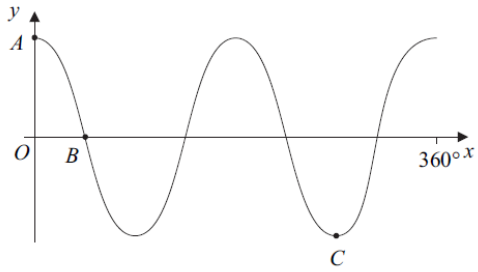
Practice examination questions for trigonometrical equations

1. Jan 2005

7 The diagram shows the graph of $y = \cos 2x$ for $0° \leqslant x \leqslant 360°$.



(a) Write down the coordinates of the points A, B and C marked on the diagram. *(4 marks)*

(b) Describe the single geometrical transformation by which the curve with equation $y = \cos 2x$ can be obtained from the curve with equation $y = \cos x$. *(2 marks)*

(c) Solve the equation

$$\cos 2x = 0.37$$

giving all solutions to the nearest 0.1° in the interval $0° \leqslant x \leqslant 360°$. (No credit will be given for simply reading values from a graph.) *(5 marks)*

2. June 2005

8 (a) (i) Show that the equation

$$4 \tan \theta \sin \theta = 15$$

can be written as

$$4 \sin^2 \theta = 15 \cos \theta$$ *(1 mark)*

(ii) Use an appropriate identity to show that the equation

$$4 \sin^2 \theta = 15 \cos \theta$$

can be written as

$$4 \cos^2 \theta + 15 \cos \theta - 4 = 0$$ *(2 marks)*

(b) (i) Solve the equation $4c^2 + 15c - 4 = 0$. *(2 marks)*

(ii) Hence explain why the only value of $\cos \theta$ which satisfies the equation

3. Jan 2006

6 (a) Describe the geometrical transformation that maps the curve with equation $y = \sin x$ onto the curve with equation:

(i) $y = 2 \sin x$; *(2 marks)*

(ii) $y = -\sin x$; *(2 marks)*

(iii) $y = \sin(x - 30°)$. *(2 marks)*

(b) Solve the equation $\sin(\theta - 30°) = 0.7$, giving your answers to the nearest $0.1°$ in the interval $0° \leqslant \theta \leqslant 360°$. *(3 marks)*

(c) Prove that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$. *(4 marks)*

4. June 2007

7 (a) Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) Write down the **two** solutions of the equation $\tan x = \tan 61°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) (i) Given that $\sin\theta + \cos\theta = 0$, show that $\tan\theta = -1$. *(1 mark)*

(ii) Hence solve the equation $\sin(x - 20°) + \cos(x - 20°) = 0$ in the interval $0° \leqslant x \leqslant 360°$. *(4 marks)*

(d) Describe the single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x - 20°)$. *(2 marks)*

(e) The curve $y = \tan x$ is stretched in the x-direction with scale factor $\frac{1}{4}$ to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(1 mark)*

5. Jan 2008

9 (a) Given that

$$\frac{3 + \sin^2\theta}{\cos\theta - 2} = 3\cos\theta$$

show that

$$\cos\theta = -\frac{1}{2}$$ *(4 marks)*

(b) Hence solve the equation

$$\frac{3 + \sin^2 3x}{\cos 3x - 2} = 3\cos 3x$$

giving all solutions in degrees in the interval $0° < x < 180°$. *(4 marks)*

6. June 2008

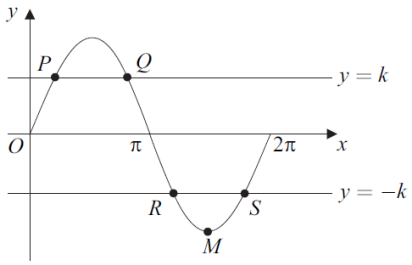
9 (a) Solve the equation $\sin 2x = \sin 48°$, giving the values of x in the interval $0° \leqslant x < 360°$. *(4 marks)*

(b) Solve the equation $2\sin\theta - 3\cos\theta = 0$ in the interval $0° \leqslant \theta < 360°$, giving your answers to the nearest $0.1°$. *(4 marks)*

7. Jan 2009

7 (a) Solve the equation $\sin x = 0.8$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of the curve $y = \sin x$, $0 \leqslant x \leqslant 2\pi$ and the lines $y = k$ and $y = -k$.



