Practice examination questions for roots of quadratics

1. Jan 2005

1 The equation

$$x^2 - 5x - 2 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2\beta + \alpha\beta^2$. *(2 marks)*

(c) Find a quadratic equation which has roots

$$\alpha^2\beta \quad \text{and} \quad \alpha\beta^2$$

(3 marks)

2. June 2005

6 The equation

$$x^2 - 4x + 13 = 0$$

has roots α and β.

(a) (i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(ii) Deduce that $\alpha^2 + \beta^2 = -10$. *(2 marks)*

(iii) Explain why the statement $\alpha^2 + \beta^2 = -10$ implies that α and β cannot both be real. *(2 marks)*

(b) Find in the form $p + \mathrm{i}q$ the values of:

(i) $(\alpha + \mathrm{i}) + (\beta + \mathrm{i})$; *(1 mark)*

(ii) $(\alpha + \mathrm{i})(\beta + \mathrm{i})$. *(2 marks)*

(c) Hence find a quadratic equation with roots $(\alpha + \mathrm{i})$ and $(\beta + \mathrm{i})$. *(2 marks)*

3. June 2006

1 The quadratic equation

$$3x^2 - 6x + 2 = 0$$

has roots α and β.

(a) Write down the numerical values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) (i) Expand $(\alpha + \beta)^3$. *(1 mark)*

(ii) Show that $\alpha^3 + \beta^3 = 4$. *(3 marks)*

(c) Find a quadratic equation with roots α^3 and β^3, giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers. *(3 marks)*

4. Jan 2007

3 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = 1$. *(3 marks)*

(c) Find the value of $\alpha^4 + \beta^4$. *(3 marks)*

5. June 2007

4 The quadratic equation

$$2x^2 - x + 4 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{4}$. *(2 marks)*

(c) Find a quadratic equation with integer coefficients such that the roots of the equation are

$$\frac{4}{\alpha} \text{ and } \frac{4}{\beta}$$ *(3 marks)*

6. Jan 2008

8 (a) (i) It is given that α and β are the roots of the equation

$$x^2 - 2x + 4 = 0$$

Without solving this equation, show that α^3 and β^3 are the roots of the equation

$$x^2 + 16x + 64 = 0$$ *(6 marks)*

(ii) State, giving a reason, whether the roots of the equation

$$x^2 + 16x + 64 = 0$$

are real and equal, real and distinct, or non-real. *(2 marks)*

(b) Solve the equation

$$x^2 - 2x + 4 = 0$$ *(2 marks)*

(c) Use your answers to parts (a) and (b) to show that

$$(1 + i\sqrt{3})^3 = (1 - i\sqrt{3})^3$$ *(2 marks)*

7. June 2008

1 The equation

$$x^2 + x + 5 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Show that $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=-\frac{9}{5}$. *(2 marks)*

(d) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. *(2 marks)*

8. June 2009

1 The equation

$$2x^2+x-8=0$$

has roots α and β.

(a) Write down the values of $\alpha+\beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2+\beta^2$. *(2 marks)*

(c) Find a quadratic equation which has roots $4\alpha^2$ and $4\beta^2$. Give your answer in the form $x^2+px+q=0$, where p and q are integers. *(3 marks)*

9. Jan 2010

1 The quadratic equation

$$3x^2-6x+1=0$$

has roots α and β.

(a) Write down the values of $\alpha+\beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^3+\beta^3=6$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. *(4 marks)*

10.June 2010

8 The quadratic equation

$$x^2-4x+10=0$$

has roots α and β.

(a) Write down the values of $\alpha+\beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{2}{5}$. *(2 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots $\alpha + \frac{2}{\beta}$ and $\beta + \frac{2}{\alpha}$. *(6 marks)*

11.Jan 2011

1 The quadratic equation $x^2 - 6x + 18 = 0$ has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find a quadratic equation, with integer coefficients, which has roots α^2 and β^2. *(4 marks)*

(c) Hence find the values of α^2 and β^2. *(1 mark)*

12.June 2011

2 The equation

$$4x^2 + 6x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = \frac{3}{4}$. *(2 marks)*

(c) Find an equation, with integer coefficients, which has roots

$$3\alpha - \beta \quad \text{and} \quad 3\beta - \alpha$$

(5 marks)

13.Jan 2012

1 The quadratic equation

$$2x^2 + 7x + 8 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = \frac{17}{4}$. *(2 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}$$ *(5 marks)*

14.June 2012

1 The quadratic equation

$$5x^2 - 7x + 1 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{39}{5}$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\alpha + \frac{1}{\alpha} \quad \text{and} \quad \beta + \frac{1}{\beta}$$ *(5 marks)*

