Practice examination questions for geometric series

1. June 2005

5 The sum to infinity of a geometric series is four times the first term of the series.

(a) Show that the common ratio, r, of the geometric series is $\frac{3}{4}$. *(3 marks)*

(b) The first term of the geometric series is 48. Find the sum of the first 10 terms of the series, giving your answer to four decimal places. *(2 marks)*

(c) The nth term of the geometric series is u_n and the $(2n)$th term of the series is u_{2n}.

(i) Write u_n and u_{2n} in terms of n. *(2 marks)*

(ii) Hence show that $\log_{10}(u_n) - \log_{10}(u_{2n}) = n\log_{10}\left(\frac{4}{3}\right)$. *(3 marks)*

(iii) Hence show that the value of

$$\log_{10}\left(\frac{u_{100}}{u_{200}}\right)$$

is 12.5 correct to three significant figures. *(2 marks)*

2. Jan 2006

3 (a) Use logarithms to solve the equation $0.8^x = 0.05$, giving your answer to three decimal places. *(3 marks)*

(b) An infinite geometric series has common ratio r. The sum to infinity of the series is five times the first term of the series.

(i) Show that $r = 0.8$. *(3 marks)*

(ii) Given that the first term of the series is 20, find the least value of n such that the nth term of the series is less than 1. *(3 marks)*

3. Jan 2007

5 The second term of a geometric series is 48 and the fourth term is 3.

(a) Show that one possible value for the common ratio, r, of the series is $-\frac{1}{4}$ and state the other value. *(4 marks)*

(b) In the case when $r = -\frac{1}{4}$, find:

(i) the first term; *(1 mark)*

(ii) the sum to infinity of the series. *(2 marks)*

4. June 2007

2 The nth term of a geometric sequence is u_n, where

$$u_n = 3 \times 4^n$$

(a) Find the value of u_1 and show that $u_2 = 48$. *(2 marks)*

(b) Write down the common ratio of the geometric sequence. *(1 mark)*

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is $4^k - 4$, where k is an integer. *(3 marks)*

(ii) Hence find the value of $\sum_{n=2}^{12} u_n$. *(1 mark)*

5. June 2008

3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + \ldots$$

(a) Find the common ratio of the series. *(1 mark)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. *(2 marks)*

(d) Prove that the nth term of the series is 25×0.8^n. *(2 marks)*

6. June 2009

7 A geometric series has second term 375 and fifth term 81.

(a) (i) Show that the common ratio of the series is 0.6. *(3 marks)*

(ii) Find the first term of the series. *(2 marks)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) The nth term of the series is u_n. Find the value of $\sum_{n=6}^{\infty} u_n$. *(4 marks)*

7. June 2010

5 (a) An infinite geometric series has common ratio r.

The first term of the series is 10 and its sum to infinity is 50.

(i) Show that $r = \frac{4}{5}$. *(2 marks)*

(ii) Find the second term of the series. *(2 marks)*

(b) The first and second terms of the geometric series in part **(a)** have the same values as the 4th and 8th terms respectively of an arithmetic series.

(i) Find the common difference of the arithmetic series. *(3 marks)*

(ii) The nth term of the arithmetic series is u_n. Find the value of $\sum_{n=1}^{40} u_n$. *(4 marks)*

8. Jan 2011

6 A geometric series has third term 36 and sixth term 972.

(a) (i) Show that the common ratio of the series is 3. *(2 marks)*

(ii) Find the first term of the series. *(2 marks)*

(b) The nth term of the series is u_n.

(i) Show that $\sum_{n=1}^{20} u_n = 2(3^{20} - 1)$. *(2 marks)*

(ii) Find the least value of n such that $u_n > 4 \times 10^{15}$. *(3 marks)*

9. June 2011

9 The first term of a geometric series is 12 and the common ratio of the series is $\frac{3}{8}$.

(a) Find the sum to infinity of the series. *(2 marks)*

(b) Show that the sixth term of the series can be written in the form $\frac{3^6}{2^{13}}$. *(3 marks)*

(c) The nth term of the series is u_n.

(i) Write down an expression for u_n in terms of n. *(1 mark)*

(ii) Hence show that

$$\log_a u_n = n \log_a 3 - (3n - 5) \log_a 2$$ *(4 marks)*

10.June 2012

4 The nth term of a geometric series is u_n, where $u_n = 48\left(\frac{1}{4}\right)^n$.

(a) Find the value of u_1 and the value of u_2. *(2 marks)*

(b) Find the value of the common ratio of the series. *(1 mark)*

(c) Find the sum to infinity of the series. *(2 marks)*

(d) Find the value of $\sum_{n=4}^{\infty} u_n$. *(3 marks)*

11.Jan 2013

6 (a) A geometric series begins $420 + 294 + 205.8 + \ldots$.