The line $y = k$ intersects the curve at the points P and Q, and the line $y = -k$ intersects the curve at the points R and S.

The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

Write down the x-coordinate of the point Q in terms of π and α. *(1 mark)*

(iii) Find the length of RS in terms of π and α, giving your answer in its simplest form. *(2 marks)*

(c) Sketch the graph of $y = \sin 2x$ for $0 \leqslant x \leqslant 2\pi$, indicating the coordinates of points where the graph intersects the x-axis and the coordinates of any maximum points. *(5 marks)*

8. June 2009

8 (a) Given that $\dfrac{\sin\theta - \cos\theta}{\cos\theta} = 4$, prove that $\tan\theta = 5$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

$$2\cos^2 x - \sin x = 1$$

can be written as

$$2\sin^2 x + \sin x - 1 = 0$$ *(2 marks)*

(ii) Hence solve the equation

$$2\cos^2 x - \sin x = 1$$

giving all solutions in the interval $0° \leqslant x \leqslant 360°$. *(5 marks)*

9. Jan 2010

8 (a) Solve the equation $\tan(x + 52°) = \tan 22°$, giving the values of x in the interval $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) (i) Show that the equation

$$3\tan\theta = \frac{8}{\sin\theta}$$

can be written as

$$3\cos^2\theta + 8\cos\theta - 3 = 0$$ *(3 marks)*

(ii) Find the value of $\cos\theta$ that satisfies the equation

$$3\cos^2\theta + 8\cos\theta - 3 = 0$$ *(2 marks)*

(iii) Hence solve the equation

$$3\tan 2x = \frac{8}{\sin 2x}$$

giving all values of x to the nearest degree in the interval $0° \leqslant x \leqslant 180°$. *(4 marks)*

10.June 2010

7 (a) Sketch the graph of $y = \cos x$ in the interval $0 \leqslant x \leqslant 2\pi$. State the values of the intercepts with the coordinate axes. *(2 marks)*

(b) (i) Given that

$$\sin^2\theta = \cos\theta(2 - \cos\theta)$$

prove that $\cos\theta = \frac{1}{2}$. *(2 marks)*

(ii) Hence solve the equation

$$\sin^2 2x = \cos 2x(2 - \cos 2x)$$

in the interval $0 \leqslant x \leqslant \pi$, giving your answers in radians to three significant figures. *(4 marks)*

11.Jan 2011

9 (a) Solve the equation $\tan x = -3$ in the interval $0° \leqslant x \leqslant 360°$, giving your answers to the nearest degree. *(3 marks)*

(b) (i) Given that

$$7\sin^2\theta + \sin\theta\cos\theta = 6$$

show that

$$\tan^2\theta + \tan\theta - 6 = 0$$ *(3 marks)*

(ii) Hence solve the equation $7\sin^2\theta + \sin\theta\cos\theta = 6$ in the interval $0° \leqslant \theta \leqslant 360°$, giving your answers to the nearest degree. *(4 marks)*

12.June 2011

8 Prove that, for all values of x, the value of the expression

$$(3\sin x + \cos x)^2 + (\sin x - 3\cos x)^2$$

is an integer and state its value. *(4 marks)*

13.Jan 2012

8 (a) Given that $2\sin\theta = 7\cos\theta$, find the value of $\tan\theta$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

$$6\sin^2 x = 4 + \cos x$$

can be written as

$$6\cos^2 x + \cos x - 2 = 0$$ *(2 marks)*

(ii) Hence solve the equation $6\sin^2 x = 4 + \cos x$ in the interval $0° < x < 360°$, giving your answers to the nearest degree. *(6 marks)*

14.June 2012

7 It is given that $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$.

(a) Find the possible values of $\tan\theta$. *(4 marks)*

(b) Hence solve the equation $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$, giving all solutions for θ, in degrees, in the interval $0° \leqslant \theta \leqslant 180°$. *(3 marks)*

15.Jan 2013

9 (a) Write down the two solutions of the equation $\tan(x + 30°) = \tan 79°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x + 30°)$. *(2 marks)*

(c) (i) Given that $5 + \sin^2\theta = (5 + 3\cos\theta)\cos\theta$, show that $\cos\theta = \frac{3}{4}$. *(5 marks)*

