Practice examination questions for linear laws

1. Jan 2005

7 [**Figure 1**, printed on the insert, is provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y^3 = ax^2 + b$$

where a and b are constants.

Experimental evidence has provided the following approximate values:

x	1.5	4.0	5.0	6.5	8.0
y	5.0	6.3	7.0	8.0	9.0

(a) On **Figure 1**, draw a linear graph connecting the variables X and Y, where

$$X = x^2 \quad \text{and} \quad Y = y^3$$ *(5 marks)*

(b) From your graph, find approximate values for the constants a and b. *(3 marks)*

2. Jan 2006

6 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y = kx^n$$

where k and n are constants.

Experimental evidence has provided the following approximate values:

x	4	17	150	300
y	1.8	5.0	30	50

(a) Complete the table in **Figure 1**, showing values of X and Y, where

$$X = \log_{10} x \quad \text{and} \quad Y = \log_{10} y$$

Give each value to two decimal places. *(3 marks)*

(b) Show that if $y = kx^n$, then X and Y must satisfy an equation of the form

$$Y = aX + b$$ *(3 marks)*

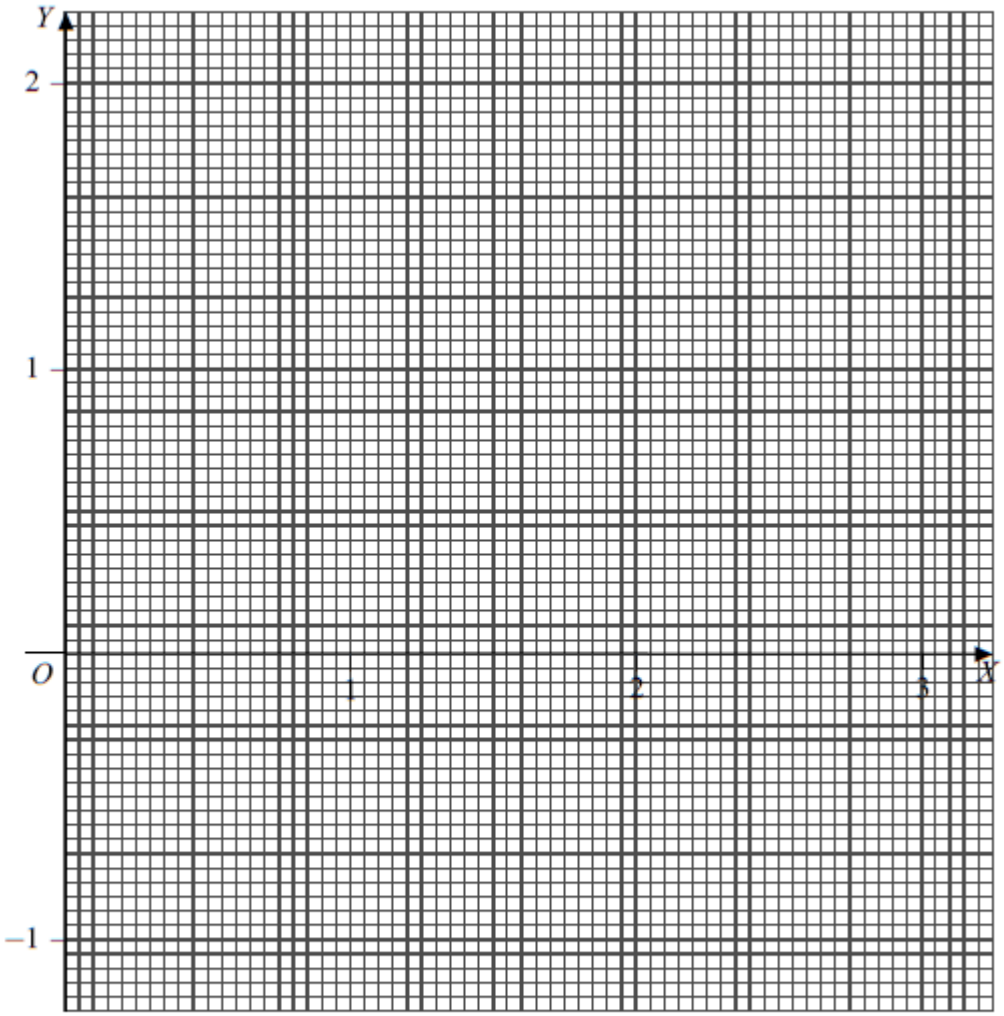
(c) Draw on **Figure 2** a linear graph relating X and Y. *(3 marks)*

