 marks)*

(b) Find the **number** of roots of the equation which lie between 0 and 2π. *(2 marks)*

2. June 2005

5 Find the general solutions of the following equations, giving your answers in terms of π:

(a) $\tan 3x=\sqrt{3}$; *(3 marks)*

(b) $\tan\left(3x-\frac{\pi}{3}\right)=-\sqrt{3}$. *(4 marks)*

3. Jan 2006

3 Find the general solution, in **degrees**, for the equation

$$\sin(4x+10^\circ)=\sin 50^\circ$$ *(5 marks)*

4. June 2006

4 Find, in **radians**, the general solution of the equation

$$\cos 3x=\frac{\sqrt{3}}{2}$$

giving your answers in terms of π. *(5 marks)*

5. Jan 2007

7 The function f is defined for all real numbers by

$$\mathrm{f}(x)=\sin\left(x+\frac{\pi}{6}\right)$$

(a) Find the general solution of the equation $\mathrm{f}(x)=0$. *(3 marks)*

(b) The quadratic function g is defined for all real numbers by

$$\mathrm{g}(x)=\frac{1}{2}+\frac{\sqrt{3}}{2}x-\frac{1}{4}x^2$$

It can be shown that $\mathrm{g}(x)$ gives a good approximation to $\mathrm{f}(x)$ for small values of x.

(i) Show that $\mathrm{g}(0.05)$ and $\mathrm{f}(0.05)$ are identical when rounded to four decimal places. *(2 marks)*

(ii) A chord joins the points on the curve $y=\mathrm{g}(x)$ for which $x=0$ and $x=h$. Find an expression in terms of h for the gradient of this chord. *(2 marks)*

(iii) Using your answer to part (b)(ii), find the value of $\mathrm{g}'(0)$. *(1 mark)*

6. June 2007

6 Find the general solution of the equation

$$\sin\left(2x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π. *(6 marks)*

7. Jan 2008

3 Find the general solution of the equation

$$\tan 4\left(x - \frac{\pi}{8}\right) = 1$$

giving your answer in terms of π. *(5 marks)*

8. June 2008

5 (a) Find, in **radians**, the general solution of the equation

$$\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

giving your answer in terms of π. *(5 marks)*

(b) Hence find the smallest **positive** value of x which satisfies this equation. *(2 marks)*

9. Jan 2009

3 Find the general solution of the equation

$$\tan\left(\frac{\pi}{2} - 3x\right) = \sqrt{3}$$

(5 marks)

10.June 2009

5 (a) Find the general solution of the equation

$$\cos(3x - \pi) = \frac{1}{2}$$

giving your answer in terms of π. *(6 marks)*

(b) From your general solution, find all the solutions of the equation which lie between 10π and 11π. *(3 marks)*

11.Jan 2010

3 Find the general solution of the equation

$$\sin\left(4x + \frac{\pi}{4}\right) = 1$$ *(4 marks)*

12.June 2010

3 Find the general solution, in degrees, of the equation

$$\cos(5x - 20°) = \cos 40°$$ *(5 marks)*

13.Jan 2011

4 Find the general solution of the equation

$$\sin\left(4x - \frac{2\pi}{3}\right) = -\frac{1}{2}$$

giving your answer in terms of π. *(6 marks)*

14.June 2011

5 (a) Find the general solution of the equation

$$\cos\left(3x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π. *(5 marks)*

(b) Use your general solution to find the smallest solution of this equation which is greater than 5π. *(2 marks)*

15.Jan 2012

6 Find the general solution of each of the following equations:

(a) $\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$; *(4 marks)*

(b) $\tan^2\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{3}$. *(3 marks)*

16.June 2012

4 Find the general solution, in degrees, of the equation

$$\sin\left(70° - \frac{2}{3}x\right) = \cos 20°$$

(6 marks)

17.Jan 2013

3 (a) Find the general solution of the equation

$$\sin\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π. *(6 marks)*

(b) Use your general solution to find the exact value of the greatest solution of this equation which is less than 6π. *(2 marks)*

Answers

1. Jan 2005

6(a)	Attempt at $\cos^{-1}\frac{1}{\sqrt{2}}$	M1		Allow degrees or decimals
	$\frac{\pi}{4}$ appearing in solution	A1		Must be exact
	Introduction of $\pm$	M1		
	Introduction of $\ldots + 2n\pi$	M1		Or $360n$
	Making x the subject	M1		From $2x + \frac{\pi}{6} = kn\pi + \alpha (\text{or} \pm \alpha)$
	$x = -\frac{\pi}{12} \pm \frac{\pi}{8} + n\pi$	A1	6	OE
(b)	No of roots is 4	M1A1√	2	M1 e.g. for answer consistent with c's general solution
	Total		**8**	

