Practice examination questions for indices

1. Jan 2005

4 (a) Write $\sqrt{x}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) Hence express $\sqrt{x}(x-1)$ in the form $x^p - x^q$. *(2 marks)*

(c) Find $\int \sqrt{x}(x-1)\,dx$. *(3 marks)*

(d) Hence show that $\int_1^2 \sqrt{x}(x-1)\,dx = \frac{4}{15}\left(\sqrt{2}+1\right)$. *(3 marks)*

2. Jan 2007

3 (a) Write down the values of p, q and r given that:

(i) $64 = 8^p$;

(ii) $\frac{1}{64} = 8^q$;

(iii) $\sqrt{8} = 8^r$. *(3 marks)*

(b) Find the value of x for which

$$\frac{8^x}{\sqrt{8}} = \frac{1}{64}$$ *(2 marks)*

3. June 2007

1 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$; *(1 mark)*

(ii) $x^{\frac{3}{2}} \div x$; *(1 mark)*

(iii) $\left(x^{\frac{3}{2}}\right)^2$. *(1 mark)*

(b) (i) Find $\int 3x^{\frac{1}{2}}\,dx$. *(3 marks)*

(ii) Hence find the value of $\int_1^9 3x^{\frac{1}{2}}\,dx$. *(2 marks)*

4. June 2008

1 (a) Write $\sqrt{x^3}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) A curve, defined for $x \geqslant 0$, has equation

$$y = x^2 - \sqrt{x^3}$$

(i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Find the equation of the tangent to the curve at the point where $x = 4$, giving your answer in the form $y = mx + c$. *(5 marks)*

5. June 2009

9 (a) (i) Find the value of p for which $\sqrt{125} = 5^p$. *(2 marks)*

(ii) Hence solve the equation $5^{2x} = \sqrt{125}$. *(1 mark)*

(b) Use logarithms to solve the equation $3^{2x-1} = 0.05$, giving your value of x to four decimal places. *(3 marks)*

(c) It is given that

$$\log_a x = 2(\log_a 3 + \log_a 2) - 1$$

Express x in terms of a, giving your answer in a form not involving logarithms. *(4 marks)*

6. Jan 2011

2 (a) Write down the values of p, q and r given that:

(i) $8 = 2^p$; *(1 mark)*

(ii) $\frac{1}{8} = 2^q$; *(1 mark)*

(iii) $\sqrt{2} = 2^r$. *(1 mark)*

(b) Find the value of x for which $\sqrt{2} \times 2^x = \frac{1}{8}$. *(2 marks)*

