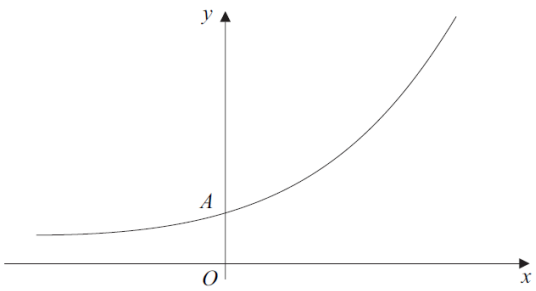
Practice examination questions for trapezium rule

1. Jan 2005

8 The diagram shows a sketch of the curve with equation $y = 3^x + 1$.



The curve intersects the y-axis at the point A.

(a) Write down the y-coordinate of point A. *(1 mark)*

(b) (i) Use the trapezium rule with five ordinates (four strips) to find an approximation for

$\int_0^1 (3^x + 1)\,dx$, giving your answer to three significant figures. *(4 marks)*

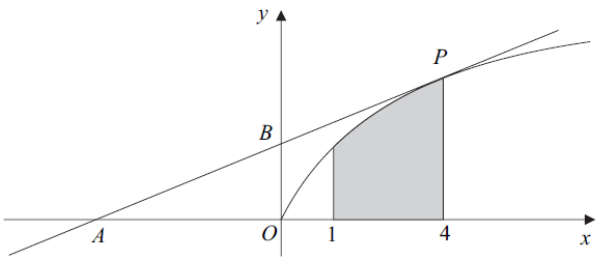
(ii) By considering the graph of $y = 3^x + 1$, explain with the aid of a diagram whether your approximation will be an overestimate or an underestimate of the true value of $\int_0^1 (3^x + 1)\,dx$. *(2 marks)*

(c) The line $y = 5$ intersects the curve $y = 3^x + 1$ at the point P. By solving a suitable equation, find the x-coordinate of the point P. Give your answer to four decimal places. *(4 marks)*

(d) The curve $y = 3^x + 1$ is reflected in the y-axis to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(1 mark)*

2. June 2005

4 The diagram shows a curve C with equation $y = \sqrt{x}$. The point O is the origin $(0, 0)$.



The region bounded by the curve C, the x-axis and the vertical lines $x = 1$ and $x = 4$ is shown shaded in the diagram.

(a) (i) Write $\sqrt{x}$ in the form x^p, where p is a constant. *(1 mark)*

(ii) Find $\int \sqrt{x}\,dx$. *(2 marks)*

(iii) Hence find the area of the shaded region. *(3 marks)*

(b) The point on C for which $x = 4$ is P. The tangent to C at the point P intersects the x-axis and the y-axis at the points A and B respectively.

(i) Find an equation for the tangent to the curve C at the point P. *(4 marks)*

(ii) Find the area of the triangle AOB. *(3 marks)*

(c) Describe the single geometrical transformation by which the curve with equation $y = \sqrt{x-1}$ can be obtained from the curve C. *(2 marks)*

(d) Use the trapezium rule with four ordinates (three strips) to find an approximation for $\int_1^4 \sqrt{x-1}\,dx$, giving your answer to three significant figures. *(4 marks)*

3. Jan 2006

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

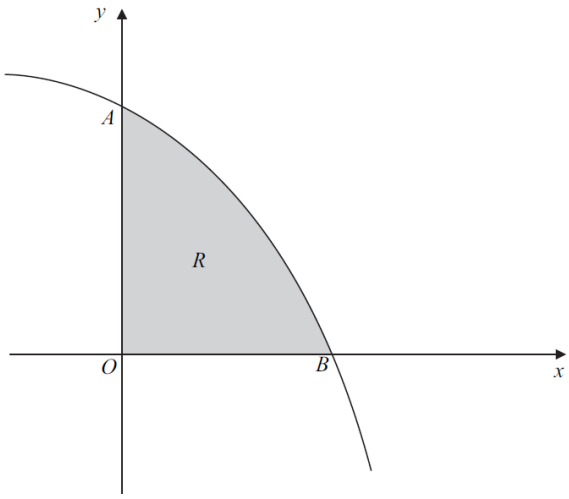
$$\int_0^4 \frac{1}{x^2+1}\,dx$$

giving your answer to four significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

4. June 2006

6 The diagram shows a sketch of the curve with equation $y = 27 - 3^x$.



The curve $y = 27 - 3^x$ intersects the y-axis at the point A and the x-axis at the point B.

(a) (i) Find the y-coordinate of point A. *(2 marks)*

(ii) Verify that the x-coordinate of point B is 3. *(1 mark)*

(b) The region, R, bounded by the curve $y = 27 - 3^x$ and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of R. *(4 marks)*

(c) (i) Use logarithms to solve the equation $3^x = 13$, giving your answer to four decimal places. *(3 marks)*

5. Jan 2007

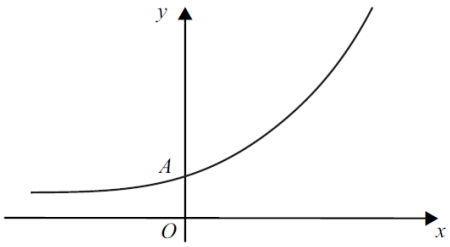
2 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{2^x}\,\mathrm{d}x$$

giving your answer to three decimal places. *(4 marks)*

6. June 2007

6 The diagram shows a sketch of the curve with equation $y = 3(2^x + 1)$.



The curve $y = 3(2^x + 1)$ intersects the y-axis at the point A.

