Practice examination questions for differentiation

1. Jan 2005

1 A curve is defined for $x > 0$ by the equation $y = x + \frac{2}{x}$.

(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Hence show that the gradient of the curve at the point P where $x = 2$ is $\frac{1}{2}$. *(1 mark)*

(b) Find an equation of the normal to the curve at this point P. *(4 marks)*

2. June 2005

7 A curve is defined, for $x > 0$, by the equation $y = f(x)$, where

$$f(x) = \frac{x^8 - 1}{x^3}$$

(a) Express $\frac{x^8 - 1}{x^3}$ in the form $x^p - x^q$, where p and q are integers. *(2 marks)*

(b) (i) Hence differentiate $f(x)$ to find $f'(x)$. *(2 marks)*

(ii) Hence show that f is an increasing function. *(2 marks)*

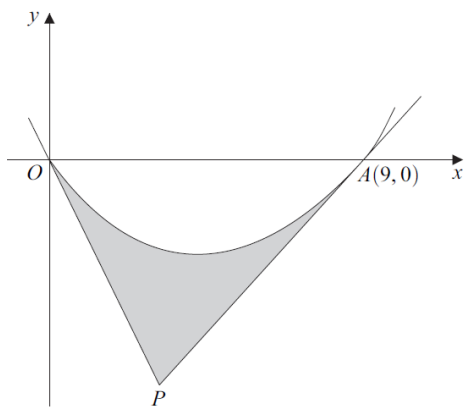
(c) Find the gradient of the normal to the curve at the point $(1, 0)$. *(3 marks)*

3. Jan 2006

1 Given that $y = 16x + x^{-1}$, find the two values of x for which $\frac{dy}{dx} = 0$. *(5 marks)*

4. Jan 2006

8 A curve, drawn from the origin O, crosses the x-axis at the point $A(9, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = x^{\frac{3}{2}} - 3x$$

(a) Find $\frac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\frac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(9, 0)$ is $2y = 3x - 27$. *(3 marks)*

(iii) Hence find the coordinates of the point P where the two tangents meet. *(3 marks)*

(c) Find $\int \left(x^{\frac{3}{2}} - 3x\right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve and the tangents OP and AP. *(5 marks)*

5. June 2006

7 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$$

(a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 4$. *(1 mark)*

(ii) Write $\frac{16}{x^2}$ in the form $16x^k$, where k is an integer. *(1 mark)*

(iii) Find $\frac{d^2y}{dx^2}$. *(3 marks)*

(iv) Hence determine whether the point where $x = 4$ is a maximum or a minimum, giving a reason for your answer. *(2 marks)*

(b) The point $P(1, 8)$ lies on the curve.

(i) Show that the gradient of the curve at the point P is 12. *(1 mark)*

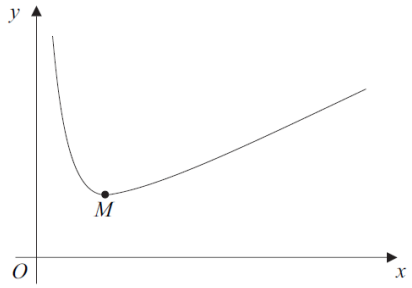
(ii) Find an equation of the normal to the curve at P. *(3 marks)*

(c) (i) Find $\int (3x^{\frac{1}{2}} + \frac{16}{x^2} - 7)\, dx$. *(3 marks)*

(ii) Hence find the equation of the curve which passes through the point $P(1, 8)$. *(3 marks)*

6. Jan 2007

6 A curve C is defined for $x > 0$ by the equation $y = x + 1 + \frac{4}{x^2}$ and is sketched below.



(a) (i) Given that $y = x + 1 + \frac{4}{x^2}$, find $\frac{dy}{dx}$. *(3 marks)*

(ii) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(iii) Find an equation of the normal to C at the point $(1, 6)$. *(4 marks)*

(b) (i) Find $\int \left(x + 1 + \frac{4}{x^2} \right) dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve C, the lines $x = 1$ and $x = 4$ and the x-axis. *(2 marks)*

7. June 2007

5 A curve is defined for $x > 0$ by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point P lies on the curve where $x = 2$.

(a) Find the y-coordinate of P. *(1 mark)*

(b) Expand $\left(1 + \frac{2}{x}\right)^2$. *(2 marks)*

(c) Find $\frac{dy}{dx}$. *(3 marks)*

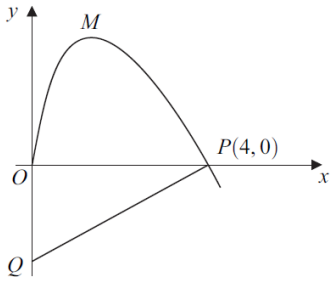
(d) Hence show that the gradient of the curve at P is -2. *(2 marks)*

(e) Find the equation of the normal to the curve at P, giving your answer in the form $x + by + c = 0$, where b and c are integers. *(4 marks)*

8. Jan 2008

5 A curve, drawn from the origin O, crosses the x-axis at the point $P(4, 0)$.

The normal to the curve at P meets the y-axis at the point Q, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Show that the gradient of the curve at $P(4, 0)$ is -2. *(2 marks)*

(iii) Find an equation of the normal to the curve at $P(4, 0)$. *(3 marks)*

(iv) Find the y-coordinate of Q and hence find the area of triangle OPQ. *(3 marks)*

(v) The curve has a maximum point M. Find the x-coordinate of M. *(3 marks)*

(b) (i) Find $\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$. *(3 marks)*

(ii) Find the total area of the region bounded by the curve and the lines PQ and QO. *(3 marks)*

9. June 2008

1 (a) Write $\sqrt{x^3}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) A curve, defined for $x \geqslant 0$, has equation

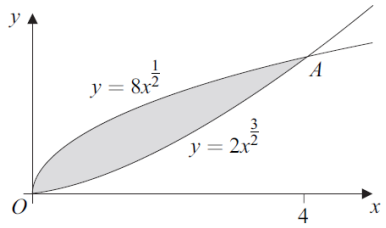
$$y = x^2 - \sqrt{x^3}$$

(i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Find the equation of the tangent to the curve at the point where $x = 4$, giving your answer in the form $y = mx + c$. *(5 marks)*

10.Jan 2009

4 The diagram shows a sketch of the curves with equations $y = 2x^{\frac{3}{2}}$ and $y = 8x^{\frac{1}{2}}$.



The curves intersect at the origin and at the point A, where $x = 4$.