(ii) **Hence** solve the equation $5 + \sin^2 2x = (5 + 3\cos 2x)\cos 2x$ in the interval $0 < x < 2\pi$, giving your values of x in radians to three significant figures. *(3 marks)*

Answers

1. Jan 2005

7(a)	$A(0^\circ, 1)$ $B(45^\circ, 0)$ $C(270^\circ,\ -1)$	B1 B1 B1, B1	4	Condone radians Condone (0.785,0) or better. B1 for 270; B1 for –1
(b)	Stretch **(I)** in x-direction **(II)** with a scale factor $\frac{1}{2}$ **(III)**	M1A1	2	More than one transformation is M0 M1 for (I) **and** either (II) or (III)
(c)	$\cos^{-1}0.37 =$ "68.284..." $(=\alpha)$ $x = \frac{\alpha}{2} = 34.1(42.)^\circ$ $x = 180 - \frac{\alpha}{2}$ $x = 180 + \frac{\alpha}{2}$ **and** $x = 180 + 180 - \frac{\alpha}{2}$	M1 A1 m1 m1		$\text{Cos}^{-1}0.37$ (PI eg by 68.3 or 1.19) Condone 34.2°, 34° or 0.596 rads OE eg $2x = 360 - \alpha$ OE Need both (OE for $2x =$) with no extras (quadrants) within the given interval
	$2x = 68.284\ldots; 291.715\ldots;$ $428.284\ldots;\ 651.715\ldots$ $x = (34.1^\circ;)$ $145.9^\circ;\ 214.1^\circ;\ 325.9^\circ$	A1	5	Dep. on **all** three method marks. Must be in degrees
	Total		**11**	

2. June 2005

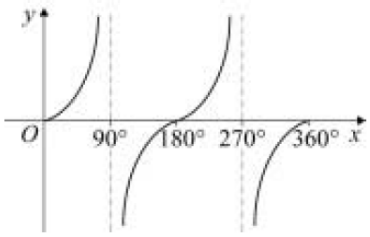
8(a)(i)	$4\frac{\sin\theta}{\cos\theta}\sin\theta = 15$ $\Rightarrow 4\sin^2\theta = 15\cos\theta$	B1	1	**AG** Be convinced
(ii)	$\sin^2\theta + \cos^2\theta = 1$ $4(1 - \cos^2\theta) = 15\cos\theta$ $4\cos^2\theta + 15\cos\theta - 4 = 0$	M1 A1	2	OE seen **AG** Be convinced
(b)(i)	$(4c-1)(c+4) = 0$ $c = -4,\ c = \frac{1}{4}$	M1 A1	2	Factorisation or formula or completion of square Both values
(ii)	Since $-1 \le \cos\theta\ \{\le 1\}$, the only possible value for $\cos\theta$ is $\frac{1}{4}$	E1√	1	**AG** convincingly explained (Condone strict inequalities) Ft provided candidates answers for c are $\frac{1}{4}$ and a value k such that $k > 1$ or $k < -1$
(iii)	$\theta = 75.5^\circ$ $\theta = 284.5^\circ$	B1 B1√	2	Ft on [360 – c's 75.5°] as only other solution in the given interval

(c)	$\ldots\ldots\ldots \Rightarrow \cos 4x = \frac{1}{4}$ $x = 19°, 71°$	M1 A1√	 2	Links with previous parts. PI Ft on (iii)/4…..(only ft if 2 answers in given range).
	Total		**10**	