15.Jan 2013

5 The roots of the quadratic equation

$$x^2 + 2x - 5 = 0$$

are α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. *(2 marks)*

(b) Calculate the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Find a quadratic equation which has roots $\alpha^3\beta + 1$ and $\alpha\beta^3 + 1$. *(5 marks)*

Answers

1. Jan 2005

1(a)	$\alpha+\beta=5, \alpha\beta=-2$	B1, B1	2	
(b)	$\alpha^2\beta+\alpha\beta^2=\alpha\beta(\alpha+\beta)=-10$	M1A1√	2	ft wrong values
(c)	$(\alpha^2\beta)(\alpha\beta^2)=(\alpha\beta)^3=-8$	M1A1√		ft wrong values
	Equation is $x^2+10x-8=0$	A1√	3	Dep on both M1s; ft wrong values; Condone omission of "= 0"
	Total		**7**	

2. June 2005

6(a)(i)	$\alpha+\beta=4,\ \alpha\beta=13$	B1B1	2	
(ii)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	M1		
	$\ldots=4^2-26=-10$	A1	2	convincingly shown (AG)
(iii)	The square of a real number is positive (or zero)	E1		
	The sum of two such squares is positive (or zero)	E1	2	
(b)(i)	$(\alpha+\mathrm{i})+(\beta+\mathrm{i})=4+2\mathrm{i}$	B1F	1	ft wrong value in (a)(i)
(ii)	$(\alpha+\mathrm{i})(\beta+\mathrm{i})=12+4\mathrm{i}$	M1A1F	2	ditto
(c)	Correct coeff of x or constant term	M1		Using c's answers in (b)
	$x^2-(4+2\mathrm{i})x+(12+4\mathrm{i})=0$	A1F	2	ft wrong answers in (b)
	Total		**11**	

3. June 2006

1(a)	$\alpha+\beta=2,\ \alpha\beta=\frac{2}{3}$	B1B1	2	**SC** 1/2 for answers 6 and 2
(b)(i)	$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$	B1	1	Accept unsimplified
(ii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1		
	Substitution of numerical values	m1		
	$\alpha^3+\beta^3=4$	A1	3	convincingly shown AG
(c)	$\alpha^3\beta^3=\frac{8}{27}$	B1		
	Equation of form $px^2\pm 4px+r=0$	M1		
	Answer $27x^2-108x+8=0$	A1√	3	ft wrong value for $\alpha^3\beta^3$
	Total		**9**	

4. Jan 2007

3(a)	$\alpha+\beta=-2,\ \alpha\beta=\frac{3}{2}$	B1B1	2	
(b)	Use of expansion of $(\alpha+\beta)^2$	M1		
	$\alpha^2+\beta^2=(-2)^2-2\left(\frac{3}{2}\right)=1$	m1A1	3	convincingly shown (**AG**); m1A0 if $\alpha+\beta=2$ used

(c)	$\alpha^4+\beta^4$ given in terms of $\alpha+\beta, \alpha\beta$ and/or $\alpha^2+\beta^2$	M1A1		M1A0 if num error made
	$\alpha^4+\beta^4=-\frac{7}{2}$	A1	3	OE
	Total		**8**	

5. June 2007

4(a)	$\alpha+\beta=\frac{1}{2},\ \alpha\beta=2$	B1B1	2	
(b)	$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$	M1		
	$\ldots=\frac{\frac{1}{2}}{2}=\frac{1}{4}$	A1	2	Convincingly shown (AG)
(c)	Sum of roots = 1	B1F		PI by term $\pm x$; ft error(s) in (a)
	Product of roots = $\frac{16}{\alpha\beta}=8$	B1F		ft wrong value of $\alpha\beta$
	Equation is $x^2-x+8=0$	B1F	3	ft wrong sum/product; "= 0" needed
	Total		**7**	

6. Jan 2008

8(a)(i)	$\alpha+\beta=2,\ \alpha\beta=4$	B1B1		
	$\alpha^3+\beta^3=(2)^3-3(4)(2)=-16$	M1A1		
	$\alpha^3\beta^3=(4)^3=64$, hence result	M1A1	6	convincingly shown (AG)
(ii)	Discriminant 0, so roots equal	B1E1	2	or by factorisation
(b)	$x=\frac{2\pm\sqrt{4-16}}{2}$	M1		or by completing square
	$\ldots=1\pm\frac{1}{2}\mathrm{i}\sqrt{12}$	A1	2	
(c)	$\alpha,\beta=1\pm\mathrm{i}\sqrt{3}$ and $\alpha^3=\beta^3$, hence result	E2	2	
	Total		**12**	