(a) (i) For the curve $y = 2x^{\frac{3}{2}}$, find the value of $\frac{dy}{dx}$ when $x = 4$. *(2 marks)*

(ii) Find an equation of the normal to the curve $y = 2x^{\frac{3}{2}}$ at the point A. *(4 marks)*

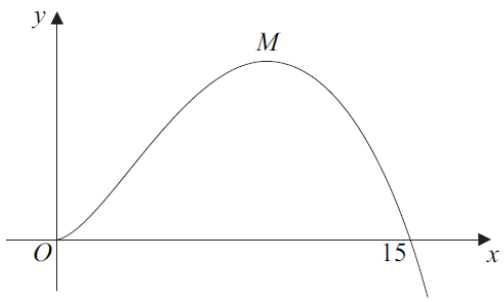
(b) (i) Find $\int 8x^{\frac{1}{2}}\,dx$. *(2 marks)*

(ii) Find the area of the shaded region bounded by the two curves. *(4 marks)*

(c) Describe a single geometrical transformation that maps the graph of $y = 2x^{\frac{3}{2}}$ onto the graph of $y = 2(x+3)^{\frac{3}{2}}$. *(2 marks)*

11.June 2009

5 The diagram shows part of a curve with a maximum point M.



The equation of the curve is

$$y = 15x^{\frac{3}{2}} - x^{\frac{5}{2}}$$

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Hence find the coordinates of the maximum point M. *(4 marks)*

(c) The point $P(1, 14)$ lies on the curve. Show that the equation of the tangent to the curve at P is $y = 20x - 6$. *(3 marks)*

(d) The tangents to the curve at the points P and M intersect at the point R. Find the length of RM. *(3 marks)*

12.Jan 2010

2 At the point (x, y) on a curve, where $x > 0$, the gradient is given by

$$\frac{dy}{dx} = 7\sqrt{x^5} - 4$$

(a) Write $\sqrt{x^5}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) Find $\int \left(7\sqrt{x^5} - 4\right) dx$. *(3 marks)*

(c) Hence find the equation of the curve, given that the curve passes through the point $(1, 3)$. *(3 marks)*

13.Jan 2010

5 A curve has equation $y = \frac{1}{x^3} + 48x$.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Hence find the equation of each of the two tangents to the curve that are parallel to the x-axis. *(4 marks)*

(c) Find an equation of the normal to the curve at the point $(1, 49)$. *(3 marks)*

14.June 2010

6 A curve C has the equation

$$y = \frac{x^3 + \sqrt{x}}{x}, \quad x > 0$$

(a) Express $\frac{x^3 + \sqrt{x}}{x}$ in the form $x^p + x^q$. *(3 marks)*

(b) (i) Hence find $\frac{dy}{dx}$. *(2 marks)*

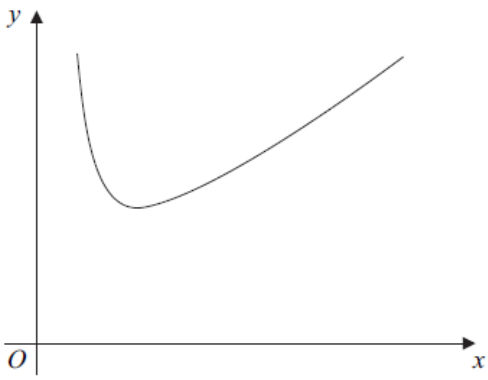
(ii) Find an equation of the normal to the curve C at the point on the curve where $x = 1$. *(4 marks)*

(c) (i) Find $\frac{d^2y}{dx^2}$. *(2 marks)*

(ii) Hence deduce that the curve C has no maximum points. *(2 marks)*

15.Jan 2011

7 A curve C is defined for $x > 0$ by the equation $y = x + 3 + \frac{8}{x^4}$ and is sketched below.



(a) Given that $y = x + 3 + \frac{8}{x^4}$, find $\frac{dy}{dx}$. *(3 marks)*

(b) Find an equation of the tangent at the point on the curve C where $x = 1$. *(3 marks)*

(c) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

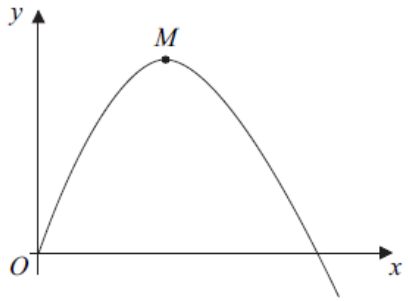
(d) (i) Find $\int \left(x + 3 + \frac{8}{x^4} \right) dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve C, the x-axis and the lines $x = 1$ and $x = 2$. *(2 marks)*

(e) The curve C is translated by $\begin{bmatrix} 0 \\ k \end{bmatrix}$ to give the curve $y = f(x)$. Given that the x-axis is a tangent to the curve $y = f(x)$, state the value of the constant k. *(1 mark)*

16.June 2011

5 The diagram shows part of a curve with a maximum point M.



The curve is defined for $x \geqslant 0$ by the equation

$$y = 6x - 2x^{\frac{3}{2}}$$

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) (i) Hence find the coordinates of the maximum point M. *(3 marks)*

(ii) Write down the equation of the normal to the curve at M. *(1 mark)*

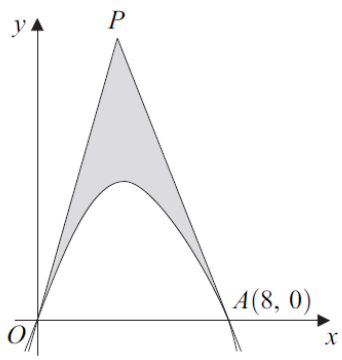
(c) The point $P\left(\frac{9}{4}, \frac{27}{4}\right)$ lies on the curve.

