Practice examination questions for integration

1. Jan 2005

4 (a) Write $\sqrt{x}$ in the form x^k, where k is a fraction. *(1 mark)*

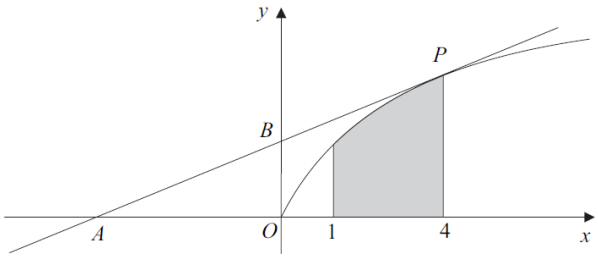
(b) Hence express $\sqrt{x}(x-1)$ in the form $x^p - x^q$. *(2 marks)*

(c) Find $\int \sqrt{x}(x-1)\,dx$. *(3 marks)*

(d) Hence show that $\int_1^2 \sqrt{x}(x-1)\,dx = \frac{4}{15}\left(\sqrt{2}+1\right)$. *(3 marks)*

2. June 2005

4 The diagram shows a curve C with equation $y = \sqrt{x}$. The point O is the origin $(0, 0)$.



The region bounded by the curve C, the x-axis and the vertical lines $x = 1$ and $x = 4$ is shown shaded in the diagram.

(a) (i) Write $\sqrt{x}$ in the form x^p, where p is a constant. *(1 mark)*

(ii) Find $\int \sqrt{x}\,dx$. *(2 marks)*

(iii) Hence find the area of the shaded region. *(3 marks)*

(b) The point on C for which $x = 4$ is P. The tangent to C at the point P intersects the x-axis and the y-axis at the points A and B respectively.

(i) Find an equation for the tangent to the curve C at the point P. *(4 marks)*

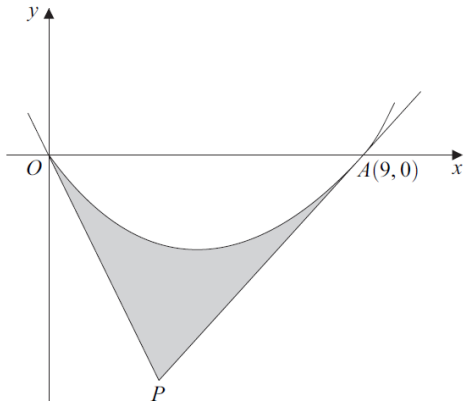
(ii) Find the area of the triangle AOB. *(3 marks)*

(c) Describe the single geometrical transformation by which the curve with equation $y = \sqrt{x-1}$ can be obtained from the curve C. *(2 marks)*

(d) Use the trapezium rule with four ordinates (three strips) to find an approximation for $\int_1^4 \sqrt{x-1}\,dx$, giving your answer to three significant figures. *(4 marks)*

3. Jan 2006

8 A curve, drawn from the origin O, crosses the x-axis at the point $A(9, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = x^{\frac{3}{2}} - 3x$$

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\dfrac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(9, 0)$ is $2y = 3x - 27$. *(3 marks)*

(iii) Hence find the coordinates of the point P where the two tangents meet. *(3 marks)*

(c) Find $\int \left(x^{\frac{3}{2}} - 3x \right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve and the tangents OP and AP. *(5 marks)*

4. June 2006

7 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$$

(a) (i) Verify that $\dfrac{dy}{dx} = 0$ when $x = 4$. *(1 mark)*

(ii) Write $\dfrac{16}{x^2}$ in the form $16x^k$, where k is an integer. *(1 mark)*

(iii) Find $\dfrac{d^2y}{dx^2}$. *(3 marks)*

(iv) Hence determine whether the point where $x = 4$ is a maximum or a minimum, giving a reason for your answer. *(2 marks)*

(b) The point $P(1, 8)$ lies on the curve.

(i) Show that the gradient of the curve at the point P is 12. *(1 mark)*

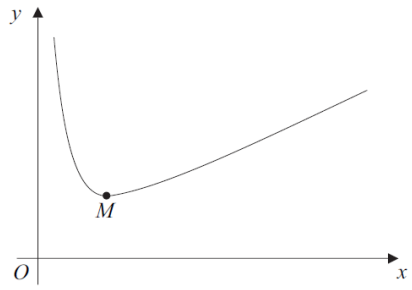
(ii) Find an equation of the normal to the curve at P. *(3 marks)*

(c) (i) Find $\int (3x^{\frac{1}{2}} + \frac{16}{x^2} - 7)\,dx$. *(3 marks)*

(ii) Hence find the equation of the curve which passes through the point $P(1, 8)$. *(3 marks)*

5. Jan 2007

6 A curve C is defined for $x > 0$ by the equation $y = x + 1 + \frac{4}{x^2}$ and is sketched below.



(a) (i) Given that $y = x + 1 + \frac{4}{x^2}$, find $\frac{dy}{dx}$. *(3 marks)*

(ii) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(iii) Find an equation of the normal to C at the point $(1, 6)$. *(4 marks)*

(b) (i) Find $\int \left(x + 1 + \frac{4}{x^2}\right) dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve C, the lines $x = 1$ and $x = 4$ and the x-axis. *(2 marks)*