(a) Find the y-coordinate of the point A. *(2 marks)*

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 3(2^x + 1)\,\mathrm{d}x$. *(4 marks)*

(c) The line $y = 21$ intersects the curve $y = 3(2^x + 1)$ at the point P.

(i) Show that the x-coordinate of P satisfies the equation

$$2^x = 6$$ *(1 mark)*

(ii) Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(3 marks)*

7. Jan 2008

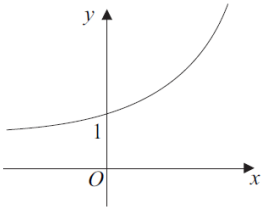
4 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{x^2 + 3}\,\mathrm{d}x$$

giving your answer to three decimal places. *(4 marks)*

8. June 2008

8 The diagram shows a sketch of the curve with equation $y = 6^x$.



(a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 6^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of $\int_0^2 6^x \, dx$. *(2 marks)*

(b) (i) Describe a single geometrical transformation that maps the graph of $y = 6^x$ onto the graph of $y = 6^{3x}$. *(2 marks)*

(ii) The line $y = 84$ intersects the curve $y = 6^{3x}$ at the point A. By using logarithms, find the x-coordinate of A, giving your answer to three decimal places. *(4 marks)*

(c) The graph of $y = 6^x$ is translated by $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ to give the graph of the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(2 marks)*

9. Jan 2009

2 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_{1.5}^{6} x^2\sqrt{x^2 - 1} \, dx$$

giving your answer to three significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

10. June 2009

4 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 \sqrt{x^3 + 1} \, dx$, giving your answer to four significant figures. *(4 marks)*

(b) The curve with equation $y = \sqrt{x^3 + 1}$ is stretched parallel to the x-axis with scale factor $\frac{1}{2}$ to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(2 marks)*

11. Jan 2010

6 (a) Sketch the curve with equation $y = 2^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

(b) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 2^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(ii) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

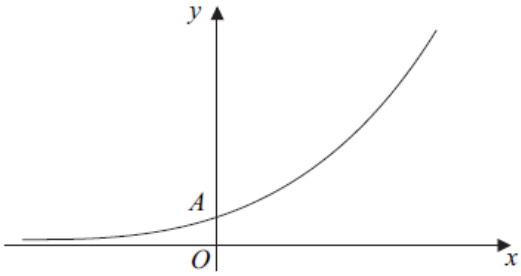
(c) Describe a geometrical transformation that maps the graph of $y = 2^x$ onto the graph of $y = 2^{x+7} + 3$. *(3 marks)*

(d) The curve $y = 2^{x+k} + 3$ intersects the y-axis at the point $A(0, 8)$.

Show that $k = \log_m n$, where m and n are integers. *(2 marks)*

12. June 2010

8 The diagram shows a sketch of the curve $y = 2^{4x}$.



The curve intersects the y-axis at the point A.

(a) Find the value of the y-coordinate of A. *(1 mark)*

(b) Use the trapezium rule with six ordinates (five strips) to find an approximate value for $\int_0^1 2^{4x} \, dx$, giving your answer to two decimal places. *(4 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = 2^{4x}$ onto the graph of $y = 2^{4x-3}$. *(2 marks)*

(d) The curve $y = 2^{4x}$ is translated by the vector $\begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix}$ to give the curve $y = g(x)$.

The curve $y = g(x)$ crosses the x-axis at the point Q. Find the x-coordinate of Q. *(4 marks)*

(e) (i) Given that

$$\log_a k = 3\log_a 2 + \log_a 5 - \log_a 4$$

show that $k = 10$. *(3 marks)*

(ii) The line $y = \frac{5}{4}$ crosses the curve $y = 2^{4x-3}$ at the point P. Show that the x-coordinate of P is $\dfrac{1}{4\log_{10} 2}$. *(3 marks)*

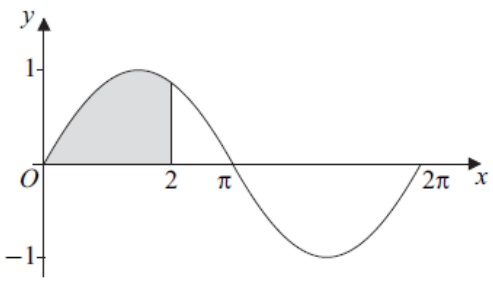
13. Jan 2011

4 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^{1.5} \sqrt{27x^3 + 4}\,dx$, giving your answer to three significant figures. *(4 marks)*

(b) The curve with equation $y = \sqrt{27x^3 + 4}$ is stretched parallel to the x-axis with scale factor 3 to give the curve with equation $y = g(x)$. Write down an expression for $g(x)$. *(2 marks)*

14. June 2011

6 A curve C, defined for $0 \leqslant x \leqslant 2\pi$ by the equation $y = \sin x$, where x is in radians, is sketched below. The region bounded by the curve C, the x-axis from 0 to 2 and the line $x = 2$ is shaded.