(i) Find an equation of the normal to the curve at the point P, giving your answer in the form $ax + by = c$, where a, b and c are positive integers. *(4 marks)*

(ii) The normals to the curve at the points M and P intersect at the point R. Find the coordinates of R. *(2 marks)*

17.Jan 2012

9 The diagram shows part of a curve crossing the x-axis at the origin O and at the point $A(8, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve has equation

$$y = 12x - 3x^{\frac{5}{3}}$$

(a) Find $\frac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\frac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(8, 0)$ is $y + 8x = 64$. *(3 marks)*

(c) Find $\int \left(12x - 3x^{\frac{5}{3}}\right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve from O to A and the tangents OP and AP. *(7 marks)*

18.June 2012

6 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - \frac{4}{x^2} - 11$$

The point $P(2, 1)$ lies on the curve.

(a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 2$. *(1 mark)*

(ii) Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$. *(4 marks)*

(iii) Hence state whether P is a maximum point or a minimum point, giving a reason for your answer. *(1 mark)*

(b) Find the equation of the curve. *(4 marks)*

19.Jan 2013

5 The point $P(2, 8)$ lies on a curve, and the point M is the only stationary point of the curve.

The curve has equation $y = 6 + 2x - \frac{8}{x^2}$.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Show that the normal to the curve at the point $P(2, 8)$ has equation $x + 4y = 34$. *(3 marks)*

(c) (i) Show that the stationary point M lies on the x-axis. *(3 marks)*

(ii) Hence **write down** the equation of the tangent to the curve at M. *(1 mark)*

(d) The tangent to the curve at M and the normal to the curve at P intersect at the point T. Find the coordinates of T. *(2 marks)*

Answers

1. Jan 2005

1(a)(i)	$y = x + 2x^{-1}$ $\frac{dy}{dx} = 1 - 2x^{-2}$	B1 M1 A1	 3	PI by sight of $-2x^{-2}$ One term correct OE
(ii)	When $x = 2$, $\frac{dy}{dx} = 1 - \frac{2}{4} = \frac{1}{2}$	A1	1	CSO AG (be convinced)
(b)	When $x = 2$, $y = 3$ gradient of normal = − 2 Equation normal $y - 3 = -2(x-2)$	B1 M1 M1 A1	 4	For $y = 3$ $m \times m' = -1$ used $y - "3" = m(x-2)$ OE Award at 1st correct form
	Total		**8**	

2. June 2005

7(a)	$\ldots\ldots\ldots = x^5 - x^{-3}$	M1 A1	 2	One power correct Accept $p = 5$, $q = -3$
(b)(i)	$f'(x) = 5x^4$ $+3x^{-4}$	B1√ B1√	 2	ft on px^{p-1} ft on $-qx^{q-1}$ provided $q < 0$
(ii)	$f'(x) \left\{ = 5x^4 + \frac{3}{x^4} \right\} > 0$ $\Rightarrow$ f is increasing {function}	M1 A1	 2	M1 Considers sign of $f'(x)$; a statement "$f'(x) > 0$ OE" with 'f increasing'. A1 needs $f'(x)$ of the form $ax^4 + \frac{b}{x^4}$, where a and b both > 0 and no incorrect statements based on $f'(x)$ at different values of x
(c)	At (1,0), $f'(1) = 5 + 3 = 8$ $\Rightarrow$ grad. of normal $= -\frac{1}{8}$	M1 m1 A1√	 3	Attempts to find $f'(1)$ Use of $m \times m' = -1$ PI ft on wrong $f'(x)$
	Total		**9**	

3. Jan 2006

1	$y'(x) = 16 - x^{-2}$ $y'(x) = 16 - \frac{1}{x^2}$ $y'(x) = 0 \Rightarrow 16x^2 = 1$; $\Rightarrow x = \pm\frac{1}{4}$	M1 A1 B1 M1 A1	 5	One term correct Both correct $x^{-2} = \frac{1}{x^2}$ OE PI c's $y'(x)=0$ and one relevant further step Both answers required.
	Total		**5**	

4. Jan 2006

8(a)	$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 3$	M1 A1	 2	One term correct Both correct
(b)(i)	When $x = 0$, $\frac{dy}{dx} = -3$ Eqn of tangent at O is $y = -3x$	B1F√ B1F√	 2	Ft provided answer < 0. OE Ft on $y'(0)$
(ii)	At (9,0) $\frac{dy}{dx} = \frac{3}{2}(9)^{\frac{1}{2}} - 3$ Eqn tangent at A is $y - 0 = y'(9)[x - 9]$ $\Rightarrow y = \frac{3}{2}(x-9) \Rightarrow 2y = 3x - 27$	M1 m1 A1	 3	Attempt to find $y'(9)$ OE **CSO**. AG
(iii)	Eliminating $y \Rightarrow -6x = 3x - 27$ $9x = 27 \Rightarrow x = 3$ When $x = 3$, $y = -9$. $\{P(3, -9)\}$	M1 A1F A1F	 3	OE method to one variable (eg $2y = -y - 27$) [A1F for each coordinate; only ft on $y = kx$ tangent in (b)(i) for $k < 0$]
(c)	$\int \left(x^{\frac{3}{2}} - 3x\right) dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{3x^2}{2} \ (+c)$	M1 A2,1,0	 3	One power correct Condone absence of "+c" and unsimplified forms
(d)	$\int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx =$ $= \frac{2}{5} \times 9^{\frac{5}{2}} - \frac{3}{2} \times 9^2 - 0$ $= -24.3$ Area of triangle $OPA = \frac{1}{2} \times 9 \times \lvert y_P \rvert$ Sh.Area $= \frac{1}{2} \times 9 \times \lvert y_P \rvert - \left\lvert \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx \right\rvert$ $= 40.5 - 24.3 = 16.2$	B1 M1 M1 M1 A1	 5	PI Correct use of limits following integration OE
	Total		**18**	