2. June 2005

5(a)	Use of $\tan\frac{\pi}{3} = \sqrt{3}$	M1		Degrees used - 1-mark penalty
	GS is $\frac{1}{3}(\frac{\pi}{3} + n\pi)$	m1A1	3	m1A0 for $\frac{\pi}{9} + n\pi$ or for correct answer plus extra solutions
(b)	One solution is 0	M1A1		OE
	GS is $\frac{1}{3}n\pi$	m1A1F	4	m1A0 for $n\pi$ or for correct answer plus extra solns; ft wrong first solution
	Total		**7**	

3. Jan 2006

3	One solution is $x = 10°$	B1		PI by general formula
	Use of $\sin 130° = \sin 50°$	M1		OE
	Second solution is $x = 30°$	A1		OE
	Introduction of $90n°$, or $360n°$ or $180n°$	M1		Or $\pi n/2$ or $2\pi n$ or πn
	GS $(10 + 90n)°, (30 + 90n)°$	A1√	5	OE; ft one numerical error or omission of 2nd soln
	Total		**5**	

4. June 2006

4	$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ stated or used	B1		Condone decimals and/or degrees until final mark
	Appropriate use of $\pm$	B1		
	Introduction of $2n\pi$	M1		
	Division by 3	M1		Of $\alpha + kn\pi$ or $\pm\alpha + kn\pi$
	$x = \pm\frac{\pi}{18} + \frac{2}{3}n\pi$	A1	5	
	Total		**5**	

5. Jan 2007

7(a)	Particular solution, eg $-\frac{\pi}{6}$ or $\frac{5\pi}{6}$	B1		Degrees or decimals penalised in 3rd mark only
	Introduction of $n\pi$ or $2n\pi$	M1		
	GS $x = -\frac{\pi}{6} + n\pi$	A1F	3	OE(accept unsimplified); ft incorrect first solution

(b)(i)	f (0.05) ≈ 0.542 66 g (0.05) ≈ 0.542 68	B1 B1	2	either value AWRT 0.5427 both values correct to 4DP
(ii)	$\frac{g(h)-g(0)}{h}=\frac{\sqrt{3}}{2}-\frac{1}{4}h$	M1A1	2	M1A0 if num error made
(iii)	As $h \to 0$ this gives $g'(0)=\frac{\sqrt{3}}{2}$	A1F	1	AWRT 0.866; ft num error
	Total		**8**	

6. June 2007

6	One value of $2x-\frac{\pi}{2}$ is $\frac{\pi}{3}$	B1		OE (PI); degrees/decimals penalised in 6th mark only
	Another value is $\pi-\frac{\pi}{3}=\frac{2\pi}{3}$	B1F		OE (PI); ft wrong first value
	Introduction of $2n\pi$ or $n\pi$	M1		
	General solution for x	m1		
	GS $x=\frac{5\pi}{12}+n\pi$ or $x=\frac{7\pi}{12}+n\pi$	A2,1	6	OE; A1 if one part correct
	Total		**6**	

7. Jan 2008

3	Use of $\tan\frac{\pi}{4}=1$	B1		Degrees or decimals penalised in last mark only
	Introduction of $n\pi$	M1		or kn at any stage
	Division of all terms by 4	m1		
	Addition of $\pi/8$	m1		OE
	GS $x=\frac{3\pi}{16}+\frac{n\pi}{4}$	A1	5	OE
	Total		**5**	

8. June 2008

5(a)	$\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ stated or used	B1		Degrees or decimals penalised in 5th mark only
	Appropriate use of ±	B1		OE
	Introduction of $2n\pi$	M1		OE
	Subtraction of $\frac{\pi}{3}$ and multiplication by 2	m1		All terms multiplied by 2
	$x=-\frac{2\pi}{3}\pm\frac{\pi}{2}+4n\pi$	A1	5	OE
5(b)	$n=1$ gives min pos $x=\frac{17\pi}{6}$	M1A1	2	**NMS** 1/2 provided (a) correct
	Total		**7**	