(d) Find an estimate for the value of n. *(2 marks)*

Figure 1 (for use in Question 6)

X	0.60			2.48
Y	0.26			1.70

Figure 2 (for use in Question 6)



3. Jan 2007

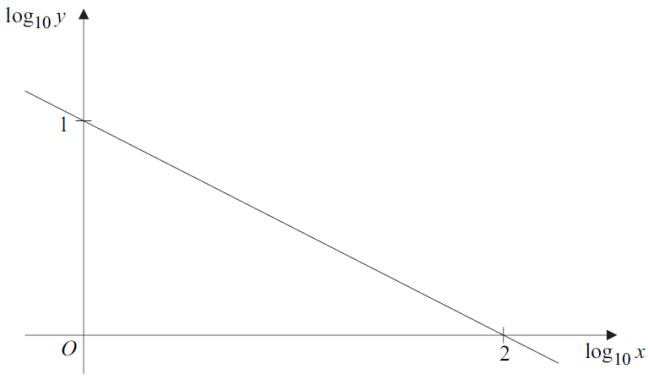
4 The variables x and y are related by an equation of the form

$$y = ax^b$$

where a and b are constants.

(a) Using logarithms to base 10, reduce the relation $y = ax^b$ to a linear law connecting $\log_{10} x$ and $\log_{10} y$. *(2 marks)*

(b) The diagram shows the linear graph that results from plotting $\log_{10} y$ against $\log_{10} x$.



Find the values of a and b. *(4 marks)*

(a) Complete the table in **Figure 1**, showing values of x and Y, where $Y = \log_{10} y$. Give each value of Y to three decimal places. *(2 marks)*

(b) Show that, if $y = ab^x$, then x and Y must satisfy an equation of the form

$$Y = mx + c$$ *(3 marks)*

(c) Draw on **Figure 2** a linear graph relating x and Y. *(2 marks)*

(d) Hence find estimates for the values of a and b. *(4 marks)*

4. June 2007

5 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y = ab^x$$

where a and b are constants.

The following approximate values of x and y have been found.

x	1	2	3	4
y	3.84	6.14	9.82	15.7

(a) Complete the table in **Figure 1**, showing values of x and Y, where $Y = \log_{10} y$. Give each value of Y to three decimal places. *(2 marks)*

(b) Show that, if $y = ab^x$, then x and Y must satisfy an equation of the form

$$Y = mx + c$$ *(3 marks)*

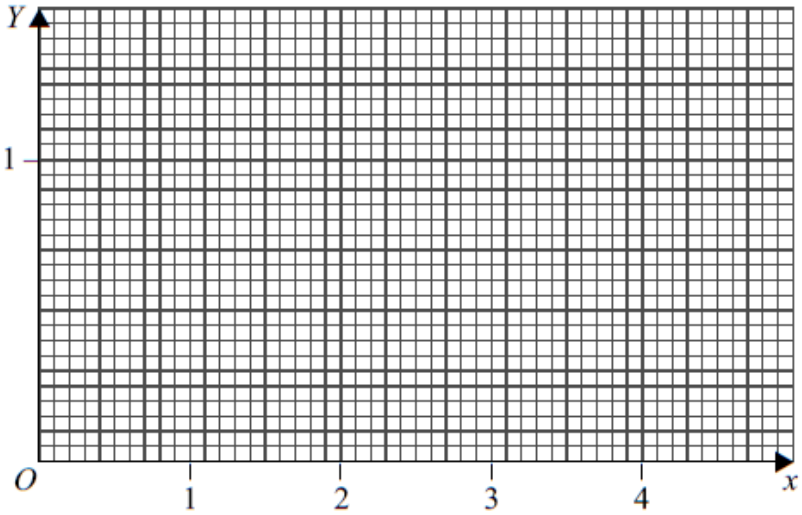
(c) Draw on **Figure 2** a linear graph relating x and Y. *(2 marks)*

(d) Hence find estimates for the values of a and b. *(4 marks)*

Figure 1 (for use in Question 5)

x	1	2	3	4
Y	0.584			

Figure 2 (for use in Question 5)



5. June 2008

4 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are related by an equation of the form

$$y = ax + \frac{b}{x+2}$$

where a and b are constants.