5. June 2006

7(a)(i)	When $x = 4$, $\frac{dy}{dx} = 3(2) + \frac{16}{16} - 7 = 0$	B1	1	AG Be convinced
(ii)	$\frac{16}{x^2} = 16x^{-2}$	B1	1	Accept $k = -2$
(iii)	$\frac{d^2y}{dx^2} = 3 \times \frac{1}{2}x^{-\frac{1}{2}} + 16 \times (-2)x^{-3} - 0$ $\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{1}{2}}; \quad -32x^{-3}$	M1 A1; A1√	 3	A power decreased by 1 candidate's negative integer k [−1 for >2 term(s)]
(iv)	When $x = 4$, $\frac{d^2y}{dx^2} = \frac{3}{4} - \frac{32}{64} = \frac{1}{4}$ Minimum since $y''(4) > 0$	M1 E1√	 2	Attempt to find $y''(4)$ reaching as far as two simplified terms candidate's sign of $y''(4)$
	[Alternative: Finds the sign of $y'(x)$ either side of the point where x=4, need evidence rather than just a statement: (M1) Correct ft conclusion with valid reason E1√] [In both, condone absent statement $y'(4)$=0]			

(b)(i)	At $P(1,8)$, $\frac{dy}{dx} = 3(1)^{\frac{1}{2}} + \frac{16}{1^2} - 7 = 12$	B1	1	AG Be convinced
(ii)	Gradient of normal $= -\frac{1}{12}$	M1		Use of or stating $m \times m' = -1$
	Equation of normal is $y - 8 = m[x-1]$	M1		Can be awarded even if m=12
	$y - 8 = -\frac{1}{12}(x-1) \Rightarrow 12y - 96 = -x + 1$ $\Rightarrow 12y + x = 97$	A1	3	Any correct form of the equation
(c)(i)	$\int 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7\,dx =$			
	…….. $= 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 16\frac{x^{-1}}{-1} - 7x + c$	M1 A2,1,0 √	3	One power correct. A1 if 2 of 3 terms correct candidate's negative integer k Condone absence of "+ c"
(ii)	$y = 2x^{\frac{3}{2}} - 16x^{-1} - 7x + c$ (*)	B1√		y = candidate's answer to (c)(i) with tidied coefficients and with '+c'. ('y =' PI by next line)
	When $x = 1, y = 8 \Rightarrow 8 = 2–16–7+c$	M1		Substitute. (1,8) in attempt to find constant of integration
	$y = 2x^{\frac{3}{2}} - 16x^{-1} - 7x + 29$	A1	3	Accept $c = 29$ after (*), including y =, stated
	Total		**17**	

6. Jan 2007

6(a)(i)	$y = x + 1 + 4x^{-2} \Rightarrow \frac{dy}{dx} = 1 - 8x^{-3}$	M1 A2,1,0	3	Power $p \to p–1$ (A1 if $1 + ax^n$ with $a = -8$ or $n = -3$)
(ii)	$1 - 8x^{-3} = 0$	M1		Puts c's $\frac{dy}{dx} = 0$
	$x^3 = 8$	m1		Using $x^{-k} = \frac{1}{x^k}$ to reach $x^a = b$, a>0 or correct use of logs.
	$x = 2$ When $x = 2$, $y = 4$	A1 A1ft	4	
(iii)	At (1, 6), $\frac{dy}{dx} = 1 - 8 = -7$	M1		Attempt to find $y'(1)$
	Gradient of normal $= \frac{1}{7}$	M1		Use of or stating $m \times m' = -1$
	Equation of normal is $y - 6 = m[x-1]$	M 1		m numerical
	$y - 6 = \frac{1}{7}(x-1)$ $\{\frac{y-6}{x-1} = \frac{1}{7};\ 7y = x + 41\}$	A1ft	4	OE ft on c's answer for (a)(i) provided at least A1 given in (a)(i) and previous 3M marks awarded
(b)(i)	$\int x\left(+1 + \frac{4}{x^2}\right) dx =$			
	…….. $= \frac{x^2}{2} + x - 4x^{-1}$ $\{+ c\}$	M1 A2,1,0	3	One of three terms correct. For A2 need all <u>three</u> terms as printed or better (A1 if 2 of 3 terms correct)

(ii)	{Area=} $\int_1^4 x+1+\frac{4}{x^2}\,dx =$			
	$\left[\frac{x^2}{2}+x-\frac{4}{x}\right]_1^4 = (8+4-1)-\left(\frac{1}{2}+1-4\right)$	M1		Dealing correctly with limits; F(4)–F(1) (must have integrated)
	$= 13.5$	A1	2	
	Total		**16**	