6. June 2007

1 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$; *(1 mark)*

(ii) $x^{\frac{3}{2}} \div x$; *(1 mark)*

(iii) $\left(x^{\frac{3}{2}}\right)^2$. *(1 mark)*

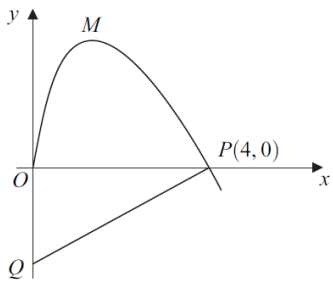
(b) (i) Find $\int 3x^{\frac{1}{2}}\,dx$. *(3 marks)*

(ii) Hence find the value of $\int_1^9 3x^{\frac{1}{2}}\,dx$. *(2 marks)*

7. Jan 2008

5 A curve, drawn from the origin O, crosses the x-axis at the point $P(4, 0)$.

The normal to the curve at P meets the y-axis at the point Q, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Show that the gradient of the curve at $P(4, 0)$ is -2. *(2 marks)*

(iii) Find an equation of the normal to the curve at $P(4, 0)$. *(3 marks)*

(iv) Find the y-coordinate of Q and hence find the area of triangle OPQ. *(3 marks)*

(v) The curve has a maximum point M. Find the x-coordinate of M. *(3 marks)*

(b) (i) Find $\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$. *(3 marks)*

(ii) Find the total area of the region bounded by the curve and the lines PQ and QO. *(3 marks)*

8. June 2008

7 (a) The expression $\left(1 + \frac{4}{x^2}\right)^3$ can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} + \frac{64}{x^6}$$

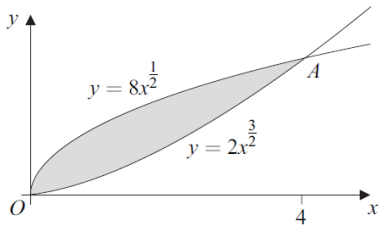
By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) (i) Hence find $\int \left(1 + \frac{4}{x^2}\right)^3 dx$. *(4 marks)*

(ii) Hence find the value of $\int_1^2 \left(1 + \frac{4}{x^2}\right)^3 dx$. *(2 marks)*

9. Jan 2009

4 The diagram shows a sketch of the curves with equations $y = 2x^{\frac{3}{2}}$ and $y = 8x^{\frac{1}{2}}$.



The curves intersect at the origin and at the point A, where $x = 4$.

(a) (i) For the curve $y = 2x^{\frac{3}{2}}$, find the value of $\frac{dy}{dx}$ when $x = 4$. *(2 marks)*

(ii) Find an equation of the normal to the curve $y = 2x^{\frac{3}{2}}$ at the point A. *(4 marks)*

(b) (i) Find $\int 8x^{\frac{1}{2}}\,dx$. *(2 marks)*

(ii) Find the area of the shaded region bounded by the two curves. *(4 marks)*

(c) Describe a single geometrical transformation that maps the graph of $y = 2x^{\frac{3}{2}}$ onto the graph of $y = 2(x+3)^{\frac{3}{2}}$. *(2 marks)*

10.June 2009

2 (a) Write down the value of n given that $\frac{1}{x^4} = x^n$. *(1 mark)*

(b) Expand $\left(1 + \frac{3}{x^2}\right)^2$. *(2 marks)*

(c) Hence find $\int \left(1 + \frac{3}{x^2}\right)^2 dx$. *(3 marks)*

(d) Hence find the exact value of $\int_1^3 \left(1 + \frac{3}{x^2}\right)^2 dx$. *(2 marks)*

11.Jan 2010

2 At the point (x, y) on a curve, where $x > 0$, the gradient is given by

$$\frac{dy}{dx} = 7\sqrt{x^5} - 4$$

(a) Write $\sqrt{x^5}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) Find $\int \left(7\sqrt{x^5} - 4\right) dx$. *(3 marks)*

(c) Hence find the equation of the curve, given that the curve passes through the point $(1, 3)$. *(3 marks)*

12.June 2010

4 (a) The expression $\left(1-\frac{1}{x^2}\right)^3$ can be written in the form

$$1+\frac{p}{x^2}+\frac{q}{x^4}-\frac{1}{x^6}$$

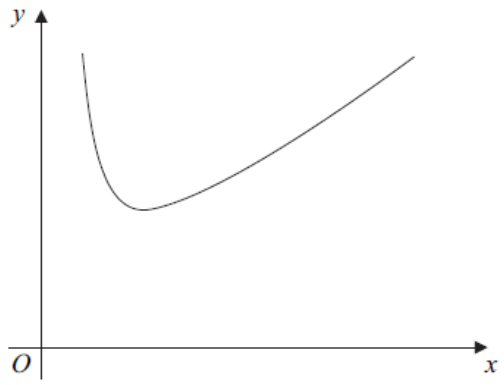
Find the values of the integers p and q. *(2 marks)*

(b) (i) Hence find $\int\left(1-\frac{1}{x^2}\right)^3 \mathrm{d}x$. *(4 marks)*

(ii) Hence find the value of $\int_{\frac{1}{2}}^{1}\left(1-\frac{1}{x^2}\right)^3 \mathrm{d}x$. *(2 marks)*

13.Jan 2011

7 A curve C is defined for $x>0$ by the equation $y=x+3+\frac{8}{x^4}$ and is sketched below.



(a) Given that $y=x+3+\frac{8}{x^4}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(a) Given that $y=x+3+\frac{8}{x^4}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(b) Find an equation of the tangent at the point on the curve C where $x=1$. *(3 marks)*

(c) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(d) (i) Find $\int\left(x+3+\frac{8}{x^4}\right)\,dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve C, the x-axis and the lines $x=1$ and $x=2$. *(2 marks)*