9. Jan 2009

3	$\tan\frac{\pi}{3}=\sqrt{3}$	B1		Decimals/degrees penalised at 5th mark
	Introduction of $n\pi$	M1		(or $2n\pi$) at any stage
	Going from $\frac{\pi}{2}-3x$ to x	m1		Including dividing all terms by 3
	$x=\frac{\pi}{18}+\frac{1}{3}n\pi$	A2,1F	5	Allow +, − or ±; A1 with dec/deg; ft wrong first solution
	Total		**5**	

10.June 2009

5(a)	$\cos\frac{\pi}{3}=\frac{1}{2}$	B1		Decimals/degrees penalised at 6th mark only
	Appropriate use of ±	B1		OE
	Introduction of $2n\pi$	M1		(or nπ) at any stage
	Going from $3x-\pi$ to x	m1		including dividing all terms by 3
	$x=\frac{\pi}{3}\pm\frac{\pi}{9}+\frac{2}{3}n\pi$	A2,1F	6	OE; A1 with decimals and/or degrees; ft wrong first solution
(b)	At least one value in given range	M1		compatible with c's GS
	Correct values $\frac{92}{9}\pi,\ \frac{94}{9}\pi,\ \frac{98}{9}\pi$	A2,1	3	A1 if one omitted or wrong values included; A0 if only one correct value given
	Total		**9**	

11.Jan 2010

3	$\sin\frac{\pi}{2}=1$ stated or used	B1		Deg/dec penalised in 4th mark
	Introduction of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $4x+\frac{\pi}{4}$ to x	m1		incl division of all terms by 4
	$x=\frac{\pi}{16}+\frac{1}{2}n\pi$	A1	4	or equivalent unsimplified form
	Total		**4**	

12.June 2010

3	Introduction of $360n°$	M1		(or $180n°$) at any stage; condone $2n\pi$ (or $n\pi$)
	$5x-20°=\pm40°\ (+360n°)$	B1		OE, eg RHS '40° or 320°'
	Going from $5x-20°$ to x	m1		including division of all terms by 5
	GS is $x=4°\pm8°+72n°$	A2,1	5	OE; A1 if radians present in answer
	Total		**5**	

13.Jan 2011

4	$\sin\left(-\frac{\pi}{6}\right)=-\frac{1}{2}$	B1		OE; dec/deg penalised at 6th mark
	$\sin\left(-\frac{5\pi}{6}\right)=-\frac{1}{2}$	B1F		OE; ft wrong first value
	Use of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $4x-\frac{2\pi}{3}$ to x	m1		including division of all terms by 4
	GS $x=\frac{\pi}{8}+\frac{1}{2}n\pi$ or $x=-\frac{\pi}{24}+\frac{1}{2}n\pi$	A1A1	6	OE
	Total		**6**	

14.June 2011

5(a)	$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$	B1		OE stated or used; deg/dec penalised at 5th mark
	$\cos(-\frac{\pi}{6})=\frac{\sqrt{3}}{2}$	B1F		OE; ft wrong first value
	Introduction of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $3x-\frac{\pi}{6}$ to x	m1		incl division of all terms by 3
	GS: $x=\frac{\pi}{18}\pm\frac{\pi}{18}+\frac{2}{3}n\pi$	A1F	5	ft wrong first value

(b)	$n = 8$ will give the required solution ... which is $\frac{16}{3}\pi$ (≈ 16.755)	M1 A1	2	GS must include $\frac{2}{3}n\pi$ for this from correct GS; allow $\frac{48}{9}\pi$ or dec approx
	Total		**7**	

15.Jan 2012

6(a)	$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$	B1		OE (PI) Stated or used. A correct angle in 1st or 3rd quadrant for $\tan^{-1}(1/\sqrt{3})$. Condone degrees / decimal equivs.
	$\left(\frac{x}{2} - \frac{\pi}{4}\right) = n\pi + \frac{\pi}{6}$;	M1		Correct use of either $n\pi$ or $2n\pi$. Eg either $n\pi + \alpha$ or **both** $2n\pi + \alpha$ and $2n\pi + \pi + \alpha$ OE where α is c's $\tan^{-1}(1/\sqrt{3})$. Condone degrees/decimals/mixture
	$x = 2\left(n\pi + \frac{\pi}{6} + \frac{\pi}{4}\right) \quad \left(= 2n\pi + \frac{5\pi}{6}\right)$	m1		Either correct rearrangement of $\frac{x}{2} - \frac{\pi}{4} = n\pi + \alpha$ to $x = \ldots$, or correct rearrangements of both the equivalents above in the M1 line involving $2n\pi$, where α is c's $\tan^{-1}(1/\sqrt{3})$. Condone degrees/decimals/mixture
		A1	4	ACF, but must now be exact and in terms of π.
(b)	$\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = \pm\sqrt{\frac{1}{3}}$	M1		PI. Taking square roots, must include the $\pm$ or evidence of its use
	$\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = -\sqrt{\frac{1}{3}}$ $\Rightarrow \frac{x}{2} - \frac{\pi}{4} = n\pi - \frac{\pi}{6}$;	m1		OE If not correct, ft on c's working in (a) with c's α replaced by $-\alpha$. Condones as in m1 above.
	$x = 2\left(n\pi + \frac{\pi}{6} + \frac{\pi}{4}\right)$, $x = 2\left(n\pi - \frac{\pi}{6} + \frac{\pi}{4}\right)$ $\{x = 2n\pi + \frac{5\pi}{6}, \; x = 2n\pi + \frac{\pi}{6}\}$	A1F	3	Any valid form, but only ft on c's exact value for $\tan^{-1}(1/\sqrt{3})$ in terms of π.
	Total		**7**	