7. June 2007

5(a)	$y_P = 4$	B1	1	
(b)	$y = 1+\frac{2}{x}+\frac{2}{x}+\frac{4}{x^2}$	B2,1,0	2	(B1 if only one error in the expansion) For B2 the last line of the candidate's solution must be correct
	$y = 1+4x^{-1}+4x^{-2}$			
(c)	$\frac{dy}{dx} = -4x^{-2}-8x^{-3}$	M1		Index reduced by 1 after differentiating x to a negative power
		A1ft		At least 1 term in x correct ft on expn
		A1	3	CSO Full correct solution. ACF
(d)	When $x = 2$, $\frac{dy}{dx} = -4\times 2^{-2}-8\times 2^{-3}$	M1		Attempt to find $y'(2)$.
	Gradient $= -1 - 1 = -2$	A1	2	AG (be convinced-no errors seen)
(e)	$-2\times m' = -1$	M1		$m_1\times m_2 = -1$ OE stated or used. PI
	$y-4 = m(x-2)$	M1		C's y_P from part (**a**) if not recovered; m must be numerical.
	$y-4 = \frac{1}{2}(x-2)$	A1ft		Ft on candidate's y_P from part (**a**) if not recovered.
	$x-2y+6 = 0$	A1	4	CAO Must be this or $0 = x-2y+6$
	Total		**12**	

8. Jan 2008

5a(i)	$\frac{dy}{dx} = 4\times\frac{1}{2}x^{-\frac{1}{2}}-\frac{3}{2}x^{\frac{1}{2}} = 2x^{-\frac{1}{2}}-\frac{3}{2}x^{\frac{1}{2}}$	M1 A1A1	3	A power decreased by 1 A1 for each correct term
(ii)	At $P(4,0)$, $\frac{dy}{dx} = \frac{2}{\sqrt{4}}-\frac{3}{2}\times 2$	M1		Attempts $\frac{dy}{dx}$ when $x = 4$
	$= 1-3 = -2$	A1	2	AG
(iii)	Gradient of normal $= \frac{1}{2}$	M1		Use of or stating $m\times m' = -1$
	Equation of normal is $y-0 = m[x-4]$	M1		m numerical; can be awarded even if $m = -2$
	$y-0 = \frac{1}{2}(x-4) \Rightarrow 2y = x-4$	A1	3	ACF of the equation
(iv)	At Q, $x = 0$, $2y = 0-4$	M1		PI
	$y_Q = -2$	A1F		Ft on a linear equation for normal provided y_Q is negative and prev A1 is lost
	Area of triangle $OPQ = 0.5\times 4\times \lvert y_Q \rvert$			
	$= 4$	B1F	3	Ft on c's negative y_Q

(v)	$2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0 \Rightarrow 2x^{-\frac{1}{2}} = \frac{3}{2}x^{\frac{1}{2}}$	M1		Puts c's $\frac{dy}{dx} = 0$ and a 1st step in attempt to solve.
	$2 = \frac{3}{2}x$; $\Rightarrow x = \frac{4}{3}$	m1; A1	 3	Valid method to $ax=b$ Condone 1.3 or better
(b)(i)	$\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx = 4\frac{x^{\frac{3}{2}}}{1.5} - \frac{x^{\frac{5}{2}}}{2.5}$ {+ c} $= \frac{8}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}$ {+ c}	M1 A1,A1	 3	One power correct Condone absence of "+c"
(ii)	Area under curve = $4\frac{4^{\frac{3}{2}}}{1.5} - \frac{4^{\frac{5}{2}}}{2.5}$ − {0} Total area = F(4) + area triangle *OPQ* Total area = $\frac{128}{15} + 4 = \frac{188}{15}$ = 12.5 (3…)	M1 m1 A1	 3	F(4) − {F(0)} Accept 3 sf if clear
	Total		**20**	

9. June 2008

1(a)	$\sqrt{x^3} = x^{\frac{3}{2}}$	B1	1	OE; accept 'k = 1.5'
(b)(i)	$\frac{dy}{dx} = 2x - \frac{3}{2}x^{\frac{1}{2}}$	M1 B1 A1F	 3	At least one index reduced by 1 and no term of the form $\sqrt{ax^2}$. For $2x$ For $-1.5\,x^{0.5}$ .Ft on ans (a) non-integer k
(ii)	When $x = 4$, $y = 8$	B1		
	$y'(4) =$; $= 2(4) - 1.5(\sqrt{4}) = 5$	M1 A1F		Attempt to find $\frac{dy}{dx}$ when $x = 4$ Ft on one earlier error provided non-integer powers in (a) and (b)(i)
	Tangent: $y - 8 = 5(x - 4)$ $y = 5x - 12$	m1 A1	 5	$y - y(4) = y'(4)[x - 4]$ OE CSO; must be $y = 5x - 12$
	Total		**9**	

10.Jan 2009

4(a)(i)	$\frac{dy}{dx} = 3x^{\frac{1}{2}}$ $= 6$ {when $x = 4$}	M1 A1cao	 2	$kx^{\frac{1}{2}}$ with or without + c Must be 6 and seen in (a)(i) 6 + c is A0
(ii)	y-coordinate of A = $2 \times 4^{\frac{3}{2}}$ (= 16)	M1		Substitute $x = 4$ in $y = 2x^{\frac{3}{2}}$
	$6 \times m' = -1$	M1		$m_1 \times m_2 = -1$ OE used with c's value of $\frac{dy}{dx}$ when $x = 4$. PI
	$y - 16 = m(x - 4)$	m1		dep on 1st M1 in (a)(ii) m must be numerical
	$y - 16 = -\frac{1}{6}(x - 4)$	A1	4	ACF