(e) The curve C is translated by $\begin{bmatrix}0\\k\end{bmatrix}$ to give the curve $y=f(x)$. Given that the x-axis is a tangent to the curve $y=f(x)$, state the value of the constant k. *(1 mark)*

14.June 2011

3 (a) The expression $(2+x^2)^3$ can be written in the form

$$8+px^2+qx^4+x^6$$

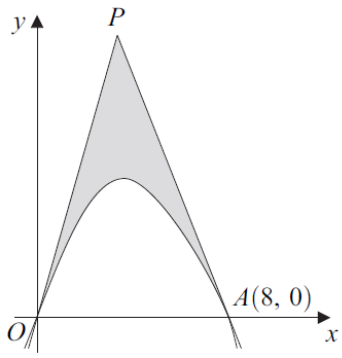
Show that $p=12$ and find the value of the integer q. *(3 marks)*

(b) (i) Hence find $\int\frac{(2+x^2)^3}{x^4}\,dx$. *(5 marks)*

(ii) Hence find the exact value of $\int_1^2\frac{(2+x^2)^3}{x^4}\,dx$. *(2 marks)*

15.Jan 2012

9 The diagram shows part of a curve crossing the x-axis at the origin O and at the point $A(8, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve has equation

$$y=12x-3x^{\frac{5}{3}}$$

(a) Find $\frac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\frac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(8, 0)$ is $y + 8x = 64$. *(3 marks)*

(c) Find $\int \left(12x - 3x^{\frac{5}{3}}\right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve from O to A and the tangents OP and AP. *(7 marks)*

16.Jan 2013

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_1^5 \frac{1}{x^2 + 1}\, dx$$

giving your answer to three significant figures. *(4 marks)*

(b) (i) Find $\int \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}}\right) dx$, giving the coefficient of each term in its simplest form. *(3 marks)*

(ii) Hence find the value of $\int_1^4 \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}}\right) dx$. *(2 marks)*

Answers

1. Jan 2005

4(a)	$\sqrt{x} = x^{\frac{1}{2}}$	B1	1	Accept $k = 0.5$
(b)	$\sqrt{x}(x-1) = x^{\frac{1}{2}}x - x^{\frac{1}{2}} = x^{\frac{3}{2}} - x^{\frac{1}{2}}$	M1 A1	2	Accept $p = 1.5$, $q = 0.5$
(c)	$\int \sqrt{x}(x-1)\,dx = \frac{x^{2.5}}{2.5} - \frac{x^{1.5}}{1.5}$ (+c)	M1 A1√ A1√	3	Increases a power of x by 1 ft non-integer p ft non-integer q
(d)	$\int_1^2 dx = \left(\frac{2^{2.5}}{2.5} - \frac{2^{1.5}}{1.5}\right) - \left(\frac{1}{2.5} - \frac{1}{1.5}\right)$	M1		Limits; F(2) – F(1)
	$\ldots = \left(\frac{4\sqrt{2}}{2.5} - \frac{2\sqrt{2}}{1.5}\right) - \left(\frac{1}{2.5} - \frac{1}{1.5}\right)$	m1		Fractional powers to surds
	$\left(\frac{24\sqrt{2}}{15} - \frac{20\sqrt{2}}{15}\right) - \left(-\frac{4}{15}\right) =$ pr. ans	A1	3	CSO AG (be convinced)
	Total		**9**	

2. June 2005

4(a)(i)	$\sqrt{x} = x^{\frac{1}{2}}$	B1	1	Accept $p = 0.5$
(ii)	$\int \sqrt{x}\ dx = \frac{x^{1.5}}{1.5}\{+c\}$	M1 A1√	2	Index raised by 1 Correct ft on p. Condone missing '+c'
(iii)	Area $= \int_1^4 \sqrt{x}\ dx$	B1		Limits 1 and 4 PI
	$\ldots\ldots = \frac{4^{1.5}}{1.5} - \frac{1}{1.5}$	M1		F(4) – F(1)
	$= \frac{14}{3}$	A1	3	Accept 4.66 or better
(b)(i)	$y = x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	M1		Index (p–1) ft
	When $x = 4$, $y'(4) = 0.25$	M1		Attempt to find $y'(4)$.
	When $x = 4$, $y = 2$	B1		
	$\ldots\ldots = \frac{4^{1.5}}{1.5} - \frac{1}{1.5}$	M1		F(4) – F(1)
	$= \frac{14}{3}$	A1	3	Accept 4.66 or better
(b)(i)	$y = x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	M1		Index (p–1) ft
	When $x = 4$, $y'(4) = 0.25$	M1		Attempt to find $y'(4)$.
	When $x = 4$, $y = 2$	B1		
	Equation of tangent: $y - 2 = \frac{1}{4}(x - 4)$	A1	4	accept other forms

(ii)	When $x = 0,\ y = 1 \quad B(0, 1)$	M1		Subs $x = 0$ **and** then $y = 0$ into equation
	When $y = 0, x = -4 \quad A(-4, 0)$	A1√		of tangent. PI Correct ft y_B and x_A (may be awarded as part of area calculation)
	Area = 0.5(1)(4) = 2	A1√	3	ft wrong sloping tangent and max of 1 further slip. Final answer must be +'ve
(c)	Translation	B1		'Translation'/'translate(d)'
	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	B1	2	Accept equivalent in words provided linked to 'translation/move/shift' (Note: B0B1 is possible)
(d)	$h = 1$ Integral = $h/2$ {……}	B1		PI
	$\{\ldots\} = f(1) + 2[f(2) + f(3)] + f(4)$	M1		OE summing of areas of the three traps
	$\{\ldots\} = 0 + 2(1 + \sqrt{2}) + \sqrt{3}$	A1		Condone 1 numerical slip
	Integral = $\frac{1}{2}\{\ 2(1+1.414...)+1.732\ \}$			
	Integral = $0.5 \times 6.560\ldots$ =3.28 to 3sf	A1	4	CAO Must be 3.28
	Total		**19**	