(a) The variables X and Y are defined by $X = x(x+2)$, $Y = y(x+2)$.

Show that $Y = aX + b$. *(2 marks)*

(b) The following approximate values of x and y have been found:

x	1	2	3	4
y	0.40	1.43	2.40	3.35

(i) Complete the table in **Figure 1**, showing values of X and Y. *(2 marks)*

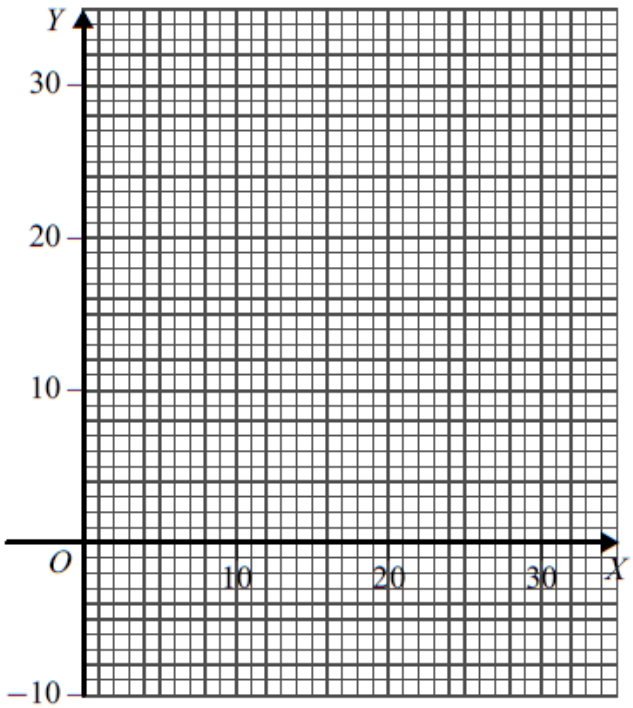
(ii) Draw on **Figure 2** a linear graph relating X and Y. *(2 marks)*

(iii) Estimate the values of a and b. *(3 marks)*

Figure 1 (for use in Question 4)

x	1	2	3	4
y	0.40	1.43	2.40	3.35
X	3			
Y	1.20			

Figure 2 (for use in Question 4)



6. June 2009

4 The variables x and y are known to be related by an equation of the form

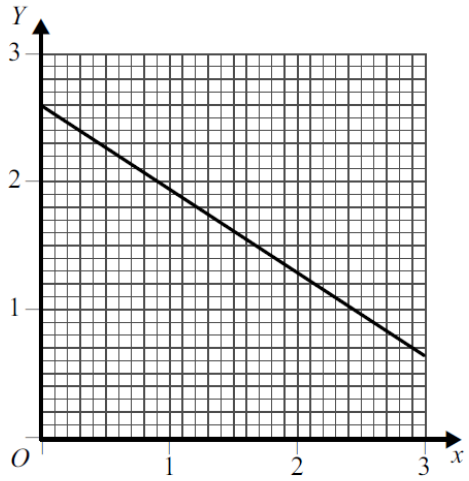
$$y = ab^x$$

where a and b are constants.

(a) Given that $Y = \log_{10} y$, show that x and Y must satisfy an equation of the form

$$Y = mx + c$$ *(3 marks)*

(b) The diagram shows the linear graph which has equation $Y = mx + c$.



Use this graph to calculate:

Use this graph to calculate:

(i) an approximate value of y when $x = 2.3$, giving your answer to one decimal place;

(ii) an approximate value of x when $y = 80$, giving your answer to one decimal place.

(You are not required to find the values of m and c.) *(4 marks)*

7. June 2010

4 The variables x and y are related by an equation of the form

$$y = ax^2 + b$$

where a and b are constants.