(b)(i)	$\int 8x^{\frac{1}{2}}\,dx = \frac{8}{3/2}x^{\frac{1}{2}+1}\{+c\}$	M1		Index raised by 1
	$= \frac{16}{3}x^{\frac{3}{2}}\ \{+c\}$	A1	2	Condone missing '+ c' Coefficient must be simplified
(ii)	$\int 2x^{\frac{3}{2}}\,dx = \frac{2}{5/2}x^{\frac{5}{2}}\{+c\}\qquad \{= \frac{4}{5}x^{\frac{5}{2}}\{+c\}\}$	B1		Can award for unsimplified form
	$\int_0^4 8x^{\frac{1}{2}}\,dx - \int_0^4 2x^{\frac{3}{2}}\,dx$	M1		Ignore limits here
	$= \frac{16}{3}(4)^{\frac{3}{2}} - 0 - \left[\frac{4}{5}(4)^{\frac{5}{2}} - 0\right]$	M1		F(4) − F(0) used in either; {F(0)=0 PI} Cand. must be using F(x) as a result of his/her integration in (b)(i) or in the (b)(ii) B1 line above
	$= \frac{256}{15}$	A1	4	Accept any value from 17.04 to 17.1 inclusive in place of 256/15
(c)	Translation	B1		Accept 'translat…' as equivalent [T or Tr is NOT sufficient]
	$\begin{bmatrix} -3 \\ 0 \end{bmatrix}$	B1	2	Accept equivalent in words provided linked to 'translation/move/shift' (B0B0 if >1 transformation)
	Total		**14**	

11.June 2009

5(a)	$\frac{dy}{dx} = \frac{45}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$	M1 A2,1,0	3	One power correctly obtained A1 for each term on the RHS coeffs simplified
(b)	$\frac{45}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} = 0$	M1		cand's (a) = 0
	$\frac{5}{2}x^{\frac{1}{2}}(9 - x) = 0$	m1		Must be solving eqn of form $ax^m + bx^n = 0$, m and n non-zero, with at least one of m and n non-integer and reaching a stage from which the non-zero value of x can be stated PI. Must deal with powers of x correctly and any squaring of kx^p terms or expressions must be correct.
	At M, $x = 9$	A1		
	$y_M = 162$	A1	4	M1 must be scored, else 0/4
(c)	At $P(1, 14)$, $\frac{dy}{dx} = \frac{45}{2} - \frac{5}{2} = 20$	M1		Attempt to find $y'(1)$
	Tangent at P: $y - 14 = m(x - 1)$	m1		m = cand's value of $y'(1)$
	$y - 14 = 20x - 20$; $y = 20x - 6$	A1	3	CSO; AG
(d)	Tangent at M: $y = 162$	B1F		ft y = cand's y_M
	At R, $162 = 20x - 6$; $x = 8.4$	M1		Solving cand's numerical $y_M = 20x - 6$ to find a value for x
	Distance $RM = \lvert x_M - x_R \rvert = 9 - 8.4 = 0.6$	A1F	3	ft on coordinates of M
	Total		**13**	

12.Jan 2010

2(a)	$\sqrt{x^5} = x^{\frac{5}{2}}$	B1	1	Accept $k = 2.5$
(b)	$\int\left(7\sqrt{x^5} - 4\right) dx = \frac{7}{3.5}x^{3.5} - 4x \ (+c)$	M1 A1F B1	 3	Index 'k' raised by 1 in integrating x^k 1st term correct follow through on non-integer k For $-4x$ as integral of -4
(c)	$y = 2x^{3.5} - 4x + c \qquad (*)$	B1F		y = c's answer to (b) with '$+ c$' ('$y =$' PI by next line)
	When $x = 1$, $y = 3 \Rightarrow 3 = 2 - 4 + c$	M1		Subst. (1, 3) in attempt to find constant of integration
	$y = 2x^{3.5} - 4x + 5$	A1	3	Accept $c = 5$ after correct eqn * which must include '$y =$' Coefficients must be tidied
	Total		**7**	

13.Jan 2010

5(a)	$\frac{1}{x^3} = x^{-3}$	B1		PI by its correct derivative
	$\frac{dy}{dx} = -3x^{-4} + 48$	M1		A power decreased by 1; could be the +48 or the ft after B0
		A1	3	
(b)	$-3x^{-4} + 48 = 0$	M1		c's answer to (a) equated to 0
	$x^{-4} = 16$	A1F		To $x^p = q$ but only ft on eqns of the form $ax^{2k} + 48 = 0$, where a and k are **negative integers**
	$x = \pm\frac{1}{2}$	A1		
	Eqns of tangents: $y = 32$ and $y = -32$	A1F	4	Only ft if answer is of the form $y = \pm k$
(c)	When $x = 1$, $\frac{dy}{dx} = -3 + 48 = 45$	M1		Attempt to find value of $\frac{dy}{dx}$ at $x = 1$
	Gradient of normal at (1, 49) is $-\frac{1}{45}$	m1		Correct use of $m \times m' = -1$ with c's value of $\frac{dy}{dx}$ when $x = 1$
	Normal at (1, 49): $y - 49 = -\frac{1}{45}(x - 1)$	A1	3	CSO. Apply ISW after ACF; accept 49.02 or better in place of $49\frac{1}{45}$
	Total		**10**	

14.June 2010

6(a)	$\sqrt{x} = x^{\frac{1}{2}}$	B1		PI
	$\frac{x^3 + \sqrt{x}}{x} = \frac{x^3}{x} + \frac{\sqrt{x}}{x} = x^2 + x^{-\frac{1}{2}}$	B1;B1	3	Accept $p = 2$; $q = -\frac{1}{2}$
(b)(i)				