(a) The area of the shaded region is given by $\int_0^2 \sin x\,dx$, where x is in radians.

Use the trapezium rule with five ordinates (four strips) to find an approximate value for the area of the shaded region, giving your answer to three significant figures. *(4 marks)*

(b) Describe the geometrical transformation that maps the graph of $y = \sin x$ onto the graph of $y = 2\sin x$. *(2 marks)*

(c) Use a trigonometrical identity to solve the equation

$$2\sin x = \cos x$$

in the interval $0 \leqslant x \leqslant 2\pi$, giving your solutions in radians to three significant figures. *(4 marks)*

15. Jan 2012

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

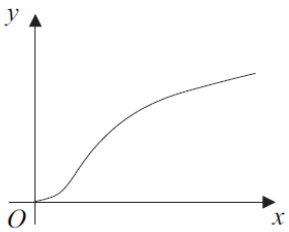
$$\int_0^4 \frac{2^x}{x+1}\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

16. June 2012

9 The diagram shows part of a curve whose equation is $y = \log_{10}(x^2 + 1)$.



(a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^1 \log_{10}(x^2 + 1)\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) The graph of $y = 2\log_{10}x$ can be transformed into the graph of $y = 1 + 2\log_{10}x$ by means of a translation. Write down the vector of the translation. *(1 mark)*

(c) (i) Show that $\log_{10}(10x^2) = 1 + 2\log_{10}x$. *(2 marks)*

(ii) Show that the graph of $y = 2\log_{10}x$ can also be transformed into the graph of $y = 1 + 2\log_{10}x$ by means of a **stretch**, and describe the stretch. *(4 marks)*

(iii) The curve with equation $y = 1 + 2\log_{10} x$ intersects the curve $y = \log_{10}(x^2 + 1)$ at the point P. Given that the x-coordinate of P is positive, find the gradient of the line OP, where O is the origin. Give your answer in the form $\log_{10}\left(\frac{a}{b}\right)$, where a and b are integers. *(4 marks)*

17. Jan 2013

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_1^5 \frac{1}{x^2 + 1}\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) (i) Find $\int \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}}\right) dx$, giving the coefficient of each term in its simplest form. *(3 marks)*

(ii) Hence find the value of $\int_1^4 \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}}\right) dx$. *(2 marks)*

Answers

1. Jan 2005

8(a)	{y-coordinate of A is} 2	B1	1	
(b)(i)	$h = 0.25$	B1		
	Integral $= \frac{h}{2}\ \{\ldots\}$			
	$\{\ldots\} =$			
	$f(0) + 2[f\left(\frac{1}{4}\right) + f\left(\frac{1}{2}\right) + f\left(\frac{3}{4}\right)] + f(1)$	M1		
	$\{\ldots\} =$			Condone one numerical slip
	$\underline{2} + 4 + 2[(2.316.. + 2.732.. + 3.279(5.)]$	A1√		Accept values to 3sf (rnd or trunc)
	$\{= 6 + 2 \times 8.3276..\}\ \{= 22.65(5\ldots)\}$			ft answer from (a) if not "2"
	Integral $= 0.125 \times 22.655.. = 2.8319..$			
	Integral $= 2.83$ to 3 sf	A1	4	CAO Must be 2.83 (NMS scores 0/4)
(c)	$5 = 3^x + 1 \Rightarrow 3^x = 4$	B1		
	$\log_{10} 3^x = \log_{10} 4$	M1		Takes ln or $\log_{10}$ on both sides of $3^x = k$, where $k > 0$
	$x \log_{10} 3 = \log_{10} 4$	m1		Use of $\log 3^x = x \log 3$
	$x = \frac{\lg 4}{\lg 3} = 1.26185\ldots = 1.2619$ to 4dp	A1	4	Accept 4dp or better [If using T&I a full justification is required; else M0m0A0]
(d)	$f(x) = 3^{-x} + 1$	B1	1	
	Total		**12**	

2. June 2005

4(a)(i)	$\sqrt{x} = x^{\frac{1}{2}}$	B1	1	Accept $p = 0.5$
(ii)	$\int \sqrt{x}\ dx = \frac{x^{1.5}}{1.5}\{+c\}$	M1		Index raised by 1
		A1√	2	Correct ft on p. Condone missing '+c'
(iii)	Area $= \int_1^4 \sqrt{x}\ dx$	B1		Limits 1 and 4 PI
	$\ldots\ldots = \frac{4^{1.5}}{1.5} - \frac{1}{1.5}$	M1		F(4) – F(1)
	$= \frac{14}{3}$	A1	3	Accept 4.66 or better
(b)(i)	$y = x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	M1		Index (p–1) ft
	When $x = 4$, $y'(4) = 0.25$	M1		Attempt to find $y'(4)$.
	When $x = 4$, $y = 2$	B1		
	Equation of tangent: $y - 2 = \frac{1}{4}(x - 4)$	A1	4	accept other forms