3. Jan 2006

6(a)(i)	Stretch **(I)** in y-direction **(II)** Scale factor 2 **(III)**	M1A1	2	>1 transformation is M0. M1 for **(I)** <u>and</u> either **(II)** or **(III)** or **(III)**
(ii)	Reflection; in x-axis	M1 A1	2	'Reflection'/ 'reflect(ed)' (or in y-axis or $y = 0$ or $x = 0$)
(iii)	Translation; $\begin{bmatrix} 30° \\ 0 \end{bmatrix}$	B1 B1	2	'Translation'/'translate(d)' Accept **full** equivalent in words provided linked to 'translation/move/shift' and **positive** x-direction (Note: B0 B1 is possible)
(b)	$\{\theta - 30° =\} \sin^{-1}(0.7) = 44.4\ldots°$ $\ldots\ldots\ldots = 180° - 44.4°$ $\theta = 74.4°,\ 165.6°$	M1 m1 A1	3	Inverse sine of 0.7 PI eg by sight of 44, 74 or better Valid method for 2nd angle Condone >1dp accuracy
(c)	$\ldots = \cos^2 x + 2\cos x \sin x + \sin^2 x +$ $\cos^2 x - 2\cos x \sin x + \sin^2 x$ $\ldots. = 2\cos^2 x + 2\sin^2 x$ $= 2(\cos^2 x + \sin^2 x) = 2\ (1)$ $= 2$	M1 A1 M1 A1	 4	 Award for either bracket expanded correctly OE $\cos^2 x + \sin^2 x = 1$ stated or used. AG (be convinced)
	Total		**13**	

4. June 2007

7(a)		M1 A1 A1	 3	Correct shape of branch from O {to 90°} **or** correct shapes of branches from 90°-360° Complete graph for $0° \le x \le 360°$ (Asymptotes not explicitly required but graphs should show 'tendency') Correct scaling on x-axis $0° \le x \le 360°$
(b)	61° ; 241°	B1 B1	2	For 61° For 241° and no 'extras' in the interval $0° \le x \le 360°$
(c)(i)	$\sin\theta = -\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = -1$ $\Rightarrow \tan\theta = -1.$	B1	1	AG; be convinced that the identity $\frac{\sin\theta}{\cos\theta} = \tan\theta$ is known and validly used
(ii)	$\Rightarrow \tan(x - 20°) = -1$ $x - 20° = \tan^{-1}(-1)$ $x - 20° = 135°, 315° \ldots$ $x = 155°$; 335°	M1 m1 A1 A1ft	 4	 Ft on (180 + "155") and no 'extras' in the given interval.

(d)	Translation $\begin{bmatrix} 20 \\ 0 \end{bmatrix}$	B1 B1	2	'Translation'/'translate(d)' Accept equivalent in words provided linked to 'translation/move/shift' (Note: B0B1 is possible)
(e)	$f(x) = \tan 4x$	B1	1	For $\tan 4x$
	Total		**13**	

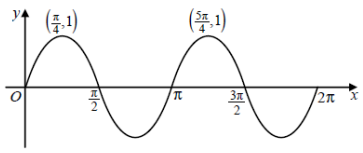
5. Jan 2008

9(a)	$\dfrac{3+\sin^2\theta}{\cos\theta-2}=3\cos\theta$			
	$\Rightarrow \dfrac{3+(1-\cos^2\theta)}{\cos\theta-2}=3\cos\theta$	M1		$\cos^2\theta + \sin^2\theta = 1$ stated or used [If cand starts with $\cos\theta = -½$ and gets $\sin^2\theta = ¾$ without explicitly finding value for θ and verifies 1st equation is true, award M1moA0]
	$\Rightarrow \dfrac{4-\cos^2\theta}{\cos\theta-2}=3\cos\theta$			
	$\Rightarrow \dfrac{(2-\cos\theta)(2+\cos\theta)}{\cos\theta-2}=3\cos\theta$	m1		Difference of two squares or division (PI by next line)
	$\Rightarrow -1(2+\cos\theta)=3\cos\theta$	A1		
	$\Rightarrow -2=4\cos\theta \Rightarrow \cos\theta=-\dfrac{1}{2}$	A1	4	CSO AG
	Alternative for (a)			
	$3+1-\cos^2\theta=3\cos^2\theta-6\cos\theta$	(M1)		$\cos^2\theta + \sin^2\theta = 1$
	$(4\cos\theta+2)(\cos\theta-2)=0$	(m1)		Factorising or formula
	$\cos\theta - 2 \neq 0$	(A1)		Indicates rejection of $\cos\theta=2$
	$\Rightarrow 4\cos\theta=-2 \Rightarrow \cos\theta=-\dfrac{1}{2}$	(A1)		AG Be convinced
(b)	$\theta = 3x \Rightarrow \cos 3x = -\dfrac{1}{2}$	M1		Uses part (a) to reach either $\cos 3x = -0.5$ or $\cos 3x = 0.5$
	$\cos^{-1}\left(-\dfrac{1}{2}\right) = 120°$	m1		Or $\cos^{-1}(0.5) = 60°$ Condone radians here
	$3x = 120°, 240°, 480°, \ldots$			
	$x = 40°, 80°, 160°$	A2,1,0	4	A1 for at least <u>two</u> correct. If >3 solutions in the interval $0° < x < 180°$, deduct 1 mark from any A marks for each extra solution. Deduct 1 mark from any A marks if answers in radians. Ignore extra values outside the given interval.
	Total		**8**	