7. June 2008

1(a)	$\alpha+\beta=-1,\ \alpha\beta=5$	B1B1	2	
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	M1		with numbers substituted
	$\ldots=1-10=-9$	A1F	2	ft sign error(s) in (a)
(c)	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$	M1		
	$\ldots=-\frac{9}{5}$	A1	2	AG: A0 if $\alpha+\beta=1$ used
(d)	Product of new roots is 1	B1		PI by constant term 1 or 5
	Eqn is $5x^2+9x+5=0$	B1F	2	ft wrong value for product
	Total		**8**	

8. June 2009

1(a)	$\alpha+\beta=-\frac{1}{2},\ \alpha\beta=-4$	B1B1	2	
(b)	$\alpha^2+\beta^2=(-\frac{1}{2})^2-2(-4)=8\frac{1}{4}$	M1A1F	2	M1 for substituting in correct formula; ft wrong answer(s) in (a)

(c)	Sum of roots = $4(8\frac{1}{4}) = 33$ Product = $16(\alpha\beta)^2 = 256$ Equation is $x^2 - 33x + 256 = 0$	B1F B1F B1F	3	ft wrong answer in (b) ft wrong answer in (a) ft wrong sum and/or product; allow '$p = -33$, $q = 256$'; condone omission of '= 0'
	Total		**7**	

9. Jan 2010

1(a)	$\alpha + \beta = 2,\ \alpha\beta = \frac{1}{3}$	B1B1	2	
(b)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $\ldots = 8 - 3(\frac{1}{3})(2) = 6$	M1 m1A1	3	or other appropriate formula m1 for substn of numerical values; A1 for result shown (AG)
(c)	Sum of roots = $\dfrac{\alpha^3 + \beta^3}{\alpha\beta}$ $\ldots = \dfrac{6}{1/3} = 18$ Product = $\alpha\beta = \frac{1}{3}$ Equation is $3x^2 - 54x + 1 = 0$	M1 A1F B1F A1F	4	 ft wrong value for $\alpha\beta$ ditto Integer coeffs and "= 0" needed; ft wrong sum and/or product
	Total		**9**	

10.June 2010

8(a)	$\alpha + \beta = 4,\ \alpha\beta = 10$	B1,B1	2	
(b)	$\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha + \beta}{\alpha\beta}$ $= \dfrac{4}{10} = \dfrac{2}{5}$	M1 A1	2	 convincingly shown (AG)
(c)	Sum of roots = $(\alpha + \beta) + 2$(ans to (b)) $= 4\dfrac{4}{5}$ Product = $\alpha\beta + 4 + \dfrac{4}{\alpha\beta}$ $= 14\dfrac{2}{5}$ Equation is $5x^2 - 24x + 72 = 0$	M1 A1F M1A1 A1F A1F	6	 ft wrong value for $\alpha + \beta$ M1 for attempt to expand product (at least two terms correct) ft wrong value for $\alpha\beta$ integer coeffs and '= 0' needed here; ft one numerical error
	Total		**10**	

11.Jan 2011

1(a)	$\alpha + \beta = 6,\ \alpha\beta = 18$	B1B1	2	
(b)	Sum of new roots = $6^2 - 2(18) = 0$ Product = $18^2 = 324$ Equation $x^2 + 324 = 0$	M1A1F B1F A1F	4	ft wrong value(s) in (a) ditto '= 0' needed here; ft wrong value(s) for sum/product
(c)	α^2 and β^2 are $\pm 18i$	B1	1	
	Total		**7**	

12.June 2011

2(a)	$\alpha+\beta=-\frac{3}{2}$, $\alpha\beta=\frac{3}{4}$	B1B1	2	
(b)	$\alpha^2+\beta^2=\left(-\frac{3}{2}\right)^2-2\left(\frac{3}{4}\right)=\frac{3}{4}$	M1A1	2	AG; A0 if $\alpha+\beta$ has wrong sign
(c)	Sum $=2(\alpha+\beta)=-3$	B1F		ft wrong value for $\alpha+\beta$
	Product $=10\alpha\beta-3\left(\alpha^2+\beta^2\right)=\frac{21}{4}$	M1A1F		ft wrong values
	$x^2-Sx+P\ (=0)$	M1		Signs must be correct for the M1
	Eqn is $4x^2+12x+21=0$	A1	5	Integer coeffs and '= 0' needed
	Total		**9**	