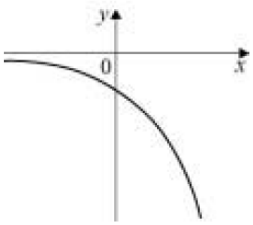
(ii)	When $x = 0$, $y = 1$ $B(0, 1)$ When $y = 0$, $x = -4$ $A(-4, 0)$ Area = 0.5(1)(4) = 2	M1 A1√ A1√	3	Subs $x = 0$ **and** then $y = 0$ into equation of tangent. PI Correct ft y_B and x_A (may be awarded as part of area calculation) ft wrong sloping tangent and max of 1 further slip. Final answer must be +'ve
(c)	Translation $\begin{bmatrix}1\\0\end{bmatrix}$	B1 B1	2	'Translation'/'translate(d)' Accept equivalent in words provided linked to 'translation/move/shift' (Note: B0B1 is possible)
(d)	$h = 1$ Integral = $h/2$ {……} {….}= f(1) + 2[f(2) + f(3)] + f(4) {….}= $0 + 2(1+\sqrt{2}) + \sqrt{3}$ Integral = $\frac{1}{2}\{\ 2(1+1.414...)+1.732\ \}$ Integral = $0.5 \times 6.560\ldots$ =3.28 to 3sf	B1 M1 A1 A1	4	PI OE summing of areas of the three traps Condone 1 numerical slip CAO Must be 3.28
	Total		**19**	

3. Jan 2006

2(a)	$h = 1$ Integral $= \frac{h}{2}\{\ldots\}$ $\{\ldots\} = f(0) + f(4) + 2[f(1) + f(2) + f(3)]$ $= \left[1 + \frac{1}{17} + 2\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)\right]$ Integral = 1.329	B1 M1 A1 A1	4	PI OE summing of areas of the four trapezia. [0.75+0.35+0.15+0.079…] Exact or to 3dp values Condone one numerical slip **CSO**. Must be 1.329
(b)	Increase the number of ordinates	E1	1	OE
	Total		**5**	

4. June 2006

6(a)(i)	y-coordinate of A is $27 - 3^0$; = 26	M1A1	2	
(ii)	When $x = 3$, $y = 27 - 3^3 = 0 \Rightarrow B(3,0)$	B1	1	AG; be convinced
(b)	$h = 1$ Area $\approx h/2\{\ldots\}$ $\{\ldots\}$= f(0)+f(3)+2[f(1)+f(2)] $\{\ldots\}$= "26" + 0 + 2(24 + 18) (Area ≈) 55	B1 M1 A1√ A1√	4	PI OE summing of areas of the 'trapezia'.. on (a)(i) (∑trap= "25"+21+9) on [42 + 0.5× "(a)(i)"]
(c)(i)	$\log_{10} 3^x = \log_{10} 13$ $x \log_{10} 3 = \log_{10} 13$	M1 m1		Takes ln or $\log_{10}$ on both or $x = \log_3 13$ Use of $\log 3^x = x \log 3$ or $\log_3 13 = \frac{\lg 13}{\lg 3}$ OE (PI by $\log_3 13$ = 2.335 or better)

	$x = \frac{\lg 13}{\lg 3} = 2.334717\ldots.$ $= 2.3347$ to 4dp	A1	3	Must show that logarithms have been used
(ii)	$\{k =\}$ 14	B1	1	Condone $y = 14$; Accept final answer 14 with only zeros after decimal point eg 14.000
(d)(i)	Translation;	B1;		'Translation'/'translate(d)' B0 if more than one transformation
	$\begin{bmatrix} 0 \\ -27 \end{bmatrix}$	B1	2	Accept full equivalent in words provided linked to 'translation/move/shift' and negative y-direction (Note: B0 B1 is possible)
(ii)		B1 B1	2	Correct shape (translation of given curve vertically downwards) Only point of intersection with coord axes is on negative y-axis and curve is asymptotic to the negative x-axis
	Total		**15**	

5. Jan 2007

2	$h = 1$ $f(x) = \sqrt{2^x}$	B1		PI
	Area $\approx h/2\{\ldots\}$ $\{\ldots\} = f(0)+f(3)+2[f(1)+f(2)]$	M1		OE summing of areas of the 'trapezia'..
	$\{\ldots\} = 1 + \sqrt{8} + 2(\sqrt{2} + 2)$	A1		OE
	(Area $\approx$) 5.3284… = 5.328 (to 3dp)	A1	4	CAO Must be 5.328
	Total		**4**	