The following approximate values of x and y have been found.

x	2	4	6	8
y	6.0	10.5	18.0	28.2

(a) Complete the table below, showing values of X, where $X = x^2$. *(1 mark)*

(b) On the diagram below, draw a linear graph relating X and y. *(2 marks)*

(c) Use your graph to find estimates, to two significant figures, for:

(i) the value of x when $y = 15$; *(2 marks)*

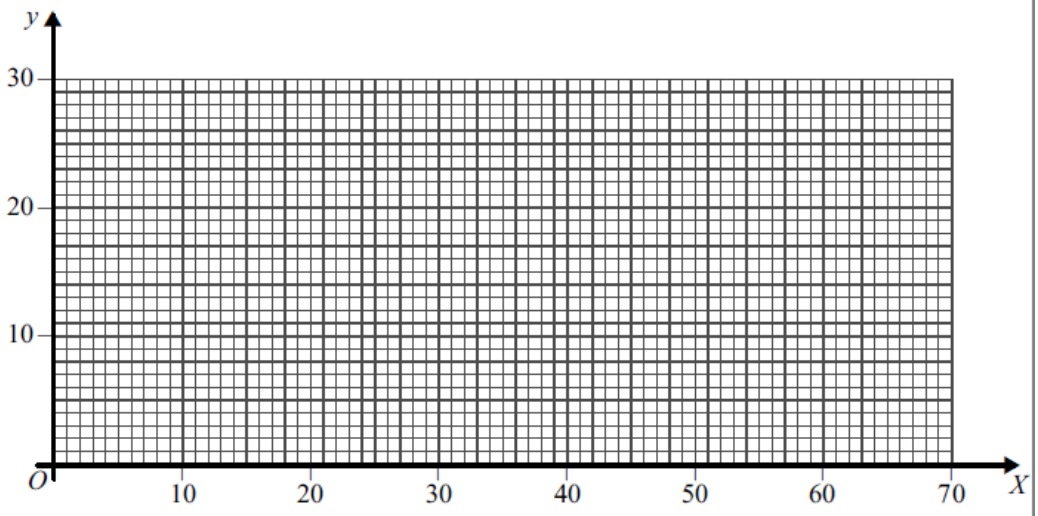
(ii) the values of a and b. *(3 marks)*

QUESTION PART REFERENCE

(a)

x	2	4	6	8
X				
y	6.0	10.5	18.0	28.2

(b)



8. June 2011

4 The variables x and Y, where $Y = \log_{10} y$, are related by the equation

$$Y = mx + c$$

where m and c are constants.

(a) Given that $y = ab^x$, express a in terms of c, and b in terms of m. *(3 marks)*

(b) It is given that $y = 12$ when $x = 1$ and that $y = 27$ when $x = 5$.

On the diagram opposite, draw a linear graph relating x and Y. *(3 marks)*

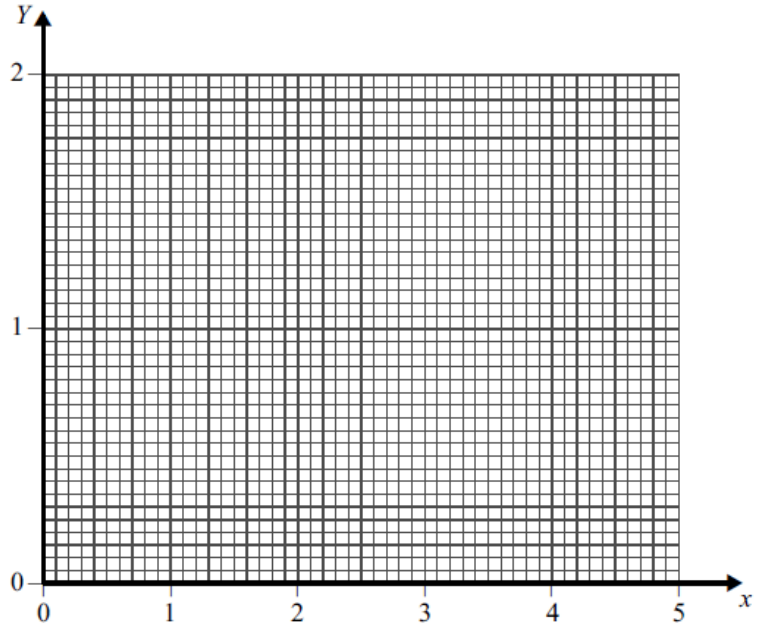
(c) Use your graph to estimate, to two significant figures:

(i) the value of y when $x = 3$; *(2 marks)*

(ii) the value of a. *(2 marks)*

QUESTION PART REFERENCE

(b)