13.Jan 2012

1(a)	$\alpha+\beta=-\frac{7}{2}$	B1		
	$\alpha\beta=4$	B1	2	
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(-\frac{7}{2}\right)^2-2(4)$	M1		Using correct identity with ft or correct substitution
	$=\frac{49}{4}-8=\frac{17}{4}$	A1	2	CSO AG. A0 if $\alpha+\beta$ has wrong sign
(c)	(Sum=) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}=\frac{17/4}{16}\left(=\frac{17}{64}\right)$	M1		Writing $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ in a correct suitable form with ft or correct substitution
	$=\frac{17}{64}$	A1F		ft wrong value for $\alpha\beta$
	(Product =) $\frac{1}{(\alpha\beta)^2}=\frac{1}{16}\left(=\frac{4}{64}\right)$	B1F		ft wrong value for $\alpha\beta$
	$x^2-Sx+P\ (=0)$	M1		Using correct general form of LHS of eqn **with** ft substitution of c's S and P values. PI
	Eqn is $64x^2-17x+4=0$	A1	5	CSO Integer coefficients and '= 0' needed
	Total		**9**	

14.June 2012

				Accept correct equivalent decimals in place of some/all fractions in the scheme
1(a)	$\alpha+\beta=\frac{7}{5}\ (=1.4)$	B1		
	$\alpha\beta=\frac{1}{5}\ (=0.2)$	B1	2	
(b)	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$	M1		OE eg $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\frac{1}{5}[7(\alpha+\beta)-1-1]}{\alpha\beta}$ scores M1 m1
	$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{\left(\frac{7}{5}\right)^2-2\left(\frac{1}{5}\right)}{\frac{1}{5}}$	m1		Correct expression for $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ in terms of either $(\alpha+\beta)$ and $\alpha\beta$ or with numerical substitution of correct/c's values from (a)

	$=\dfrac{\frac{49}{25}-2\left(\frac{1}{5}\right)}{\frac{1}{5}} = \dfrac{\frac{49}{25}-\frac{2}{5}}{\frac{1}{5}} = \dfrac{\frac{39}{25}}{\frac{1}{5}} = \dfrac{39}{5}$	A1	3	CSO AG must see some intermediate evaluation, must see one of the first three expressions A0 if $\alpha+\beta$ has wrong sign
(c)	(Sum=) $\alpha+\dfrac{1}{\alpha}+\beta+\dfrac{1}{\beta}=\alpha+\beta+\dfrac{\alpha+\beta}{\alpha\beta}$ $\left(=\dfrac{7}{5}+\dfrac{\frac{7}{5}}{\frac{1}{5}}\right)$	M1		Writing $\alpha+\dfrac{1}{\alpha}+\beta+\dfrac{1}{\beta}$ in a correct suitable form or with numerical values
	(Product =) $\alpha\beta+\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}+\dfrac{1}{\alpha\beta}$ $=\dfrac{1}{\ }+\dfrac{39}{\ }+5$	M1		Correct expression for product into which substitution of numbers attempted for all terms, at least one either correct/correct ft
	Sum = $\dfrac{42}{5}$, Product = 13	A1		OE <u>Both</u> **SC** If B0 for $\alpha+\beta=-\dfrac{7}{5}$ in (a), and (c) S= $-\dfrac{42}{5}$ oe, P = 13 award this A1
	$x^2-Sx+P\ (=0)$	M1		Using correct general form of LHS of equation **with** ft substitution of c's S and P values. PI. M0 if either $S=\alpha+\beta$ or $P=\alpha\beta$ values
	Equation is $5x^2-42x+65=0$	A1	5	CSO Integer coefficients and '= 0' needed. Dependent on B1B1 in (a) and previous 4 marks in (c) scored
	Total		**10**	

15.Jan 2013

5(a)	$\alpha+\beta=-2$ $\alpha\beta=-5$	B1 B1	2	
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(-2)^2-2(-5)$	M1		OE Using correct identity for $\alpha^2+\beta^2$ with ft or correct substitution
	$=14$	A1	2	CSO A0 if $\alpha+\beta$ has wrong sign
(c)	$\alpha^3\beta+\alpha\beta^3=\alpha\beta(\alpha^2+\beta^2)$	M1		PI Seen at least once in part (c). OE eg $\alpha^3\beta+\alpha\beta^3=\alpha\beta[(\alpha+\beta)^2-2\alpha\beta]$
	$S(\text{um})=\alpha^3\beta+\alpha\beta^3+2=(-5)(14)+2=-68$	A1F		Correct or ft c's $\alpha\beta\times$ c's [answer (b)] +2
	$P(\text{roduct})=(\alpha\beta)^4+\alpha^3\beta+\alpha\beta^3+1$ $=(-5)^4+(-5)(14)+1=556$	A1F		Correct or ft [c's $\alpha\beta]^4$ + c's $\alpha\beta\times$ c's [answer (b)] +1
	$x^2-Sx+P\ (=0)$	M1		Using correct general form of LHS of eqn **with** ft substitution of c's S and P values.
	Eqn.: $x^2+68x+556=0$	A1	5	CSO ACF
	Total		**9**	