6. June 2008

9(a)	$2x = 48$ $2x = 180 - 48$ $2x = 360 + 48$ and $2x = 360+180-48$ $x = 24°,\ 66°,\ 204°,\ 246°$	B1 M1 M1 A1	 4	PI by $x = 24°$ Accept equivalents for x Accept equivalents for x CAO; need all four, no extras in given interval
(b)	$\frac{\sin\theta}{\cos\theta} = \tan\theta$ $2\sin\theta - 3\cos\theta = 0 \Rightarrow \tan\theta = 1.5$ $\theta = 56.3°$ $\theta = 56.3° + 180° = 236.3°$	M1 A1 A1 A1F	 4	Stated or used Condone > 1dp Ft on c's PV+180° dep only on the M1 provided no 'extra' solutions in the given interval.
	Total		**8**	

7. Jan 2009

7(a)	$\{x =\}\ \sin^{-1}(0.8) = 0.927(29\ldots)\quad \{=\beta\}$ $\{x =\}\ \pi - \beta$ $x = 0.927(29\ldots),\quad 2.21(42\ldots)$	M1 m1 A1	 3	$\sin^{-1}(0.8)$ PI Both Ignore values outside interval 0–2π but A0 if 'extra' values inside the given interval
(b)(i)	$\left(\frac{3\pi}{2}, -1\right)$	B2,1	2	B1 if one coordinate correct or $\left(-1, \frac{3\pi}{2}\right)$
(ii)	$\pi - \alpha$	B1	1	
(iii)	$RS = (2\pi - \alpha) - (\pi + \alpha)$ $= \pi - 2\alpha$	M1 A1	 2	OE eg $RS = PQ = (\pi - \alpha) - \alpha$ Must be simplified
(c)	 Maximum points $\left(\frac{\pi}{4}, 1\right)$ and $\left(\frac{5\pi}{4}, 1\right)$ stated or clearly shown on the sketch	B1 B1 B1 B2,1	 5	Sine curve with positive gradient at O with at least 3 stationary points between 0 and 2π Correct shaped curve with 2 max and 2 min between 0 and 2π All 5 correct points of intersection with x-axis with $\frac{\pi}{2}$, π and $\frac{3\pi}{2}$ clearly shown B1 for either: 1 as the y-coordinate of max pt(s) or: two max pts between 0 and 2π with correct x-coordinates
	Total		**13**	

8. June 2009

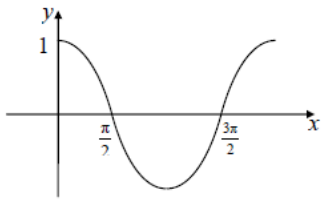
8(a)	$\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} = 4$ $\tan\theta - 1 = 4$ $\tan\theta = 5$	 M1 A1	 2	 $\tan\theta = \frac{\sin\theta}{\cos\theta}$ stated or used AG; CSO

(b)(i)	$2\cos^2 x - \sin x = 1$ $2(1-\sin^2 x) - \sin x = 1$	M1		**Use** of $\cos^2 x + \sin^2 x = 1$
	$2 - 2\sin^2 x - \sin x = 1$ $\Rightarrow 2\sin^2 x + \sin x - 1 = 0$	A1	2	AG; CSO
(ii)	$(\sin x + 1)(2\sin x - 1) = 0$	M1		Factorisation or use of formula; PI by **both** correct values for $\sin x$
	$\sin x = -1, \quad \sin x = 0.5$	A1		Need both
	$(\sin x = -1)$ so $x = 270°$	B1		
	$(\sin x = 0.5)$ so $x = 30°$	A1		30° as the only acute angle
	$x = 180 - 30 = 150°$	B1F	5	ft for 2nd angle from c's sin$x$ = non-integer
				Ignore values outside interval 0°–360° but extras inside interval lose the corresp. B, A or B1F mark. If using rads, accepting either equivalent exact vals (in terms of pi) or 2dp values instead of degrees, penalise max of 1 mark from any of the final three marks (B1A1B1F) awarded NMS: 270° (B1); 30°, 150° (B1) [max 2/5]
	Total		**9**	