9. Jan 2013

7 The variables y and x are related by an equation of the form

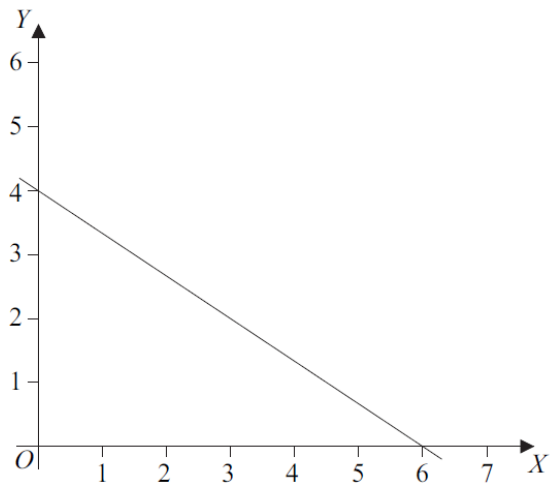
$$y = ax^n$$

where a and n are constants.

Let $Y = \log_{10} y$ and $X = \log_{10} x$.

(a) Show that there is a linear relationship between Y and X. *(3 marks)*

(b) The graph of Y against X is shown in the diagram.



Find the value of n and the value of a. *(4 marks)*

Answers

1. Jan 2005

7(a)	(X, Y) values: (2.25, 125), (16, 250), (25, 343), (42.25, 512), (64, 729)	B2,1		PI by c's graph
	Five points accurately plotted	B2,1√		ft wrong values
	Reasonable straight line drawn	B1√	5	ft errors in plotting
(b)	Calculation of gradient of line	M1		
	Value of a equal to gradient found	A1		
	Value of $b = y$-intercept of line	B1	3	
	Total		**8**	

2. Jan 2006

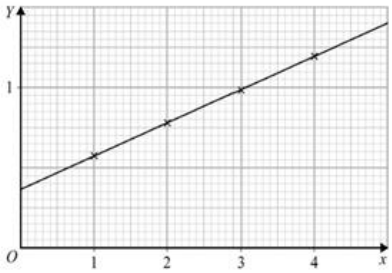
6(a)	X values 1.23, 2.18			
	Y values 0.70, 1.48	B3,2,1	3	−1 for each error
(b)	$\lg y = \lg k + \lg x^n$	M1		
	$\lg x^n = n \lg x$	M1		
	So $Y = nX + \lg k$	A1	3	
(c)	Four points plotted	B2,1√		B1 if one error here; ft wrong values in (a)
	Good straight line drawn	B1√	3	ft incorrect points (approx collinear)
(d)	Method for gradient	M1		
	Estimate for n	A1√	2	Allow AWRT 0.75 - 0.78; ft grad of candidate's graph
	Total		**11**	

3. Jan 2007

7(a)	Particular solution, eg $-\frac{\pi}{6}$ or $\frac{5\pi}{6}$	B1		Degrees or decimals penalised in 3rd mark only
	Introduction of $n\pi$ or $2n\pi$	M1		
	GS $x = -\frac{\pi}{6} + n\pi$	A1F	3	OE(accept unsimplified); ft incorrect first solution
(b)(i)	$f(0.05) \approx 0.542\,66$	B1		either value AWRT 0.5427
	$g(0.05) \approx 0.542\,68$	B1	2	both values correct to 4DP
(ii)	$\frac{g(h) - g(0)}{h} = \frac{\sqrt{3}}{2} - \frac{1}{4}h$	M1A1	2	M1A0 if num error made
(iii)	As $h \to 0$ this gives $g'(0) = \frac{\sqrt{3}}{2}$	A1F	1	AWRT 0.866; ft num error
	Total		**8**	

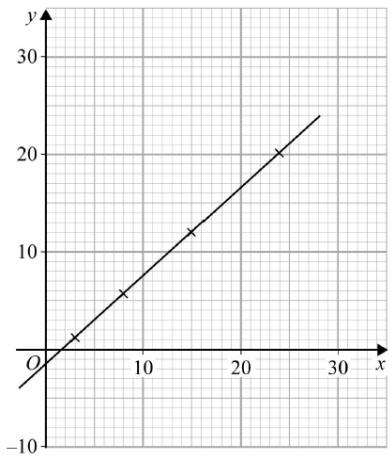
4. June 2007

5(a)	Values 0.788, 0.992, 1.196 in table	B2,1	2	B1 if one correct (or if wrong number of dp given)
(b)	$\lg ab^x = \lg a + \lg b^x$	M1		
	$\lg b^x = x \lg b$	M1		
	So $Y = (\lg b)x + \lg a$	A1	3	Allow NMS