(i) Show that the common ratio of the series is 0.7. *(1 mark)*

(ii) Find the sum to infinity of the series. *(2 marks)*

(iii) Write the nth term of the series in the form $p \times q^n$, where p and q are constants. *(2 marks)*

(b) The first term of an arithmetic series is 240 and the common difference of the series is -8.

The nth term of the series is u_n.

(i) Write down an expression for u_n. *(1 mark)*

(ii) Given that $u_k = 0$, find the value of $\sum_{n=1}^{k} u_n$. *(4 marks)*

Answers

1. June 2005

Q	Solution	Marks	Total	Comments
5(a)	$\frac{a}{1-r} = 4a$	M1		(Accept $S_\infty = \frac{a}{1-\frac{3}{4}}$)
	$\Rightarrow 1-r = \frac{a}{4a}$ or $a = 4a(1-r)$	A1		Either (or better) (or $S_\infty = 4a$ if M1 above)
	$1-r = \frac{1}{4} \Rightarrow r = \frac{3}{4}$	A1	3	**AG** CSO Be convinced. (or statement 4 times 1st term)
(b)	$(S_{10} =)\frac{48(1-r^{10})}{1-r}$	M1		Correct formula with n = 10 and one of $a = 48$ or $r = \frac{3}{4}$ OE
	$= 192(1-0.75^{10}) = 181.1878$ to 4dp	A1	2	
(c)(i)	$u_n = \underline{ar^{n-1}} = a\left(\frac{3}{4}\right)^{n-1} = 48\left(\frac{3}{4}\right)^{n-1}$	B1		
	$u_{2n} = \underline{ar^{2n-1}} = a\left(\frac{3}{4}\right)^{2n-1} = 48\left(\frac{3}{4}\right)^{2n-1}$	B1√	2	ft on candidate's $u_n = ar^{\text{function of } n}$
(ii)	$\frac{u_n}{u_{2n}} = \frac{ar^{n-1}}{ar^{2n-1}} = \frac{r^{n-1}}{r^{2n-1}}$	M1		Eliminating a (or 48) or log a
	$\log_{10} u_n - \log_{10} u_{2n} = \log_{10} \frac{u_n}{u_{2n}}$	M1		Using at least one log law
	$= \log_{10} \frac{r^{n-1}}{r^{2n-1}} = \log_{10}(r^{-n})$			
	$= -n \log_{10} \frac{3}{4} = n \log_{10} \frac{4}{3}$	A1	3	**AG** CSO Full valid completion
(iii)	$\log_{10}\left[\frac{u_{100}}{u_{200}}\right] = 100 \log_{10}\left(\frac{4}{3}\right)$	M1		
	$= 12.49\ldots = 12.5$ to 3 sf	A1	2	**AG** CSO Be convinced **SC:** Those applying 'hence' to (i) rather than to (ii) **Mark as B2**
	Total		**12**	

2. Jan 2006

3(a)	$\log 0.8^x = \log 0.05$	$x = \log_{0.8} 0.05$ **(M1)**	M1		**NMS:** **SC** B2 for 13.425 or better (B1 for 13.4 or 13.43; 13.42)
	$x \log_{10} 0.8 = \log_{10} 0.05$ oe		A1		

	$x\log_{10} 0.8 = \log_{10} 0.05$ oe		A1		
	$x = 13.425$ to 3dp	13.425 (**A2**) (**else A1** for 1 or 2dp)	A1	3	Condone greater accuracy
(b)(i)	$\frac{a}{1-r}$		M1		$S_\infty = \frac{a}{1-r}$ **used**
	$\frac{a}{1-r} = 5a \Rightarrow a = 5a(1-r)$		A1		Or better
	$\Rightarrow 1 = 5(1-r) \Rightarrow r = \frac{4}{5} = 0.8$		A1	3	AG (be convinced)
(ii)	n^{th} term $= 20\times(0.8)^{n-1}$		M1		Condone $20\times(0.8)^n$.
	n^{th} term $< 1 \Rightarrow 0.8^{n-1} < \frac{1}{20}$ oe		A1		$0.8^{n-1} < 0.05$ or $0.8^{n-1} = k$, where $k = 0.05$ or k rounds **up** to 0.050
	Least n is 15		A1F	3	If not 15, ft on integer part of [answer (a)+2] provided $n>2$ **SC** 3/3 for 15 if no error **SC** n^{th} term$=16^{n-1}$M1A0A0
		Total		**9**	