7. Jan 2012

3 (a) Write $\sqrt[4]{x^3}$ in the form x^k. *(1 mark)*

(b) Write $\dfrac{1 - x^2}{\sqrt[4]{x^3}}$ in the form $x^p - x^q$. *(2 marks)*

Answers

1. Jan 2005

4(a)(i)	$\sqrt{x} = x^{\frac{1}{2}}$	B1	1	Accept $p = 0.5$
(ii)	$\int \sqrt{x}\ \mathrm{d}x = \frac{x^{1.5}}{1.5}\{+c\}$	M1 A1√	2	Index raised by 1 Correct ft on p. Condone missing '+c'
(iii)	Area $= \int_1^4 \sqrt{x}\ \mathrm{d}x$	B1		Limits 1 and 4 PI
	$\ldots\ldots = \frac{4^{1.5}}{1.5} - \frac{1}{1.5}$	M1		F(4) – F(1)
	$= \frac{14}{3}$	A1	3	Accept 4.66 or better
(b)(i)	$y = x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	M1		Index (p–1) ft
	When x= 4, $y'(4) = 0.25$	M1		Attempt to find $y'(4)$.
	When x= 4, $y = 2$	B1		
	Equation of tangent: $y - 2 = \frac{1}{4}(x - 4)$	A1	4	accept other forms
(ii)	When $x = 0$, $y = 1$ $B(0, 1)$	M1		Subs $x = 0$ **and** then $y = 0$ into equation
	When $y = 0$, $x = -4$ $A(-4, 0)$	A1√		of tangent. PI Correct ft y_B and x_A (may be awarded as part of area calculation)
	Area = 0.5(1)(4) = 2	A1√	3	ft wrong sloping tangent and max of 1 further slip. Final answer must be +'ve
(c)	Translation	B1		'Translation'/'translate(d)'
	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	B1	2	Accept equivalent in words provided linked to 'translation/move/shift' (Note: B0B1 is possible)
(d)	$h = 1$ Integral = $h/2$ {……}	B1		PI
	$\{\ldots\} = f(1) + 2[f(2) + f(3)] + f(4)$	M1		OE summing of areas of the three traps
	$\{\ldots\} = 0 + 2(1 + \sqrt{2}) + \sqrt{3}$	A1		Condone 1 numerical slip
	Integral $= \frac{1}{2}\{\ 2(1+1.414\ldots)+1.732\ \}$			
	Integral $= 0.5 \times 6.560\ldots = 3.28$ to 3sf	A1	4	CAO Must be 3.28
	Total		**19**	

2. Jan 2007

3(a)(i)	$\{p =\}\ 2$	B1		Condone '$64=8^2$'
(ii)	$\{q =\} -2$	B1ft		Ft on '$-p$' if q not correct
(iii)	$\{r =\}\ 0.5$	B1	3	Condone '$\sqrt{8} = 8^{0.5}$'
(b)	$\frac{8^x}{8^{0.5}} = 8^{-2} \Rightarrow 8^{x-0.5} = 8^{-2}$ OE	M1		Using parts (a) **&** valid index law to stage $8^c=8^d$ (PI)
	$\Rightarrow x - 0.5 = -2 \quad \Rightarrow x = -1.5$	A1ft	2	Ft on c's $(q + r)$ if not correct (Accept correct answer without working)
	ALT: $\log 8^x = \log k$, $x\log 8 = \log k$; $x = -1.5$			(M1 A1)
	Total		**5**	

3. June 2007

1(a)(i)	x^2	B1	1	
(ii)	$x^{\frac{1}{2}} = \sqrt{x}$	B1	1	Accept either form
(iii)	x^3	B1	1	
(b)(i)	$\int 3x^{\frac{1}{2}}\,dx = \frac{3}{\frac{3}{2}}x^{\frac{3}{2}}$ $\{+c\}$	M1 A1		Index raised by 1 Simplification not yet required
	$= 2x^{\frac{3}{2}} + c$	A1	3	Need simplification **and** the $+ c$ OE
(ii)	$\int_1^9 3x^{\frac{1}{2}}\,dx = (2 \times 9^{\frac{3}{2}}) - (2 \times 1^{\frac{3}{2}})$	M1		F(9) – F(1), where F(x) is candidate's answer to (b)(i) [or clear recovery]
	$= 52$	A1ft	2	Ft on (b)(i) answer of form $kx^{1.5}$ i.e. $26k$
	Total		**8**	

4. June 2008

1(a)	$\sqrt{x^3} = x^{\frac{3}{2}}$	B1	1	OE; accept '$k = 1.5$'
(b)(i)	$\frac{dy}{dx} = 2x - \frac{3}{2}x^{\frac{1}{2}}$	M1 B1 A1F	3	At least one index reduced by 1 and no term of the form $\sqrt{ax^2}$. For $2x$ For $-1.5\,x^{0.5}$ .Ft on ans (a) non-integer k
(ii)	When $x = 4$, $y = 8$	B1		
	$y'(4) =$;	M1		Attempt to find $\frac{dy}{dx}$ when $x = 4$
	$= 2(4) - 1.5(\sqrt{4}) = 5$	A1F		Ft on one earlier error provided non-integer powers in (a) and (b)(i)
	Tangent: $y - 8 = 5(x - 4)$ $y = 5x - 12$	m1 A1	5	$y - y(4) = y'(4)[x - 4]$ OE CSO; must be $y = 5x - 12$
	Total		**9**	