9. Jan 2010

8(a)	$x + 52° = (22°)$, $180° + 22°$; $360° + 22°$ $(x = 180 + 22 - 52;\ x = 360 + 22 - 52)$	M1;M1		$x + 52 = 180 + \text{AWRT } 22,\ 360 + \text{AWRT } 22$ OE (max of M1 if extras in range) LHS could be any letter but not x unless final answer shows recovery Ms can be PI
	$x = 150°,\ 330°$	A1	3	Both CAO with no extras in $0° \le x \le 360°$ Ignore anything outside $0° \le x \le 360°$
(b)(i)	$3\tan\theta = \dfrac{8}{\sin\theta} \Rightarrow 3\dfrac{\sin\theta}{\cos\theta} = \dfrac{8}{\sin\theta}$	M1		$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ used/seen
	$\dfrac{3(1-\cos^2\theta)}{\cos\theta} = 8$	M1		$\sin^2\theta = 1 - \cos^2\theta$ **used**
(ii)	$(3\cos\theta - 1)(\cos\theta + 3) = 0$	M1		Any valid method to solve the quadratic
	$\cos\theta = \dfrac{1}{3}$	A1	2	CSO Must only be the one value
(iii)	$\cos 2x = \dfrac{1}{3}$	M1		Using (ii) OE to get or use $\cos 2x = k$ where $-1 \le k \le 1$
	$(2x =)\ 70.528..$	B1		Award for $\cos^{-1}(1/3)$ = value from 70 to 71 inclusive, even if θ used. PI
	$2x = 360° - 70.528..\ (= 289.47\ldots)$	m1		$2x = 360 - \cos^{-1}(\text{c's } k)$ OE No extras inside the range
	$x = 35°,\ 145°$ (to the nearest degree)	A1	4	Both, condoning greater accuracy, with no extras in $0° \le x \le 180°$ Ignore anything outside $0° \le x \le 180°$
				SC for (b)(iii) only when c's answer for (b)(ii) is $\cos\theta = -\dfrac{1}{3}$: max mark M1B1 (val 70-71 or val 109-110 inclusive) m1A0
	Total		**12**	

10.June 2010

7(a)		B1 B1	 2	Correct shape meeting positive y-axis and only one oscillation within interval 0 to 2π The three correct intercepts stated; Accept 1.57 for $\pi/2$ and 4.71 for $3\pi/2$ but must be evidence of radian vals. not just degrees Ignore any parts of the graph clearly indicated as outside the given interval
(b)(i)	$1-\cos^2\theta = \cos\theta(2-\cos\theta)$ $1 = 2\cos\theta \Rightarrow \cos\theta = \frac{1}{2}$	M1 A1	 2	$\cos^2\theta + \sin^2\theta = 1$ used CSO AG Completion
(ii)	$\sin^2 2x = \cos 2x(2-\cos 2x)$ $\Rightarrow \cos 2x = \frac{1}{2}$ $\{2x =\}\quad \cos^{-1}\left(\frac{1}{2}\right) = 1.04(7..)$	 M1 m1		 Uses (b)(i) PI Accept 1.05, $\frac{\pi}{3}$; Condone 60°
	$x = 0.524,\ 2.62$ $x = 0.523(59..),\ 2.61(7..)$	A2,1,0	4	Condone >3sf; Condone $x = 0.525,\ 2.62$ Accept truncated '3sf' vals x =0.523, 2.61 Deduct 1 mark for each extra (>2 solns) in the given interval from A marks to a min of A0. Ignore any solns outside the given interval 0 to π. Accept, as equivalent, the exact answers $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$ when seen and apply ISW if 'errors' converting these to decimals. If not A2 then A1 if • one soln correct. • 30°, 150° ie solns left in degrees • AWRT 0.52, 2.6 ie correct vals to only 2sf. Must see an indication that (b)(i) has been used otherwise 0/4 so just stating the two correct answers with nothing else scores 0/4.
	Total		**8**	