(c)		B1F B1F	 2	Four points plotted; ft wrong values in (a) Good straight line drawn; ft incorrect points
(d)	a = antilog of y-intercept b = antilog of gradient	M1A1 M1A1	 4	Accept 2.23 to 2.52 Accept 1.58 to 1.62
	Total		**11**	

5. June 2008

4(a)	Multiplication by $x + 2$ $Y = aX + b$ convincingly shown	M1 A1	 2	applied to all 3 terms AG
(b)(i)	X = 8, 15, 24 in table Y = 5.72, 12, 20.1 in table	B1 B1	 2	 Allow correct to 2SF

4(b)(ii)				
	Four points plotted Reasonable line drawn	B1F B1F	 2	ft incorrect values in table ft incorrect points
(iii)	Method for gradient a = gradient ≈ 0.9 b = Y-intercept ≈ −1.5	M1 A1 B1F	 3	or algebraic method for a or b Allow from 0.88 to 0.93 incl Allow from −2 to −1 inclusive; ft incorrect points/line **NMS** B1 for a, B1 for b
	Total		**9**	

6. June 2009

4(a)	$\lg(ab^x) = \lg a + \lg(b^x)$ $\ldots = \lg a + x \lg b$ Correct relationship established [SC After M0M0, B2 for correct form]	M1 M1 A1	 3	Use of one log law Use of another log law
(b)(i)	When $x = 2.3$, $Y \approx 1.1$, so $y \approx 12.6$	M1A1		Allow 12.7; allow NMS
(ii)	When $y = 80$, $Y \approx 1.90$, so $x \approx 1.1$	M1A1	4	M1 for $Y \approx 1.9$, allow NMS
	Total		**7**	

7.June 2010

4(a)	4, 16, 36, 64 entered in table	B1	1	
(b)	Four points plotted accurately	B1F		ft wrong values in (a)
	Linear graph drawn	B1	2	
(c)(i)	Finding X for $y = 15$ and taking sq root	M1		
	$x \approx 5.3$	A1	2	AWRT 5.2 or 5.3; NMS 1/2
(ii)	Calculation of gradient	M1		
	a = gradient ≈ 0.37	A1		AWRT 0.36 to 0.38; NMS 1/2
	$b = y$-intercept ≈ 4.5	B1F	3	can be found by calculation; ft c's y-intercept
	Total		**8**	

8. June 2011

4(a)	Use of one law of logs or exponentials lg $a = c$ and lg $b = m$ So $a = 10^c$ and $b = 10^m$	M1 A1 A1	 3	 OE; both needed
(b)	Points (1, 1.08), (5, 1.43) plotted Straight line drawn through points	M1A1 A1F	 3	M1 A0 if one point correct ft small inaccuracy
(c)(i)	Attempt at antilog of $Y(3)$ When $x = 3$, $Y \approx 1.25$ so $y \approx 18$	M1 A1	 2	OE Allow AWRT 18
(ii)	Attempt at a as antilog of Y-intercept $a \approx 9.3$ to 10	M1 A1	 2	OE AWRT
	Total		**10**	

9. Jan 2013

7(a)	$y = ax^n \Rightarrow \log_{10} y = \log_{10} ax^n$ $\log_{10} y = \log_{10} a + \log_{10} x^n$	M1		Take logs and apply one log law in soln. correctly PI.
	$\log_{10} y = \log_{10} a + n \log_{10} x$	m1		Apply a further log law correctly.
	$Y = \log_{10} a + nX$ (which is a linear relationship between Y and X.)	A1	3	Correct eqn. with base 10 (or lg or later evidence of use of base 10 if log without base here)
(b)	n = gradient of line	M1		Stated or used. Accept $n = \pm\frac{2}{3}$ OE as evidence
	$n = -\frac{2}{3}$	A1		$n = -\frac{2}{3}$ (OE 3sf)
	$\log_{10} a = 4$	M1		Equating c's constant term [must involve a log] in c's **(a)** eqn. to the Y-intercept value PI by correct value of a
	$a = 10^4$ (= 10 000)	A1	4	
	Total		**7**	