6. June 2007

6(a)	$y_A = 3(2^0 + 1)$ $= 6$	M1 A1	2	Substituting $x = 0$ PI
(b)	$h = 2$ Integral $= h/2\ \{\ldots\ldots\}$ $\{\ldots.\} = f(0) + 2[f(2) + f(4)] + f(6)$ $\{\} = 6 + 2[3\times5 + 3\times17] + 3\times65$ $= 6 + 2[15 + 51] + 195$ Integral = 333	B1 M1 A1 A1	4	PI OE summing of areas of the three traps. Condone 1 numerical slip {ft on (**a**) for f(0) if not recovered} [Sum of 3 traps. = 21 + 66 + 246] CAO
(c)(i)	$21 = 3(2^x + 1) \Rightarrow 2^x = 6$	B1	1	AG (be convinced)
(ii)	$\log_{10} 2^x = \log_{10} 6$ $x \log_{10} 2 = \log_{10} 6$ $x = \frac{\lg 6}{\lg 2} = 2.5849\ldots = 2.58$ to 3sf	M1 m1 A1	3	Take ln or $\log_{10}$ of both sides of $a^x = b$ or other relevant base if clear. The equation $a^x = b$ used must be correct. Use of $\log 2^x = x \log 2$ OE Both method marks must have been awarded.
	Total		**10**	

7. Jan 2008

4	$h = 1$	B1		PI
	$I \approx \frac{h}{2}\{\ldots\}$	M1		OE summing of areas of the three 'trapezia'
	$\{\ldots\} = f(0)+f(3)+2[f(1)+f(2)]$ $\{\ldots\} = \sqrt{3} + \sqrt{12} + 2[\sqrt{4} + \sqrt{7}]$	A1		($\sum$trap=1.866.+2.3228.+3.0549)
	$(I \approx) \frac{1}{2}[5.19615. + 2 \times 4.64575...]$			
	$= \frac{1}{2}[14.4876..] = 7.2438.. = 7.244$	A1	4	CAO Must be 3dp.
	Total		**4**	

8. June 2008

8(a)(i)	$h = 0.5$ Integral = $h/2$ {……}	B1		PI
	$\{..\}=f(0)+2[f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)]+f(2)$	M1		OE summing of areas of the four traps.
	$\{\}= 1+2\left[\sqrt{6}+6+6\sqrt{6}\right]+36$ $= 1+2[2.449..+ 6 + 14.6969..] + 36$ $= 37 + 2 \times 23.146.. = 83.292\ldots$	A1		Condone 1 numerical slip. Accept 3sf values if not exact.
	Integral = $0.25 \times 83.292.. = 20.8$ (3sf)	A1	4	CAO; must be 20.8
(ii)	Relevant trapezia drawn on a copy of given graph	M1		Accept single trapezium with its sloping side above the curve
	{Approximation is an}overestimate	A1	2	Dep. on 4 trapezia with each of their upper vertices lying on the curve
(b)(i)	Stretch **(I)** in x-direction **(II)**	M1		Need **(I)** and one of **(II)**, **(III)** M0 if more than one transformation
	(scale factor) $\frac{1}{3}$ **(III)**	A1	2	
(ii)	$6^{3x} = 84$	M1		PI
	$\log_{10} 6^{3x} = \log_{10} 84$	M1		Take logs of both sides of $a^x = b$, PI by 'correct' value(s) later or $3x = \log_6 84$
	$3x \log_{10} 6 = \log_{10} 84$	m1		Use of $\log 6^{3x} = 3x \log 6$ OE or $3x = \log_6 84$ seen
	$x = \frac{\lg 84}{3 \lg 6}$			
	$x = 0.82429\ldots = 0.824$ (to 3dp)	A1	4	Must see that logs have been used before any of the last 3 marks are awarded in (b)(ii). Condone > 3dp
(c)	$f(x) = 6^{x-1} - 2$	B2,1	2	B1 for either $6^{x-1}+2$ or for $6^{x+1}-2$
	Total		**14**	

9. Jan 2009

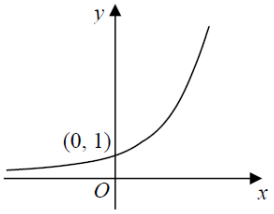
2(a)	$h = 1.5$ $f(x) = x^2\sqrt{x^2 - 1}$ Integral = $h/2$ {….}	B1		PI
	$\{\ldots\} = f(1.5) + 2[f(3) + f(4.5)] + f(6)$	M1		For the M1 covered range must be 1.5 to 6 OE summing of areas of the three traps.

	{….} = 2.51(5..)+2[25.4(5..)+88.8(4..)]+212(.9..)	A1		Check at least 3sf values, rounded or truncated, or award if a combined value WRT 444 is seen or final answer is 333 or rounds to 333 Condone one numerical slip
	Integral = 0.75 × 444.1 = 333 to 3sf	A1cao	4	Must have 333 Treat using 4 strips as a MR and mark with max of B0M1A1A1cao as follows: h =1.125 B0 {….} =f(1.5)+2[f(2.625)+f(3.75)+f(4.875)]+f(6) M1 =2.51(5)+2[16.7(2)+50.8(2)+113(.3)]+212(.9) A1 or award if a combined value WRT 577 is seen or final answer is 325 or rounds to 325. Condone one numerical slip. Answer = 325 A1cao Must have 325
(b)	Increase the number of ordinates	E1	1	OE eg increase the number of strips
	Total		**5**	