11.Jan 2011

9(a)	$\tan x = -3$ $\Rightarrow x = \tan^{-1}(-3) \qquad (= -71.56...)°$ $x = 108°,\ 288°$	M1 A1,A1	 3	PI eg by 71(.56..) or −71(.56..) seen Condone more accurate answers. (108.4349…, 288.4349…). [Ignore answers outside interval; If more than 2 answers inside interval −1 from A marks for each extra to a min of 0]

(b)(i)	$7\sin^2\theta+\sin\theta\cos\theta=6(\cos^2\theta+\sin^2\theta)$ $7\sin^2\theta-6\sin^2\theta+\sin\theta\cos\theta-6\cos^2\theta=0$ $\Rightarrow \sin^2\theta+\sin\theta\cos\theta-6\cos^2\theta=0$ $\Rightarrow \dfrac{\sin^2\theta}{\cos^2\theta}+\dfrac{\sin\theta}{\cos\theta}-6=0$ $\Rightarrow \tan^2\theta+\tan\theta-6=0$	M1 M1 A1	 3	$\cos^2\theta+\sin^2\theta=1$ used; OE $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$ used CSO AG
(ii)	$(\tan\theta+3)(\tan\theta-2)=0$ $\tan\theta=-3$ or $\tan\theta=2$ $\theta=108°,\ 288°;\quad \theta=63°,\ 243°;$	M1 A1 B2F,1F	 4	Factorise or other valid method to solve quadratic Need both **Only** ft on (a) for the c's two +'ve $\tan^{-1}(-3)$ vals. [B1 if 3 correct (ft)] Condone more accurate answers. (108.4349…, 288.4349…; 63.4349…, 243.4349…) [Ignore answers outside interval; If more than 2 answers for each inside interval, −1 for each extra from Bs to a min of 0]
	Total		**10**	

12.June 2011

8	$\ldots = 9\sin^2 x+6\sin x\cos x+\cos^2 x+$ $\sin^2 x-6\sin x\cos x+9\cos^2 x$	M1		Attempt at expanding both sets of brackets. Minimum requirement either one of the two expansions correct or 4 of these 6 terms seen. Expanding and simplifying the given expression in one step to get the correct two terms scores this M1 and next A1
	$\ldots = 10\cos^2 x+10\sin^2 x$	A1		Either correct pair of expansions and simplification to remove $\sin x\cos x$ terms or full collecting of like terms within the original correct expansion
	$= 10(1-\sin^2 x)+10\sin^2 x$	M1		$\cos^2 x+\sin^2 x=1$ clearly used. If identity is applied correctly, but not directly, it must be stated at the relevant point in the proof.
	$= 10$ (which is an integer)	A1	4	CSO [all previous 3 marks must have been scored] Condone absence of statement after 10 obtained correctly
	Total		**4**	

13.Jan 2012

8(a)	$2\sin\theta=7\cos\theta\Rightarrow\dfrac{\sin\theta}{\cos\theta}=\dfrac{7}{2}$	M1		$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ clearly used to reach either $2\tan\theta=7$ or $2/7\tan\theta=1$ or $\tan\theta=3.5$ or even $\tan\theta=2/7$ after seeing $\dfrac{\sin\theta}{\cos\theta}=\dfrac{2}{7}$
	$\Rightarrow \tan\theta=\dfrac{7}{2}$	A1	2	$\dfrac{7}{2}$ OE eg 3.5

(b)(i)	$6\sin^2 x = 4 + \cos x$ $6(1-\cos^2 x) = 4 + \cos x$ $6 - 6\cos^2 x = 4 + \cos x$ $\Rightarrow\ 6\cos^2 x + \cos x - 2 = 0$	M1 A1	2	$\cos^2 x + \sin^2 x = 1$ used CSO AG Be convinced.
(ii)	$6\sin^2 x = 4 + \cos x \Rightarrow$ $6\cos^2 x + \cos x - 2 = 0$ $(3\cos x + 2)(2\cos x - 1) \quad (=0)$	M1 m1 A1		Uses (b)(i) $(3c \pm 2)(2c \pm 1)$ or by formula Correct factorisation or quadratic formula with $b^2 - 4ac$ evaluated correctly. (PI by both correct values for cos x)
	$\cos x = -\frac{2}{3}$, $\cos x = \frac{1}{2}$ x = 132°, 228°, 60°, 300°	A1 B2,1,0	6	CSO Both values for cos x correct. Accept 3sf rounded or truncated. B1 for any 3 of the 4 values correct. Condone greater accuracy (131.810..; 228.189..). Ignore answers outside the given interval. Deduct 1 mark from these two B marks for each extra solution if more than 4 answers in the given interval to a min of B0 NMS: max possible is B2
	Total		**10**	