16.June 2012

				Watch out for the many correct different forms of the general solutions
4	$\sin\left(70° - \frac{2}{3}x\right) = \cos 20° = \sin 70°$	B1		OE cos20 = sin70; or cos20 = sin110 etc PI
	$\sin\left(70° - \frac{2}{3}x\right) = \sin 110°$	B1		OE; Use of a correct angle, in degrees, in other relevant quadrant PI
	$70° - \frac{2}{3}x = 360n° + "70°"$ $70° - \frac{2}{3}x = 360n° + "110°"$	M1		OE; Either one, showing a correct use of $360n$ in forming a general solution. Condone $2n\pi$ in place of $360n$
	$x = \frac{3}{2}(70° - 70° - 360n°)$			Rearrangement of $70 - \frac{2}{3}x = 360n + \alpha$

	$x=\frac{3}{2}\left(70^\circ-110^\circ-360n^\circ\right)$ $x=-540n^\circ;\ \ x=-540n^\circ-60^\circ$	m1 A2,1,0	 6	OE to $x=-\frac{3}{2}\left(\pm 360n+\alpha-70\right)$ OE, where α is from c's $\sin\alpha=\cos 20$ Condone $2n\pi$ in place of $360n$ OE eg $540n^\circ$, $540n^\circ-60^\circ$. Condone $0\pm 540n$ for $\pm 540n$. If not A2, award (i) A1 for either correct unsimplified full general solution or (ii) A1F for correct ft full general solution, ft c's wrong angle(s) after award of B0, may be left in unsimplified form(s) or (iii) A1 for 'correct' simplified full general solution but with radians present A0 for only a partial correct solution
	Total		**6**	

17.Jan 2013

3(a)	$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$ $\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$ $2x+\frac{\pi}{4}=2n\pi+\frac{\pi}{3}$; $2x+\frac{\pi}{4}=2n\pi+\frac{2\pi}{3}$ $x=\frac{1}{2}\left(2n\pi+\frac{\pi}{3}-\frac{\pi}{4}\right)$; $x=\frac{1}{2}\left(2n\pi+\frac{2\pi}{3}-\frac{\pi}{4}\right)$	B1 B1F M1 m1		OE (PI) Stated or used. A correct angle in 1st or 2nd quadrant for $\sin^{-1}(\sqrt{3}/2)$. OE (PI) Stated or used. A correct ft angle in remaining quadrant for $\sin^{-1}(\sqrt{3}/2)$. B0F if angle used is in an incorrect quadrant OE Either. Ft on c's $\sin^{-1}(\sqrt{3}/2)$. Either. Correct rearrangement of $2x+\frac{\pi}{4}=2n\pi+\alpha$ to $x=\ldots$, where α is c's $\sin^{-1}(\sqrt{3}/2)$.
(b)	GS: $x=n\pi+\frac{\pi}{24}$; $x=n\pi+\frac{5\pi}{24}$ $n=5$ (gives greatest soln $<6\pi$) $=5\pi+\frac{5\pi}{24}$ $=\frac{125\pi}{24}$	A2,1,0 M1 A1	6 2	Both in ACF, but must now be exact and in terms of π for A2. A1 if decimal approx used. Applying a correct value for n which gives greatest soln.$<6\pi$ for c's GS dep on GS, using above method, having two expressions of the form $n\pi+\lambda$, for different λ and m1 scored in (a). Dep on correct full GS.
	Total		**8**	