(b)(i)	$\frac{dy}{dx} = 2x \quad -\frac{1}{2}x^{-\frac{3}{2}}$	M1 A1	2	Reduces both powers by 1 ACF
(ii)	When $x = 1$, $y = 2$	B1		PI if not stated explicitly eg the '2' may appear in the correct posn. in later eqn.
	When $x = 1$, $\frac{dy}{dx} = 2 - \frac{1}{2} = \frac{3}{2}$	M1		Attempt to find $\frac{dy}{dx}$ when $x = 1$ PI
	Gradient of normal $= -\frac{2}{3}$	m1		$-1/$(c's value of dy/dx when $x = 1$) either stated as the gradient of the normal or used as the gradient in the equation of the normal
	Equation of normal: $y - 2 = -\frac{2}{3}(x-1)$	A1F	4	Only ft on c's $\frac{dy}{dx}$ in part (b)(i).
(c)(i)	$\frac{d^2y}{dx^2} = 2 + \frac{3}{4}x^{-\frac{5}{2}}$	M1 A1F	2	ACftF Reduces both powers by 1. Ft on (b)(i) provided at least one power to be differentiated is both negative and fractional
(ii)	(Since x>0,) $\frac{d^2y}{dx^2}$>0 For a maximum point $\frac{d^2y}{dx^2}$ is **not** positive so C has no maximum points	E2,1,0	2	E1 for attempt to find the sign of $\frac{d^2y}{dx^2}$; either in general terms or at the pt(s) where c's $dy/dx = 0$ for the remaining E mark a correct justification for why $\frac{d^2y}{dx^2}$>0 and also a full correct concluding statement must be made.
	Total		**13**	

15.Jan 2011

7(a)	$y = x + 3 + \frac{8}{x^4} = x + 3 + 8x^{-4}$	B1		For $\frac{8}{x^4} = 8x^{-4}$ PI by correct differentiation of 3rd term
	$\frac{dy}{dx} = 1 - 32x^{-5}$ or $1 - \frac{32}{x^5}$	M1 A1	3	kx^{-5} OE For either
(b)	When $x = 1$, $y = 12$	B1		
	When $x = 1$, $\frac{dy}{dx} = 1 - 32 = -31$	M1		Attempt to find value of $\frac{dy}{dx}$ when x=1
	Tangent: $y - 12 = -31(x-1)$	A1F	3	Only ft on c's answer to (a). Any correct (ft on c's (a)) form.
(c)	$1 - 32x^{-5} = 0$	M1		$1 - 32x^{-5} = 0$ or c's $\frac{dy}{dx} = 0$
	$\Rightarrow x^5 = 32$	m1		Attempt to form x^n = const ($\neq$0). PI by next line
	$\Rightarrow x = 2$	A1		CSO
	(Coordinates of M) (2, 5.5)	A1	4	CSO

(d)(i)	$\int \left(x+3+\frac{8}{x^4}\right)\,dx$			
	$=\frac{x^2}{2}+3x-\frac{8}{3}x^{-3}+c$	M1		Power −3 correctly obtained
		A1		$-\frac{8}{3}x^{-3}$
		B1	3	$\frac{x^2}{2}+3x+c$
(ii)	Area $=\left[\frac{x^2}{2}+3x-\frac{8}{3}x^{-3}\right]_1^2$			
	$=\left(2+6-\frac{1}{3}\right)-\left(\frac{1}{2}+3-\frac{8}{3}\right)$	M1		Attempting to calculate F(2) − F(1) where F(x) is c's answer to part (d)(i) provided F is not just the c's integrand (x+3+8/x^4)
	$=\frac{9}{2}+\frac{7}{3}=\frac{41}{6}$	A1	2	OE Accept 6.83 or better provided d(i) used
(e)	$k=-5.5$	B1F	1	Ft on $-y_M$ from part (c).
	Total		**16**	

16.June 2011

5(a)		B1		For either 6 or $6x^0$
	$\frac{dy}{dx}=6-3x^{\frac{1}{2}}$	M1		$Ax^{\frac{3}{2}-1}$, $A\neq 0$ OE
		A1	3	$6-3x^{\frac{1}{2}}$ or $6-3\sqrt{x}$ with no '+c' [If unsimplified here, A1 can be awarded retrospectively if correct simplified expression is seen explicitly in (b)(i).]
(b)(i)	$6-3x^{\frac{1}{2}}=0$	M1		Equating c's $\frac{dy}{dx}$ to 0 PI by correct ft rearrangement of c's dy/dx=0
	$x^{\frac{1}{2}}=2\Rightarrow x=2^2$	m1		$x^{\frac{1}{2}}=k$ (k>0), to $x=k^2$. PI by correct value of x if no error seen
	$M(4, 8)$	A1	3	SC If M0 award B1 for (4, 8)
(ii)	Eqn of normal at M is $x=4$	B1F	1	Ft on x = c's x_M
(c)(i)	When $x=\frac{9}{4}$, $\frac{dy}{dx}=6-3\times\frac{3}{2}=\frac{3}{2}$	M1		Attempt to find $\frac{dy}{dx}$ when $x=\frac{9}{4}$
	Gradient of normal at $P=-\frac{2}{3}$	m1		$m\times m'=-1$ used
	Eqn of normal: $y-\frac{27}{4}=-\frac{2}{3}\left(x-\frac{9}{4}\right)$	A1		ACF eg $y=-\frac{2}{3}x+\frac{33}{4}$
	$12y-81=-8x+18\Rightarrow 8x+12y=99$	A1	4	Coeffs and constant must now be positive integers, but accept different order eg $12y+8x=99$
(ii)	$8(4)+12y=99$	M1		Solving c's answer (b)(ii), (must be in form x = positive const), with c's answer (c)(i). PI by correct earlier work and correct coordinates for R.
	$R\left(4,\frac{67}{12}\right)$	A1	2	Accept 5.58 or better as equivalent to $\frac{67}{12}$
	Total		**13**	