10. June 2009

4(a)	$h = 2$ $g(x) = \sqrt{x^3+1}$ I ≈ $h/2\{\ldots\}$ {…} = g(0) + g(6) + 2[g(2) + g(4)]	B1 M1		PI OE summing of areas of the 'trapezia'.. Can award even if MR expression for g(x) but must be using from 0 to 6
	{…} = 1 + √217 + 2(3 + √65) 1 + 14.73... +2(3 + 8.06…)	A1		OE Accept 2dp evidence for surds
	(I ≈) 37.8554… = 37.86 (to 4sf)	A1	4	Must be 37.86
(b)	$f(x) = \sqrt{(2x)^3+1} = \sqrt{8x^3+1}$	M1 A1	 2	$\sqrt{kx^3+1}$, $k \neq 1$ or 0 or f(x) = g(2x) Either form acceptable
	Total		**6**	

11. Jan 2010

6(a)		B1 B1	 2	Shape with some indication of asymptotic behaviour in 2nd quadrant below pt of intersection with y-axis Only intersection is with y-axis at (0, 1) stated/indicated … (accept 1 on y-axis as equivalent)
(b)(i)	$h = 0.5$ $f(x) = 2^x$ I ≈ $h/2\{\ldots\}$ {…} = f(0)+f(2)+2[f(0.5)+f(1)+f(1.5)]	B1 M1		PI OE summing of areas of the 4 'trapezia'
	{…} = 1 + 4 + 2(√2 + 2 + √8) = 5 + 2 × 6.2426… = 17.485…	A1		OE Accept 2dp (rounded or truncated) as evidence for surds
	(I ≈) 4.3713… = 4.37 (to 3sf)	A1	4	CAO Must be 4.37 **SC** for those who use 5 strips, max possible is B0M1A1A0

(ii)	Increase the number of ordinates	E1	1	OE
(c)	Translation;	B1;		Accept 'translat…' as equivalent [T or Tr is NOT sufficient]
	$\begin{bmatrix} -7 \\ 3 \end{bmatrix}$	B1;B1	3	B1 for each component of the vector. Condone if the equiv 2 vectors are given. Accept **full** equivalent to vector(s) in words provided linked to 'translation/ move/shift' and **correct** directions. (No marks if **different** transformations)
(d)	$8 = 2^k + 3 \Rightarrow 2^k = 5$	M1		Correct subst. and an attempted rearrangement to $2^k = N$. PI by $k = \frac{\log 5}{\log 2}$
	$k = \log_2 5$	A1	2	Accept $m = 2$, $n = 5$
	Total		**12**	

12. June 2010

8(a)	$(y =)\ 1$	B1	1	
(b)	$h = 0.2$	B1		PI
	$f(x) = 2^{4x}$			
	$I \approx h/2\{\ldots\}$			
	$\{.\}=f(0)+f(1)+2[f(0.2)+f(0.4)+f(0.6)+f(0.8)]$	M1		OE summing of areas of the 'trapezia'..
	$\{.\} = 1 + 16+2(2^{0.8} + 2^{1.6} +2^{2.4} +2^{3.2})$ $=1+16+2(1.741..+3.031..+5.278..+9.1895..)$ $= [17+2\times19.24...]$	A1		OE Accept 2dp rounded or truncated evidence
	$I = 5.55$ (to2dp)	A1	4	Must be 5.55
(c)	Stretch(**I**) in y-direction(**II**) scale factor $\frac{1}{8}$ (**III**)	M1 A1	 2	Need (**I**) **and** either (**II**) or (**III**) Need (**I**) and (**II**) and (**III**)
	<u>ALTn</u>: Translation with an indication that the translation is in the x-direction (B1) $\begin{bmatrix} \frac{3}{4} \\ 0 \end{bmatrix}$ (B1)			Combination of **<u>different</u>** transformations scores 0/2
(d)	$g(x) = 2^{4(x-1)} - \frac{1}{2}$	B2,1,0		B1 for either $2^{4(x+1)} - \frac{1}{2}$ or for $2^{4(x-1)} + \frac{1}{2}$ or for $2^{4x-1} - \frac{1}{2}$
	At Q, $y = 0 \Rightarrow 2^{4(x-1)} = 2^{-1}$	M1		Reaches a stage from which linear eqn can be stated directly eg an alternative stage is $4(x-1)\log 2 = -\log 2$
	$\Rightarrow 4x - 4 = -1 \Rightarrow x = 0.75$	A1	4	NMS mark as 4 or 0
(e)(i)	$\log_a k = \log_a 2^3 + \log_a 5 - \log_a 4$	M1		One law of logs used
	$\log_a k = \log_a (2^3 \times 5) - \log_a 4$	M1		A second law of logs used; could be $\log_a k = \log_a 2^3 + \log_a (\frac{5}{4})$
	$\log_a k = \log_a (\frac{2^3 \times 5}{4}) = \log_a 10 \Rightarrow k = 1$	A1	3	CSO AG