3. Jan 2006

8(a)	$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 3$	M1 A1	2	One term correct Both correct
(b)(i)	When $x = 0$, $\frac{dy}{dx} = -3$	B1F√		Ft provided answer < 0.
	Eqn of tangent at O is $y = -3x$	B1F√	2	OE Ft on $y'(0)$
(ii)	At (9,0) $\frac{dy}{dx} = \frac{3}{2}(9)^{\frac{1}{2}} - 3$	M1		Attempt to find $y'(9)$
	Eqn tangent at A is $y - 0 = y'(9)[x - 9]$	m1		OE
	$\Rightarrow y = \frac{3}{2}(x - 9) \Rightarrow 2y = 3x - 27$	A1	3	**CSO**. AG
(iii)	Eliminating $y \Rightarrow -6x = 3x - 27$	M1		OE method to one variable (eg $2y = -y - 27$)
	$9x = 27 \Rightarrow x = 3$	A1F		[A1F for each coordinate; only ft on $y = kx$ tangent in (b)(i) for $k < 0$]
	When $x = 3,\ y = -9.\ \{P(3, -9)\}$	A1F	3	
(iii)	Eliminating $y \Rightarrow -6x = 3x - 27$	M1		OE method to one variable (eg $2y = -y - 27$)
	$9x = 27 \Rightarrow x = 3$	A1F		[A1F for each coordinate; only ft on $y = kx$ tangent in (b)(i) for $k < 0$]
	When $x = 3,\ y = -9.\ \{P(3, -9)\}$	A1F	3	
(c)	$\int \left(x^{\frac{3}{2}} - 3x\right) dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{3x^2}{2}\ (+c)$	M1 A2,1,0	3	One power correct Condone absence of "+c" and unsimplified forms

(d)	$\int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx =$	B1		PI
	$= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0$ $= -24.3$	M1		Correct use of limits following integration
	Area of triangle $OPA = \frac{1}{2} \times 9 \times \lvert y_P \rvert$	M1		
	Sh.Area $= \frac{1}{2} \times 9 \times \lvert y_P \rvert - \left\lvert \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx \right\rvert$	M1		OE
	$= 40.5 - 24.3 = 16.2$	A1	5	
	Total		**18**	

4. June 2006

7(a)(i)	When $x = 4$, $\frac{dy}{dx} = 3(2) + \frac{16}{16} - 7 = 0$	B1	1	AG Be convinced
(ii)	$\frac{16}{x^2} = 16x^{-2}$	B1	1	Accept $k = -2$
(iii)	$\frac{d^2y}{dx^2} = 3 \times \frac{1}{2} x^{-\frac{1}{2}} + 16 \times (-2)x^{-3} - 0$	M1		A power decreased by 1
	$\frac{d^2y}{dx^2} = \frac{3}{2} x^{-\frac{1}{2}}; \quad -32x^{-3}$	A1; A1√	3	candidate's negative integer k [−1 for >2 term(s)]
(iv)	When $x = 4$, $\frac{d^2y}{dx^2} = \frac{3}{4} - \frac{32}{64} = \frac{1}{4}$	M1		Attempt to find $y''(4)$ reaching as far as two simplified terms
	Minimum since $y''(4) > 0$	E1√	2	candidate's sign of $y''(4)$
	[Alternative: Finds the sign of $y'(x)$ either side of the point where x=4, need evidence rather than just a statement: (M1) Correct ft conclusion with valid reason E1√] [In both, condone absent statement $y'(4)$=0]			
(b)(i)	At $P(1,8)$, $\frac{dy}{dx} = 3(1)^{\frac{1}{2}} + \frac{16}{1^2} - 7 = 12$	B1	1	AG Be convinced
(ii)	Gradient of normal $= -\frac{1}{12}$	M1		Use of or stating $m \times m' = -1$
	Equation of normal is $y - 8 = m[x - 1]$	M1		Can be awarded even if m=12
	$y - 8 = -\frac{1}{12}(x - 1) \Rightarrow 12y - 96 = -x + 1$ $\Rightarrow 12y + x = 97$	A1	3	Any correct form of the equation
(c)(i)	$\int 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7\, dx =$			
	…….. $= 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 16\frac{x^{-1}}{-1} - 7x + c$	M1 A2,1,0 √	3	One power correct. A1 if 2 of 3 terms correct candidate's negative integer k Condone absence of "+ c"
(ii)	$y = 2x^{\frac{3}{2}} - 16x^{-1} - 7x + c \qquad (*)$	B1√		y = candidate's answer to (c)(i) with tidied coefficients and with '+c'. ('y =' PI by next line)

	When $x = 1$, $y = 8 \Rightarrow 8 = 2–16–7+c$	M1		Substitute. (1,8) in attempt to find constant of integration
	$y = 2x^{\frac{3}{2}} - 16x^{-1} - 7x + 29$	A1	3	Accept $c = 29$ after (*), including $y =$, stated
	Total		**17**	