17.Jan 2012

9(a)	$\frac{dy}{dx} = 12 - 5x^{\frac{2}{3}}$	M1 A1	 2	$kx^{\frac{2}{3}}$ term. ACF
(b)(i)	When x=0, $\frac{dy}{dx} = 12$ Eqn of tangent at O is $y = 12x$	B1F B1F	 2	Ft on c's y' evaluated correctly at x=0 OE Ft on c's value for $y'(0)$ provided $y'(0)>0$.
(ii)	When $x = 8$, $\frac{dy}{dx} = 12 - 5 \times (8)^{\frac{2}{3}}$ Equation of tangent at (8, 0) is $y - 0 = y'(8)[x - 8]$ $y = -8(x-8) \Rightarrow y + 8x = 64$	M1 m1 A1	 3	Attempt to find $\frac{dy}{dx}$ when $x = 8$ $y = y'(8)[x - 8]$ OE CSO AG
(c)	$\int \left(12x - 3x^{\frac{5}{3}}\right) dx = \frac{12x^2}{2} - \frac{3x^{\frac{8}{3}}}{\frac{8}{3}}$ $(+c)$	M1		$kx^{\frac{5}{3}+1}$ term after integrating, condone k left unsimplified for this M mark.
	$= 6x^2 - \frac{9}{8}x^{\frac{8}{3}}$ $(+c)$	B1 A1	 3	For $6x^2$ OE eg $(12x^2/2)$ For $-\frac{9}{8}x^{\frac{8}{3}}$ OE
(d)	Area bounded by curve and x-axis $= \int_0^8 \left(12x - 3x^{\frac{5}{3}}\right) dx = 6 \times 8^2 - \frac{9}{8} \times (8)^{\frac{8}{3}}$ $= 384 - 288 = 96$ At P, $12x + 8x = 64$ $(x_P = 3.2)$ $y_P = 38.4$ Area of triangle $OPA = \frac{1}{2} \times 8 \times y_P$	 M1 A1 M1 A1 M1		 $\pm$ F(8) {$-$ F(0)} PI following integration PI by correct final answer if evaluation not seen here Solving $y + 8x = 64$ and c's $y=kx$, $k>0$, down to an eqn in one variable… [$y+2y/3=64$] For $y_P = 38.4$ OE [If using integration to find area of triangle, award A1 if both '$x_P = 3.2$' and correct integration of correct eqns of the 2 lines] OE Need perpendicular ht to be linked to $y_P > 0$.
	Area of triangle $OPA = \frac{1}{2} \times 8 \times y_P$ Area of shaded region =Area Δ $OPA - \int_0^8 \left(12x - 3x^{\frac{5}{3}}\right) dx$ $= 153.6 - 96 = 57.6$	M1 M1 A1	 7	OE Need perpendicular ht to be linked to $y_P > 0$. M0 if evaluated to a value <0 OE eg 288/5
	Total		**17**	

18.June 2012

6(a)(i)	(When $x = 2$) $\frac{dy}{dx} = 12 - 1 - 11 = 0$	B1	1	AG Must see intermediate evaluations
(ii)	$\frac{4}{x^2} = 4x^{-2}$ {so $\frac{dy}{dx} = 3x^2 - 4x^{-2} - 11$} $\frac{d^2y}{dx^2} = 6x + 8x^{-3}$ When $x = 2$, $\frac{d^2y}{dx^2} = 12 + 8/8 = 13$	B1 M1 A1 A1	 4	$\frac{4}{x^2} = 4x^{-2}$, seen in (a)(ii) or earlier. PI by $\pm 8x^{-3}$ term in answer Correct powers of x correctly obtained from differentiating the first two terms $6x + 8x^{-3}$ ACF

(iii)	Since $\frac{d^2y}{dx^2} > 0$, P is a minimum point.	E1F	1	Ft on c's value of $y''(2)$ in (a)(ii) but must see reference to sign of $y''(2)$ either explicitly or as inequality, as well as the correct ft conclusion
(b)	$\int\left(3x^2 - \frac{4}{x^2} - 11\right)dx = x^3 + 4x^{-1} - 11x(+c)$	M1		Attempt to integrate $\frac{dy}{dx}$ with at least two of the three terms integrated correctly
	$(y =)\ x^3 + 4x^{-1} - 11x\ (+\ c)$	A1		For $x^3 + 4x^{-1} - 11x$ OE even unsimplified
	When $x = 2, y = 1 \Rightarrow 1 = 8 + 2 - 22 + c$	M1		Substituting. $x = 2$, $y = 1$ into $y = F(x) +$ 'c' in attempt to find constant of integration, where $F(x)$ follows attempted integration of expression for $\frac{dy}{dx}$
	$y = x^3 + 4x^{-1} - 11x + 13$	A1	4	ACF
	Total		**10**	

19.Jan 2013

5(a)	$\frac{8}{x^2} = 8x^{-2}$	B1		PI by its derivative as $16x^{-3}$ or $-16x^{-3}$
	$\frac{dy}{dx} = 2 + 16x^{-3}$	M1		Differentiating either $6+2x$ correctly or differentiating $-8/x^2$ correctly.
		A1	3	$2 + 16x^{-3}$ OE
(b)	At $P(2, 8)$, $\frac{dy}{dx} = 2 + 16 \times 2^{-3}$ $(= 4)$	M1		Attempt to find $\frac{dy}{dx}$ when $x = 2$
	Gradient of normal at $P = -\frac{1}{4}$	m1		$m \times m' = -1$ used
	Eqn. of normal at P: $y - 8 = -\frac{1}{4}(x - 2) \Rightarrow x + 4y = 34$	A1	3	CSO AG
(c)(i)	At St. Pt $\frac{dy}{dx} = 0$, $2 + 16x^{-3} = 0$	M1		Equating c's $\frac{dy}{dx}$ to 0 Accept '$\frac{dy}{dx} = 0$ so $x = -2$' stated with no errors seen
	$(16x^{-3} = -2)$ $x = -2$	A1		$x = -2$
	When $x = -2$, $y = 6-4-2=0$; $M(-2,0)$ lies on x-axis	A1	3	Need statement and correct coords.
(c)(ii)	Tangent at M has equation $y = 0$	B1	1	$y = 0$ OE
(d)	Intersects normal at P when $x + 0 = 34$	M1		PI Solving c's eqn. of tangent with ans (b) as far as correctly eliminating one variable.
	$T\ (34, 0)$	A1	2	Accept $x = 34$, $y = 0$
	Total		**12**	