(ii)	$2^{4x-3} = \frac{5}{4}$ so			Equate y's, take logs (to any base) of both sides **and** apply 3rd law of logs.
	$(4x-3)\log_{10} 2 = \log_{10}\frac{5}{4}$	M1		Altn $4x\log 2 = \log\left(\frac{5}{4}\times 2^3\right)$
	$x = \dfrac{3\log_{10} 2 + \log_{10}\left(\frac{5}{4}\right)}{4\log_{10} 2}$			Rearrange correctly to $x = \ldots$. Altn $4x\log 2 = \log 10$
		m1		In both cases, log term(s) must have same base and expressions must be in an exact form, ie not approx. dec. vals
	$x = \frac{\log_{10} 10}{4\log_{10} 2}$ so $x = \frac{1}{4\log_{10} 2}$	A1	3	CSO AG Must be clear evidence that base 10 is used, also be convinced
	Total		**17**	

13. Jan 2011

4(a)	$h = 0.5$	B1		PI
	$f(x) = \sqrt{27x^3+4}$ $I \approx h/2\{\ldots\}$ $\{\ldots\} = f(0) + f(1.5) + 2[f(0.5) + f(1)]$	M1		OE summing of areas of the 'trapezia'..
	$\{\ldots\} = \sqrt{4} + \sqrt{95.125} + 2(\sqrt{7.375} + \sqrt{31})$ $= 2 + 9.7532\ldots + 2(2.7156\ldots + 5.5677\ldots)$	A1		OE Accept 2dp rounded or truncated as evidence for surds
	$(I \approx)\ 0.25\times 28.32012\ldots = 7.08$ (to 3sf)	A1	4	Must be 7.08
(b)	$g(x) = \sqrt{27\left(\frac{1}{3}x\right)^3 + 4} = \sqrt{x^3+4}$	M1		Any form which simplifies to $\sqrt{kx^3+4}$, $k\neq 27$, $k\neq 0$ or which simplifies to x^3+4
		A1	2	ACF
	Total		**6**	

14. June 2011

6(a)	$h = 0.5$	B1		$h = 0.5$ stated or used. (PI by x-values 0, 0.5, 1, 1.5, 2 provided no contradiction)
	$f(x) = \sin x$ $I \approx h/2\{\ldots\}$ $\{.\} = f(0) + f(2) + 2[f(0.5) + f(1) + f(1.5)]$	M1		OE summing of areas of 'trapezia'..
	$\{.\} =$ $0 + 0.90929.. + 2[0.4794.. + 0.84147.. + 0.99749..]$	A1		Min. of 2dp values rounded or truncated. Can be implied by later correct work provided >1 term or a single term which rounds to 1.39
	$\{= 0.90929.. + 2[2.318..]\} = \{0.90929.. + 4.636..\}$			
	$(I \approx)\ 0.25[5.546..] = 1.3865.. = 1.39$ (to 3sf)	A1	4	CAO Must be 1.39
(b)	Stretch(**I**) in y-direction(**II**) scale factor 2(**III**)	M1 A1	2	Need (**I**) **and** either (**II**) or (**III**) All correct. Need (**I**) and (**II**) and (**III**) [>1 transformation scores 0/2]
(c)	$\frac{\sin x}{\cos x} = \frac{1}{2}$; $\tan x = 0.5$	M1		$\frac{\sin x}{\cos x} = \tan x$ used to get $\tan x = k$ or identity $\cos^2 x + \sin^2 x = 1$ used to get either $\sin^2 x = p$ or to get $\cos^2 x = q$, (p and q must be between 0 and 1)
	$\tan x = 0.5$	A1		Either $\tan x = \frac{1}{2}$ or $\cos x = \pm\sqrt{\frac{4}{5}}$ (=±0.894..)
				or $\sin x = \pm\sqrt{\frac{1}{5}}$ (=±0.447..)

	$x = \alpha$ or $\pi + \alpha$ where $\alpha = \tan^{-1}(k)$	m1		Correct method to find 2nd angle. Any in wrong ft quadrants then m0. In case of squaring method candidates must also have rejected the extra 'quadrants' for the m1. Condone degrees or mixture
	$x = 0.464, 3.61$	A1	4	Both. Condone>3sf [0.463(6..), 3.60(5..or 6.)] Accept **pair** of truncated values [0.463, 3.60] Ignore any answers outside interval 0 to 6.28
	Total		**10**	

15. Jan 2012

2(a)	$h = 1$	B1		$h = 1$ stated or used. (PI by x-values 0,1,2,3,4 provided no contradiction)
	$f(x) = \dfrac{2^x}{x+1}$ $I \approx h/2\{\ldots\}$ $\{.\}$=f(0)+f(4)+2[f(1)+f(2)+f(3)]	M1		OE summing of areas of the 'trapezia'..
	$\{.\} = 1 + \dfrac{16}{5} + 2\left(\dfrac{2}{2} + \dfrac{4}{3} + \dfrac{8}{4}\right)$ =1+3.2+2(1+1.33…+2)	A1		OE Accept 1dp evidence. Can be implied by later correct work provided >1 term or a single term which rounds to 6.43
	(I ≈) 0.5[4.2+2×4.333..] =6.43 (to 3sf)	A1	4	CAO Must be 6.43
(b)	Increase the number of ordinates	E1	1	OE eg increase the number of strips.
	Total		**5**	