3. Jan 2007

5(a)	$ar = 48;\quad ar^3 = 3$	B1		For either. OE
	$\Rightarrow 16r^2 = 1$	M1		Elimination of a OE
	$r^2 = \frac{1}{16} \Rightarrow r = -\frac{1}{4}$	A1		CSO **AG** Full valid completion. **SC** Clear explicit verification (max B2 out of 3.)
	or $r = \frac{1}{4}$	B1	4	
(b)(i)	$a = -192$	B1	1	
(b)(i)	$a = -192$	B1	1	
(ii)	$\frac{a}{1-r} = \frac{a}{1-\left(-\frac{1}{4}\right)}$	M1		$\frac{a}{1-r}$ **used**
	$S_\infty = \frac{-768}{5}$ $(= -153.6)$	A1ft	2	Ft on candidate's value for a. i.e. $\frac{4}{5}a$ **SC** candidate uses $r = 0.25$, gives $a = 192$ and sum to infinity = 256. (max. B0 M1A1)
	Total		**7**	

4. June 2007

2(a)	$u_1 = 12$ $u_2 = 3 \times 4^2 = 48$	B1 B1	2	CSO AG (be convinced)
(b)	$r = 4$	B1	1	

(c)(i)	$\{S_{12} =\}\dfrac{a\left(1-r^{12}\right)}{1-r}$	M1		OE Using a correct formula with $n = 12$
	$= \dfrac{12\left(1-4^{12}\right)}{1-4}$	A1ft		Ft on answer for u_1 in (a) and r in (b)
	$= \dfrac{12\left(1-4^{12}\right)}{-3} = -4(1-4^{12}) = 4^{13}-4$	A1	3	CAO Accept $k = 13$ for 4^{13} term
(ii)	$\sum_{n=2}^{12} u_n = (4^{13}-4) - u_1$ $= 67108848$	B1	1	
	Total		**7**	

5. June 2008

3(a)	$r = 16 \div 20 = 0.8$	B1	1	OE
(b)	$\dfrac{a}{1-r} = \dfrac{20}{1-0.8}$	M1		OE Using a correct formula with $a = 20$ or $r =$ c's 0.8
	$= 100$	A1F	2	ft on c's value of r provided $\lvert r \rvert < 1$
(c)	$\{S_{20} =\}\dfrac{a\left(1-r^{20}\right)}{1-r}$	M1		OE Using a correct formula with $n = 20$
	$= 100\left(1-0.8^{20}\right) = 98.847\{07..\}$	A1	2	Condone > 3dp
(d)	nth term $= 20\,r^{n-1} = 20(0.8)^{n-1}$	M1		Ft on c's r . Award even if 16^{n-1} seen
	$= 20\times0.8^{-1}\times0.8^{n}$ $= 25\times0.8^{n}$	A1	2	CSO; AG
	Total		**7**	

6. June 2009

7(a)(i)	$ar = 375;\quad ar^4 = 81$	B1		For either OE or PI by next line
	$\Rightarrow 375r^3 = 81$	M1		Elimination of a OE
	$r^3 = \dfrac{81}{375} = \dfrac{27}{125} = 0.216 \Rightarrow r = 0.6$	A1	3	CSO AG Full valid completion SC: Clear explicit verification, with statement max B1 out of 3. (If considers uniqueness then 3 is possible)
(ii)	$0.6a = 375$ $a = 625$	M1 A1	2	OE; PI

(b)	$\dfrac{a}{1-r} = \dfrac{a}{1-0.6}$	M1		$\dfrac{a}{1-r}$ **used** with $\lvert$value of $r\rvert < 1$
	$S_\infty = \dfrac{625}{0.4} = 1562.5$	A1F	2	ft on cand's value for a…. ie $2.5 \times a$
(c)	$\sum_{n=6}^{\infty} u_n = \sum_{n=1}^{\infty} u_n - \sum_{n=1}^{5} u_n$	M1		
	$u_3 = 0.6\,u_2$ (= 225) and $u_4 = 0.6^2\,u_2$ (= 135)	M1		Valid method to either find u_3 and u_4 or use of $\{S_n =\}\dfrac{a\left(1-r^n\right)}{1-r}$ for either $n = 5$ or $n = 6$