5. Jan 2007

6(a)(i)	$y = x + 1 + 4x^{-2} \Rightarrow \frac{dy}{dx} = 1 - 8x^{-3}$	M1 A2,1,0	3	Power $p \to p–1$ (A1 if $1 + ax^n$ with $a = -8$ or $n = -3$)
(ii)	$1 - 8x^{-3} = 0$	M1		Puts c's $\frac{dy}{dx} = 0$
	$x^3 = 8$	m1		Using $x^{-k} = \frac{1}{x^k}$ to reach $x^a = b$, $a>0$ or correct use of logs.
	$x = 2$ When $x = 2$, $y = 4$	A1 A1ft	4	
(iii)	At (1, 6), $\frac{dy}{dx} = 1 - 8 = -7$	M1		Attempt to find $y'(1)$
	Gradient of normal $= \frac{1}{7}$	M1		Use of or stating $m \times m' = -1$
	Equation of normal is $y - 6 = m[x - 1]$	M 1		m numerical
	$y - 6 = \frac{1}{7}(x - 1)$ $\{\frac{y-6}{x-1} = \frac{1}{7};\ 7y = x + 41\}$	A1ft	4	OE ft on c's answer for (a)(i) provided at least A1 given in (a)(i) and previous 3M marks awarded
(b)(i)	$\int x\left(+1 + \frac{4}{x^2}\right) dx =$ $\ldots\ldots = \frac{x^2}{2} + x - 4x^{-1}\ \{+c\}$	M1 A2,1,0	3	One of three terms correct. For A2 need all <u>three</u> terms as printed or better (A1 if 2 of 3 terms correct)
(ii)	{Area=} $\int_1^4 x + 1 + \frac{4}{x^2} dx =$ $\left[\frac{x^2}{2} + x - \frac{4}{x}\right]_1^4 = (8 + 4 - 1) - \left(\frac{1}{2} + 1 - 4\right)$	M1		Dealing correctly with limits; F(4)–F(1) (must have integrated)
	$= 13.5$	A1	2	
	Total		**16**	

6. June 2007

1(a)(i)	x^2	B1	1	
(ii)	$x^{\frac{1}{2}} = \sqrt{x}$	B1	1	Accept either form
(iii)	x^3	B1	1	

(b)(i)	$\int 3x^{\frac{1}{2}}\,dx = \frac{3}{\frac{3}{2}}x^{\frac{3}{2}} \quad \{+c\}$	M1 A1		Index raised by 1 Simplification not yet required
	$= 2x^{\frac{3}{2}} + c$	A1	3	Need simplification **and** the + c OE
(ii)	$\int_1^9 3x^{\frac{1}{2}}\,dx = (2\times 9^{\frac{3}{2}}) - (2\times 1^{\frac{3}{2}})$	M1		F(9) – F(1), where F(x) is candidate's answer to (b)(i) [or clear recovery]
	= 52	A1ft	2	Ft on (b)(i) answer of form $kx^{1.5}$ i.e. $26k$
	Total		**8**	

7. Jan 2008

5a(i)	$\frac{dy}{dx} = 4\times\frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$	M1 A1A1	3	A power decreased by 1 A1 for each correct term
(ii)	At $P(4,0)$, $\frac{dy}{dx} = \frac{2}{\sqrt{4}} - \frac{3}{2}\times 2$	M1		Attempts $\frac{dy}{dx}$ when $x = 4$
	$= 1-3 = -2$	A1	2	AG
(iii)	Gradient of normal = $\frac{1}{2}$	M1		Use of or stating $m\times m' = -1$
	Equation of normal is $y - 0 = m[x-4]$	M1		m numerical; can be awarded even if $m = -2$
	$y - 0 = \frac{1}{2}(x-4) \Rightarrow \quad 2y = x - 4$	A1	3	ACF of the equation
(iv)	At Q, $x = 0$, $2y = 0 - 4$ $y_Q = -2$	M1 A1F		PI Ft on a linear equation for normal provided y_Q is negative and prev A1 is lost
	Area of triangle $OPQ = 0.5\times 4\times \lvert y_Q \rvert$ $= 4$	B1F	3	Ft on c's negative y_Q

(v)	$2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0 \Rightarrow 2x^{-\frac{1}{2}} = \frac{3}{2}x^{\frac{1}{2}}$	M1		Puts c's $\frac{dy}{dx} = 0$ and a 1st step in attempt to solve.
	$2 = \frac{3}{2}x$; $\quad \Rightarrow x = \frac{4}{3}$	m1; A1	3	Valid method to $ax=b$ Condone 1.3 or better
(b)(i)	$\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx = 4\frac{x^{\frac{3}{2}}}{1.5} - \frac{x^{\frac{5}{2}}}{2.5} \quad \{+c\}$ $= \frac{8}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} \{+c\}$	M1 A1,A1	3	One power correct Condone absence of "+c"
(ii)	Area under curve = $4\frac{4^{\frac{3}{2}}}{1.5} - \frac{4^{\frac{5}{2}}}{2.5} - \{0\}$	M1		F(4) − {F(0)}
	Total area = F(4) + area triangle OPQ	m1		
	Total area = $\frac{128}{15} + 4 = \frac{188}{15}$ = 12.5 (3…)	A1	3	Accept 3 sf if clear
	Total		**20**	