16. June 2012

9(a)	$h = 0.25$ $f(x) = \log_{10}(x^2 + 1)$ $I \approx h/2\{\ldots\}$	B1		PI
	$\{.\} = f(0) + f(1) +$ $2\,[f(0.25) + f(0.5) + f(0.75)]$	M1		OE summing of areas of the 'trapezia'
	$\{.\} =$ $\log 1 + \log 2 + 2\left[\log\dfrac{17}{16} + \log\dfrac{5}{4} + \log\dfrac{25}{16}\right]$ $= 0 + 0.3010... +$ $2\,(0.0263... + 0.0969... + 0.1938...)$	A1		OE Accept 1sf evidence
	$= 0.3010...+2(0.317058...) = 0.935147\ldots$ (I ≈) 0.125 [0.935147…] = 0.117 (to 3SF)	A1	4	CAO Must be 0.117
(b)	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	B1	1	
(c)(i)	$\log_{10}(10x^2) = \log_{10} 10 + \log_{10} x^2$	M1		Condone missing bases for M mark. Accept $\log x^2$ replaced by 2logx in M1 line AG. Bases must be included or statement '$\log_{10} 10 = 1$' given.
	$= 1 + 2\log_{10} x$	A1	2	
				Condone missing bases in (c)(ii) & (c)(iii)
(ii)	$y = 1 + 2\log_{10} x = \log_{10}(10x^2)$	M1		PI
	Either $y = 2\log_{10}(\sqrt{10}\,x)$ (to compare $y = 2\log x$) or both $y = \log_{10} x^2$ and $y = \log_{10}(\sqrt{10}\,x)^2$	A1		Writing in correct form so that stretch details can be stated directly B2 for correct direction and scale factor ACF

	(Stretch) parallel to x-axis, sf $\frac{1}{\sqrt{10}}$ OE	B2,1,0	4	(B1 for correct exact scale factor ACF) (or B1 for 'x-direction, scale factor 1/10 ') (or B1for 'x-direction, scale factor $\sqrt{10}$ ') Apply ISW if dec follows exact values. (OE scale factor must be in exact form)
(iii)	$\log_{10}(10x^2) = \log_{10}(x^2+1)$ $(10x^2 = x^2+1,\ 9x^2 = 1$	M1		PI by $10x^2 = x^2+1$ or correct x
	and since $x > 0$) $x = \frac{1}{3}$	A1		$x = \frac{1}{3}$ OE stated or used; accept $\sqrt{\frac{1}{9}}, \frac{1}{\sqrt{9}}$
	(y-coordinate of P) $y = \log_{10}\frac{10}{9}$ Or $y = \log\left(\frac{1}{9}+1\right)$	A1		PI by $3\log\frac{10}{9}$ OE for the gradient of OP
	Gradient of OP = $3\log_{10}\frac{10}{9} = \log_{10}\frac{1000}{729}$	A1	4	$\log\frac{1000}{729}$; Accept 'a=1000, b=729'
	Total		**15**	

17. Jan 2013

2(a)	$h = 1$ $f(x) = \frac{1}{x^2+1}$	B1		PI
	$I \approx \frac{h}{2}\{f(1)+f(5)+2[f(2)+f(3)+f(4)]\}$	M1		$\frac{h}{2}\{f(1)+f(5)+2[f(2)+f(3)+f(4)]\}$ OE summing of areas of the four 'trapezia'…
	$\frac{h}{2}$ with $\{\ldots\} = \frac{1}{2}+\frac{1}{26}+2\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{17}\right)$ $= 0.5 + 0.03(84...)+2[0.2+0.1+0.05(88...)]$ $= 0.538(46\ldots)+2[0.358(82...)]=1.256(108\ldots)$	A1		OE Accept 2dp (rounded or truncated) for non-terminating decs. equiv.
	$(I \approx)$ $0.628054\ldots = \frac{694}{1105} = 0.628$ (to 3sf)	A1	4	CAO Must be 0.628 **SC** for those who use 5 strips, max possible is B0M1A1A0
(b)(i)	$\int\left(x^{-\frac{3}{2}}+6x^{\frac{1}{2}}\right)dx = \frac{x^{-\frac{1}{2}}}{-1/2}+\frac{6x^{\frac{3}{2}}}{3/2}$ $(+c)$	M1 A1		One term correct (even unsimplified) Both terms correct (even unsimplified)
	$= -2x^{-0.5}+4x^{1.5}$ $(+c)$	A1	3	Must be simplified.
(ii)	$\int_1^4\left(x^{-\frac{3}{2}}+6x^{\frac{1}{2}}\right)dx$ $= [-2(4^{-0.5})+4(4^{1.5})] - [-2(1^{-0.5})+4(1^{1.5})]$	M1		Attempt to calculate F(4)−F(1) where F(x) follows integration and is not just the integrand
	$= (-1+32) - (-2+4) = 29$	A1	2	Since 'Hence' NMS scores 0/2
	Total		**9**	