	Alternative for (c):			
	Recognise that the sum to infinity with first term u_6 is required	(M1)		
	Valid method to find u_6 $(= 0.6u_5)$	(M1)		
	$\sum_{n=6}^{\infty} u_n = \frac{81\times 0.6}{1-0.6}$	(A1)		
	$= 121.5$	(A1)		
	Total		**11**	

7. June 2010

5(a)(i)	$\{S_\infty =\}\ \frac{a}{1-r} = \frac{10}{1-r}$	M1		$\frac{a}{1-r}$ used
	$\frac{10}{1-r} = 50$ so $1-r = \frac{10}{50} \Rightarrow r = \frac{4}{5}$	A1	2	AG Condone verification with the correct final statement but be convinced.
(ii)	2nd term $= ar$	M1		ar stated or used for the 2nd term. PI by ans '8'
	$= 8$	A1	2	
(b)(i)	4th term $= a + 3d$; 8th term $= a + 7d$	M1		Uses $a + (n-1)d$ correctly at least once
	$a + 3d = 10$, $a + 7d = 8$	A1F		Both eqns. correct ft on c's (a)(ii) OE eg $8 = 10 + 4d$
	$\Rightarrow 4d = -2 \quad \Rightarrow d = -0.5$	A1	3	OE fraction.
(ii)	$a + 3(-0.5) = 10$	M1		An appreciation that a is required in **(b)(ii)** and a valid method to find a anywhere or PI if $a = 11.5$ seen/used
	$\Rightarrow a = 11.5$	A1F		Ft on c's non-zero value for d ie using $a = 10-3d$ or a= c's $8-7d$. [c's 8 is candidate's answer to (a)(ii)]
	$\sum_{n=1}^{40} u_n = S_{40} = \frac{40}{2}[2a + (40-1)d]$	M1		$\frac{40}{2}[2a + (40-1)d]$ OE
	$= 70$	A1	4	
	Total		**11**	

8. Jan 2011

6(a)(i)	$ar^2 = 36;\ \ ar^5 = 972;$	M1		For $ar^2 = 36$ or $ar^5 = 972$ or for seeing $36r^3 = 972$
	$r^3 = \frac{972}{36}\ (= 27) \ \Rightarrow r = 3$	A1	2	CSO AG Full valid completion.
(ii)	$a \times 3^2 = 36$	M1		OE. PI
	$a = 4$	A1	2	Correct answer without working scores the two marks
(b)(i)	$\sum_{n=1}^{20} u_n = S_{20} = \frac{a(1-r^{20})}{1-r}$	M1		OE
	$= \frac{4(1-3^{20})}{-2} = -2(1-3^{20}) = 2(3^{20}-1)$	A1	2	CSO AG Be convinced

(ii)	$u_n = a\times 3^{n-1}$	B1		Seen or used
	$4\times 3^{n-1} > 4\times 10^{15} \Rightarrow 3^{n-1} > 10^{15}$			
	$(n-1)\log 3$ (>) $\log 10^{15}$	M1		Or finds values of u_n for appropriate adjacent integer values of n so that u_n's are either side of 4×10^{15}
	$n-1 > \dfrac{15}{\log_{10} 3}$; $n-1 > 31.4...$			
	($n > 32.4...$ and n is an integer so least value of n is) $n = 33$	A1	3	CSO
	Total		**9**	

9. June 2011

9(a)	$\{S_\infty =\}\ \dfrac{a}{1-r} = \dfrac{12}{1-\frac{3}{8}}$	M1		$\dfrac{a}{1-r}$ used
	$\{S_\infty =\}$ 19.2	A1	2	19.2 OE NMS mark as 2/2 or 0/2
(b)	{6th term = } ar^{6-1}	M1		Stated or used
	$= 12\times\left(\dfrac{3}{8}\right)^5 = 2\times 2\times 3\times\left(\dfrac{3}{2\times 2\times 2}\right)^5$	m1		Changing 8 and 12 in correct expression to correct products/powers of 2 and 3
	$= \dfrac{2\times 2\times 3\times 3^5}{(2^3)^5} = \dfrac{2^2\times 3^6}{2^{15}} = \dfrac{3^6}{2^{13}}$	A1	3	AG Be convinced