8. June 2008

7(a)	$\left(1+\frac{4}{x^2}\right)^3 =$			
	$\left[1^3\right]+3(1^2)\left(\frac{4}{x^2}\right)+3(1)\left(\frac{4}{x^2}\right)^2+\left[\left(\frac{4}{x^2}\right)^3\right]$	M1		Any valid method as far as term(s) in $1/x^2$ and term(s) in $1/x^4$
	$=[1]+\frac{12}{x^2}+\frac{48}{x^4}+\left[\frac{64}{x^6}\right]$	A1		$p = 12$ Accept $\frac{12}{x^2}$ even within a series
		A1	3	$q = 48$ Accept $\frac{48}{x^4}$ even within a series
(b)(i)	$\int\left(1+\frac{4}{x^2}\right)^3 \mathrm{d}x$			
	$=\int\ (1+\frac{p}{x^2}+\frac{q}{x^4}+\frac{64}{x^6})\ \mathrm{d}x$	M1		Integral of an 'expansion', at least 3 terms PI by the next line
	$= x-px^{-1}-\frac{q}{3}x^{-3}-\frac{64}{5}x^{-5}\ (+c)$	m1 A2F,1	4	At least two powers correctly obtained Ft on c's non-zero integer values for p and q (A1F for two terms correct; can be unsimplified) Condone missing c but check that signs have been simplified at some stage before the award of both A marks.
	$= x-12x^{-1}-16x^{-3}-\frac{64}{5}x^{-5}\ (+c)$			
(ii)	$\left(2-\frac{p}{2}-\frac{q}{3(8)}-\frac{64}{5(32)}\right)-$			
	$\left(1-p-\frac{q}{3}-\frac{64}{5}\right)$	M1		F(2) − F(1), where F(x) is cand's answer or the correct answer to (b)(i).
	$= 33.4$	A1	2	CSO
	Total		**9**	

9. Jan 2009

4(a)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=3x^{\frac{1}{2}}$	M1		$kx^{\frac{1}{2}}$ with or without $+c$
	$=6$ {when $x = 4$}	A1cao	2	Must be 6 and seen in (a)(i) $6 + c$ is A0
(ii)	y-coordinate of $A = 2\times4^{\frac{3}{2}}$ (= 16)	M1		Substitute $x = 4$ in $y=2x^{\frac{3}{2}}$
	$6 \times m' = -1$	M1		$m_1\times m_2=-1$ OE used with c's value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 4$. PI
	$y - 16 = m(x - 4)$	m1		dep on 1st M1 in (a)(ii) m must be numerical
	$y-16=-\frac{1}{6}(x-4)$	A1	4	ACF

(b)(i)	$\int 8x^{\frac{1}{2}}\,dx = \frac{8}{3/2}x^{\frac{1}{2}+1}\ \{+c\}$	M1		Index raised by 1
	$= \frac{16}{3}x^{\frac{3}{2}}\ \{+c\}$	A1	2	Condone missing '+ c' Coefficient must be simplified
(ii)	$\int 2x^{\frac{3}{2}}\,dx = \frac{2}{5/2}x^{\frac{5}{2}}\ \{+c\}\qquad \{= \frac{4}{5}x^{\frac{5}{2}}\ \{+c\}\}$	B1		Can award for unsimplified form
	$\int_0^4 8x^{\frac{1}{2}}\,dx - \int_0^4 2x^{\frac{3}{2}}\,dx$	M1		Ignore limits here
	$= \frac{16}{3}(4)^{\frac{3}{2}} - 0 - \left[\frac{4}{5}(4)^{\frac{5}{2}} - 0\right]$	M1		F(4) − F(0) used in either; {F(0)=0 PI} Cand. must be using F(x) as a result of his/her integration in (b)(i) or in the (b)(ii) B1 line above
	$= \frac{256}{15}$	A1	4	Accept any value from 17.04 to 17.1 inclusive in place of 256/15
(c)	Translation	B1		Accept 'translat…' as equivalent [T or Tr is NOT sufficient]
	$\begin{bmatrix} -3 \\ 0 \end{bmatrix}$	B1	2	Accept equivalent in words provided linked to 'translation/move/shift' (B0B0 if >1 transformation)
	Total		**14**	

10.June 2009

2(a)	$(n =) -4$	B1	1	Accept x^{-4}
(b)	$\left(1+\frac{3}{x^2}\right)^2 = 1+\frac{6}{x^2}+\frac{9}{x^4}$	B2,1,0	2	Apply ISW after B2 stage (B1 if correct but unsimplified seen)
(c)	$\int\left(1+\frac{3}{x^2}\right)^2 dx = x - 6x^{-1} - 3x^{-3} + c$	M1 A2,1,0	3	At least one power of x correctly obtained in the integration of an expansion A2 terms correct **and** '+ c' (A1F two terms in x correct ft on expansion provided integrating x to a negative power)
(d)	$\int_1^3\left(1+\frac{3}{x^2}\right)^2 dx = \left[x - \frac{6}{x} - \frac{3}{x^3}\right]_1^3$			
	$= \left(3 - \frac{6}{3} - \frac{3}{27}\right) - (1 - 6 - 3)$	M1		Dealing correctly with limits; F(3) − F(1) (must have attempted integration to get F)
	$= 8\frac{8}{9}$	A1	2	CSO; OE provided value is **exact**, eg $\frac{80}{9}$, $\frac{240}{27}$; ISW dec value after exact value seen NMS scores 0/2
	Total		**8**	