5. June 2009

9(a)(i)	$\sqrt{125} = \sqrt{25\times 5} = 5\sqrt{5}$	M1		OE eg $\sqrt{125} = \sqrt{5^3}$ or $5^{1.5}$ seen
	$5^p = \sqrt{125} \Rightarrow p = 1.5$	A1	2	Correct value of p must be explicitly stated
	Alternative for (a)(i):			
	$p\log 5 = \frac{1}{2}\log 125$	(M1)		OE eg $p\log 5 = \log 11.18$ or eg $p = \log_5\sqrt{125}$
	$p\log 5 = \frac{3}{2}\log 5 \Rightarrow p = \frac{3}{2}$	(A1)		Correct value of p must be explicitly stated
(ii)	$5^{2x} = \sqrt{125} = 5^p \Rightarrow x = 0.5p = 0.75$	B1F	1	Must be 0.5 × c's value of p SC: $x = 0.75$ with c's ans (a)(i) $5^{1.5}$ scores B1F
(b)	$3^{2x-1} = 0.05$ $(2x-1)\log 3 = \log 0.05$	M1		Take logs of both sides and use 3rd law of logs. PI eg by $2x - 1 = \log_3 0.05$ seen
	$x = \frac{\log_{10} 0.05}{2\log_{10} 3} + \frac{1}{2}$	m1		Correct rearrangement to $x = \ldots$ PI
	$= -0.8634(165\ldots) = -0.8634$ to 4dp	A1	3	Condone > 4dp. Must see logs clearly **used** in solution, so NMS scores 0/3
(c)	$\log_a x = 2(\log_a 3 + \log_a 2) - 1$			
	$= 2\log_a(3\times 2) - 1$	M1		A valid law of logs used
	$= \log_a(6^2) - 1$	M1		Another valid law of logs used
	$= \log_a 36 - \log_a a$	B1		$\log_a a = 1$ quoted or used or $\log_a \frac{x}{k} = -1 \Rightarrow \frac{x}{k} = a^{-1}$ OE
	$\log_a x = \log_a\left(\frac{36}{a}\right) \Rightarrow x = \frac{36}{a}$	A1	4	CSO Must be $x = \frac{36}{a}$ or $x = 36a^{-1}$

6. Jan 2011

2(a)(i)	$(p =)\ 3$	B1	1	
(ii)	$(q =) -3$	B1F	1	If not correct, ft on $-p$
(iii)	$(r =) \frac{1}{2}$	B1	1	OE
(b)	$2^{\frac{1}{2}} \times 2^x = 2^{-3} \Rightarrow 2^{\frac{1}{2}+x} = 2^{-3}$	M1		Using a law of indices or logs correctly to combine at least two of the powers of 2 PI
	$\Rightarrow x = -3\frac{1}{2}$	A1F	2	If not correct, ft on $x = q - r$ provided method shown
	Total		**5**	

7. Jan 2012

3(a)	$\sqrt[4]{x^3} = x^{\frac{3}{4}}$	B1	1	Accept $k = ¾$ OE
(b)	$\frac{1-x^2}{\sqrt[4]{x^3}} = \frac{1}{\sqrt[4]{x^3}} - \frac{x^2}{\sqrt[4]{x^3}}$ $= x^{-k} - \frac{x^2}{\sqrt[4]{x^3}}$ [or $\frac{1}{\sqrt[4]{x^3}} - x^{2-k}$]	M1		Split followed by at least one correct index law used to remove denominator.
	$= x^{-\frac{3}{4}} - x^{\frac{5}{4}}$	A1F	2	If incorrect, ft on c's non-integer k value answer to part (a), provided M1 has been awarded. Accept answer given in form of values for p and q.
			3	