(c)(i)	$\{u_n =\}\ 12\times\left(\dfrac{3}{8}\right)^{n-1}$	B1	1	OE. eg $32(3/8)^n$
(ii)	$\log u_n = \log 12 + \log\left(\dfrac{3}{8}\right)^{n-1}$			Log laws log(PQ)=logP + logQ ; $\log\left(\dfrac{P}{Q}\right) = \log P - \log Q$ $\log(P)^k = k\log P$
	$\log u_n = \log 12 + (n-1)\log\left(\dfrac{3}{8}\right)$			
	$\log u_n = \log 12 + (n-1)[\log 3 - \log 8]$	M1		Using (c)(i) and taking logs: one log law used correctly, on a correct expression for u_n.
		M1		a second different log law used correctly, indep of prev M error and ft on cand's (c)(i) provided cand's u_n expression has a power involving n.
	$\log u_n = \log 3 + 2\log 2 + (n-1)[\log 3 - 3\log 2]$	m1		A third different log law used correctly (or equivalent valid step) to reach a correct RHS whose terms are all multiples of log2 and log3. Dep on both prev two Ms
	$\log u_n = n\log 3 - 3n\log 2 + 5\log 2$ $\log_a u_n = n\log_a 3 - (3n-5)\log_a 2$	A1	4	CSO AG Be convinced, no slips although we will condone the absence of the bases a even in the final line.
	Total		**10**	

10.June 2012

4(a)	$u_1 = 12$	B1		CAO Must be 12
	$u_2 = 48 \times \frac{1}{16} = 3$	B1F	2	If not correct, ft on c's $u_1 \times \frac{1}{4}$
(b)	$r = \frac{1}{4}$	B1F	1	Only ft on r = (c's u_2)÷ (c's u_1) if $\lvert r \rvert < 1$. Answers may be in equivalent fraction form or exact decimal form. If other notation used award the mark if correct or ft value confirmed in (c)
(c)	$(S_\infty =) \frac{u_1}{1-r} = \frac{12}{1-\frac{1}{4}}$	M1		Use of $\frac{a}{1-r}$, ft on c's u_1 and c's r in (a) and (b) if not recovered, provided $\lvert r \rvert < 1$
	$= 16$	A1F	2	If not 16, ft on c's u_1 and c's r in (a) and (b) provided $\lvert r \rvert < 1$.
(d)	$\sum_{n=4}^{\infty} u_n = S_\infty - \sum_{n=1}^{3} u_n$	M1		OE eg RHS $S_\infty - (u_1 + u_2 + u_3)$
	$u_3 = \frac{3}{4}$ (or $\sum_{n=1}^{3} u_n = \frac{12(1-0.25^3)}{1-0.25}$)	B1		Either result, or better eg $\sum_{n=1}^{3} u_n = 15.75$
	$\sum_{n=4}^{\infty} u_n = 0.25$	A1	3	NMS scores 0/3 **SC** For c's scoring 0/3 in (d); Award B1 to candidates who used $S_\infty - S_4$ for $\sum_{n=4}^{\infty} u_n$ and obtained the answer $\frac{1}{16}$ OE
	(Alternative) $(\sum_{n=4}^{\infty} u_n = \frac{u_4}{1-r})$	(M1)		
	$(u_4 = \frac{3}{16}$ (= 0.1875))	(B1)		
	$(\sum_{n=4}^{\infty} u_n = \frac{3}{16} \div \frac{3}{4} = \frac{1}{4})$	(A1)	(3)	(NMS scores 0/3)
	Total		**8**	

11.Jan 2013

6(a)(i)	$r = \frac{294}{420} = 0.7$	B1	1	AG. Accept any valid justification to the given answer
(ii)	$\{S_\infty =\} \frac{a}{1-r} = \frac{420}{1-0.7}$	M1		$\frac{a}{1-r}$ used
	$\{S_\infty =\}$ 1400	A1	2	1400 NMS mark as 2/2 or 0/2
(iii)	nth term = $600 \times (0.7)^n$	B2	2	If not B2 award B1 for $420 \times (0.7)^{n-1}$ OE
(b)(i)	$\{u_n =\} 248 - 8n$	B1	1	Accept ACF
(ii)	$u_k = 0 \Rightarrow 8k = 248$	M1		$248-8k=0$ OE e.g. $240+(k-1)(-8)=0$ ft if no recovery, on c's (b)(i) answer

	$k = 31$	A1		
	$\sum_{n=1}^{k} u_n = 240+232+\ldots+0 = \frac{k}{2}[240+0]$	M1		For $\frac{k}{2}[240+0]$ or for $\frac{k}{2}[\text{c's } u_1 + 0]$ OE e.g. $\frac{k}{2}[2 \times \text{c's } u_1 + (k-1)(-8)]$
	$\sum_{n=1}^{k} u_n$ (= 15.5×240) = 3720	A1	4	3720
	Total		**10**	