11.Jan 2010

2(a)	$\sqrt{x^5} = x^{\frac{5}{2}}$	B1	1	Accept $k = 2.5$
(b)	$\int\left(7\sqrt{x^5} - 4\right) dx = \frac{7}{3.5}x^{3.5} - 4x \ (+c)$	M1 A1F B1	 3	Index 'k' raised by 1 in integrating x^k 1st term correct follow through on non-integer k For $-4x$ as integral of -4
(c)	$y = 2x^{3.5} - 4x + c \qquad (*)$	B1F		y = c's answer to (b) with '+ c' ('y =' PI by next line)
	When $x = 1$, $y = 3 \Rightarrow 3 = 2 - 4 + c$	M1		Subst. (1, 3) in attempt to find constant of integration
	$y = 2x^{3.5} - 4x + 5$	A1	3	Accept $c = 5$ after correct eqn * which must include 'y =' Coefficients must be tidied
	Total		**7**	

12.June 2010

4(a)	$p = -3$; $q = 3$	B1;B1	2	Accept even if just embedded in the expansion
(b)(i)	$\int\left(1-\frac{1}{x^2}\right)^3 dx =$ $\int\left(1-3x^{-2}+3x^{-4}-x^{-6}\right) dx$	M1		Uses (a) with indication of integration and indication of $\frac{1}{x^n} = x^{-n}$ PI
	$= x + 3x^{-1} - x^{-3} + \frac{1}{5}x^{-5}$ $\{+c\}$	m1 A2F,1F	 4	At least three powers of x correctly obtained Ft on c's non-zero integers p and q. A1F if 3 of the 4 terms are correct (ft) or if all correct (ft) but left unsimplified Condone missing $+c$.
(ii)	$\int_{\frac{1}{2}}^{1}\left(1-\frac{1}{x^2}\right)^3 dx =$ $\left(1+3-1+\frac{1}{5}\right)-\left(\frac{1}{2}+6-8+\frac{32}{5}\right)$	M1		Attempting to calculate F(1)−F(1/2) where F is c's answer to part (b)(i) provided F is not the integrand or the c's equivalent of the integrand $(1-\frac{1}{x^2})^3$.
	$= -\frac{17}{10}$	A1	2	OE exact answer eg −1.7
	Total		**8**	

13.Jan 2011

7(a)	$y = x + 3 + \frac{8}{x^4} = x + 3 + 8x^{-4}$	B1		For $\frac{8}{x^4} = 8x^{-4}$ PI by correct differentiation of 3rd term
	$\frac{dy}{dx} = 1 - 32x^{-5}$ or $1 - \frac{32}{x^5}$	M1 A1	 3	kx^{-5} OE For either

(b)	When $x=1,\ y=12$	B1		
	When $x=1,\ \frac{dy}{dx}=1-32=-31$	M1		Attempt to find value of $\frac{dy}{dx}$ when x=1
	Tangent: $y-12=-31(x-1)$	A1F	3	Only ft on c's answer to (a). Any correct (ft on c's (a)) form.
(c)	$1-32x^{-5}=0$	M1		$1-32x^{-5}=0$ or c's $\frac{dy}{dx}=0$
	$\Rightarrow x^5=32$	m1		Attempt to form $x^n=$ const ($\neq 0$). PI by next line
	$\Rightarrow x=2$	A1		CSO
	(Coordinates of M) (2, 5.5)	A1	4	CSO
(d)(i)	$\int \left(x+3+\frac{8}{x^4}\right)\,dx$			
	$=\frac{x^2}{2}+3x-\frac{8}{3}x^{-3}+c$	M1		Power −3 correctly obtained
		A1		$-\frac{8}{3}x^{-3}$
		B1	3	$\frac{x^2}{2}+3x+c$
(ii)	Area $=\left[\frac{x^2}{2}+3x-\frac{8}{3}x^{-3}\right]_1^2$			
	$=\left(2+6-\frac{1}{3}\right)-\left(\frac{1}{2}+3-\frac{8}{3}\right)$	M1		Attempting to calculate F(2) − F(1) where F(x) is c's answer to part (d)(i) provided F is not just the c's integrand (x+3+8/x^4)
	$=\frac{9}{2}+\frac{7}{3}=\frac{41}{6}$	A1	2	OE Accept 6.83 or better provided d(i) used
(e)	$k=-5.5$	B1F	1	Ft on $-y_M$ from part (c).
	Total		**16**	

14.June 2011

3(a)	$(2+x^2)^3$ $=[(2)^3]+3(2)^2(x^2)+3(2)(x^2)^2\ [+(x^2)^3]$	M1		For either (1),3,3,(1) OE unsimplified or $\binom{3}{1}2^2x^2+\binom{3}{2}2(x^2)^2$ OE. PI
	$p=3(2)^2=12$	A1		AG Be convinced. Condone left as $12x^2$
	$q=6$	B1	3	Accept left as $6x^4$
(b)(i)	$\int \frac{(2+x^2)^3}{x^4}\,dx=\int x^{-4}(8+12x^2+qx^4+x^6)\,dx$ or $\int\left(\frac{8}{x^4}+\frac{12}{x^2}+q+x^2\right)dx$	M1		Uses (a) and either an indication that $\frac{1}{x^n}=x^{-n}$ in a product PI or cancelling to get at least 3 correct ft terms
	$\int (8x^{-4}+12x^{-2}+q+x^2)\,dx$	A1F		Ft on c's non-zero q. PI by next line in solution
	$=\frac{8x^{-3}}{-3}+\frac{12x^{-1}}{-1}+qx+\frac{x^3}{3}\ \{+c\}$	M1		Correct integration of either $8x^{-4}$ or $12x^{-2}$; accept unsimplified
		A1		Correct integration of both $8x^{-4}$ and $12x^{-2}$; accept unsimplified coefficients

(b)(ii)	$\int_1^2 \frac{(2+x^2)^3}{x^4}\,dx = \{-\frac{8}{3}(2)^{-3} - 12(2^{-1}) + 6(2) + \frac{2^3}{3}\} - \{-\frac{8}{3}(1)^{-3} - 12(1)^{-1} + 6(1) + \frac{1^3}{3}\}$	M1		Dealing correctly with limits; F(2)−F(1) (must have attempted integration to get F ie c's F is not just the integrand)
	$= \left(-\frac{1}{3} - 6 + 12 + \frac{8}{3}\right) - \left(-\frac{8}{3} - 12 + 6 + \frac{1}{3}\right)$			
	$= 16\frac{2}{3}$	A1	2	OE **exact** answer eg 50/3. NMS scores 0
	Total		**10**	

15.Jan 2012

9(a)	$\frac{dy}{dx} = 12 - 5x^{\frac{2}{3}}$	M1 A1	 2	$kx^{\frac{2}{3}}$ term. ACF
(b)(i)	When x=0, $\frac{dy}{dx} = 12$	B1F		Ft on c's y' evaluated correctly at x=0
	Eqn of tangent at O is $y = 12x$	B1F	2	OE Ft on c's value for $y'(0)$ provided $y'(0)>0$.
(ii)	When $x = 8$, $\frac{dy}{dx} = 12 - 5\times(8)^{\frac{2}{3}}$	M1		Attempt to find $\frac{dy}{dx}$ when $x = 8$
	Equation of tangent at (8, 0) is $y - 0 = y'(8)[x-8]$	m1		$y = y'(8)[x-8]$ OE
	$y = -8(x-8) \Rightarrow y + 8x = 64$	A1	3	CSO AG
(c)	$\int\left(12x - 3x^{\frac{5}{3}}\right)dx = \frac{12x^2}{2} - \frac{3x^{\frac{8}{3}}}{\frac{8}{3}}$ (+c)	M1		$kx^{\frac{5}{3}+1}$ term after integrating, condone k left unsimplified for this M mark.
	$= 6x^2 - \frac{9}{8}x^{\frac{8}{3}}$ (+c)	B1 A1	 3	For $6x^2$ OE eg $(12x^2/2)$ For $-\frac{9}{8}x^{\frac{8}{3}}$ OE
(d)	Area bounded by curve and x-axis $= \int_0^8\left(12x - 3x^{\frac{5}{3}}\right)dx = 6\times 8^2 - \frac{9}{8}\times(8)^{\frac{8}{3}}$	M1		± F(8) {− F(0)} PI following integration
	$= 384 - 288 = 96$	A1		PI by correct final answer if evaluation not seen here
	At P, $12x + 8x = 64$	M1		Solving $y + 8x = 64$ and c's $y=kx$, $k>0$, down to an eqn in one variable… [$y+2y/3=64$]
	$(x_P = 3.2)$ $y_P = 38.4$	A1		For $y_P = 38.4$ OE [If using integration to find area of triangle, award A1 if both '$x_P = 3.2$' and correct integration of correct eqns of the 2 lines]
	Area of triangle $OPA = \frac{1}{2}\times 8\times y_P$	M1		OE Need perpendicular ht to be linked to $y_P > 0$.
	Area of shaded region =Area Δ $OPA - \int_0^8\left(12x - 3x^{\frac{5}{3}}\right)dx$	M1		M0 if evaluated to a value <0
	= 153.6 −96 = 57.6	A1	7	OE eg 288/5
	Total		**17**	

16.Jan 2013

2(a)	$h = 1$ $f(x) = \dfrac{1}{x^2+1}$ $I \approx \dfrac{h}{2}\{f(1)+f(5)+2[f(2)+f(3)+f(4)]\}$ $\dfrac{h}{2}$ with $\{\ldots\} = \dfrac{1}{2}+\dfrac{1}{26}+2\left(\dfrac{1}{5}+\dfrac{1}{10}+\dfrac{1}{17}\right)$ $= 0.5 + 0.03(84...)+2[0.2+0.1+0.05(88...)]$ $= 0.538(46\ldots)+2[0.358(82...)]=1.256(108\ldots)$ $(I \approx)\ \ 0.628054\ldots = \dfrac{694}{1105} = 0.628$ (to 3sf)	B1 M1 A1 A1	4	PI $\dfrac{h}{2}\{f(1)+f(5)+2[f(2)+f(3)+f(4)]\}$ OE summing of areas of the four 'trapezia'… OE Accept 2dp (rounded or truncated) for non-terminating decs. equiv. CAO Must be 0.628 **SC** for those who use 5 strips, max possible is B0M1A1A0
(b)(i)	$\int\left(x^{-\frac{3}{2}}+6x^{\frac{1}{2}}\right)dx = \dfrac{x^{-\frac{1}{2}}}{-1/2}+\dfrac{6x^{\frac{3}{2}}}{3/2}\quad(+c)$ $= -2x^{-0.5}+4x^{1.5}\quad(+c)$	M1 A1 A1	3	One term correct (even unsimplified) Both terms correct (even unsimplified) Must be simplified.
(ii)	$\int_1^4\left(x^{-\frac{3}{2}}+6x^{\frac{1}{2}}\right)dx$ $= [-2(4^{-0.5})+4(4^{1.5})] - [-2(1^{-0.5})+4(1^{1.5})]$ $= (-1+32) - (-2+4) = 29$	M1 A1	2	Attempt to calculate F(4)−F(1) where F(x) follows integration and is not just the integrand Since 'Hence' NMS scores 0/2
	Total		**9**	