14.June 2012

7(a)	$\tan\theta = -1$ $\sin^2\theta = 3\cos^2\theta$ $\frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$ $\tan^2\theta = 3$ $\tan\theta = \pm\sqrt{3}$	B1 M1 A1 A1	4	 $\frac{\sin\theta}{\cos\theta} = \tan\theta$ used on $\sin^2\theta - 3\cos^2\theta$ or forms and solves a correct quadratic in sin or cos and then uses to find $\tan\theta$ $\tan^2\theta = 3$ or $\tan^2\theta - 3 = 0$ or $(\tan\theta + \sqrt{3})(\tan\theta - \sqrt{3}) = 0$ or $\tan\theta = \sqrt{3}$ or $\tan\theta = -\sqrt{3}$ Both
(b)	$\tan\theta = -1$, $\tan\theta = \sqrt{3}$, $\tan\theta = -\sqrt{3}$ (θ =) 135°, (θ =) 60°, (θ =) 120°	M1 A2,1,0	3	Uses part (a), at least as far as attempting to solve $\tan\theta = k$, where k is any one of c's values for $\tan\theta$ If not A2 for all three correct, award A1 for two values correct **Special Case** If $\tan^2\theta = \frac{1}{3}$ in part (a) and M1 scored in (a) and in (b) then apply ft in part (b) ie A2F for θ = 135°, 30°, 150°. (A1F if two of these ft values) **Special Case**: If M0 then award B1 for any two correct values provided no incorrect extras in given interval. If > 3 answers in the given interval, deduct 1 mark for each extra in the given interval from any A marks awarded in (b). Ignore any answers outside $0 \le \theta \le 180$
	Total		**7**	

15.Jan 2013

9(a)	$x + 30° = 79°$, $x + 30° = 180° + 79°$			
	$x = 49°$	B1		49 as the only solution in the interval $0° \le x < 90°$
	$x = 229°$	B1	2	AWRT 229. Not given if any other soln. in the interval $90° \le x \le 360°$. Ignore anything outside $0° \le x \le 360°$
(b)	Translation;	B1		Accept 'translat…' as equivalent. [T or Tr is NOT sufficient]
	$\begin{bmatrix} -30° \\ 0 \end{bmatrix}$	B1	2	OE Accept **full** equivalent to vector in words provided linked to 'translation/ move/shift' and **correct** direction. (0/2 if >1 transformation).
(c)(i)	$5 + \sin^2\theta = (5 + 3\cos\theta)\cos\theta$			
	$\Rightarrow 5 + \sin^2\theta = 5\cos\theta + 3\cos^2\theta$	B1		Correct RHS.
	$5 + 1 - \cos^2\theta = 5\cos\theta + 3\cos^2\theta$	M1		$\sin^2\theta = 1 - \cos^2\theta$ used to get a quadratic in cos .
	$6 = 5\cos\theta + 4\cos^2\theta$ or $4\cos^2\theta + 5\cos\theta - 6\ (= 0)$	A1		ACF with like terms collected.
	$\Rightarrow \quad (4\cos\theta - 3)(\cos\theta + 2) \quad (= 0)$	m1		**Correct** quadratic and $(4c \pm 3)(c \pm 2)$ or by formula OE PI by 'correct' 2 values for $\cos\theta$.
	Since $\cos\theta \ne -2$, $\cos\theta = \frac{3}{4}$	A1	5	CSO AG. Must show that the 'soln' $\cos\theta = -2$ has been considered and rejected
(ii)	$5 + \sin^2 2x = (5 + 3\cos 2x)\cos 2x$			
	$\Rightarrow \cos 2x = \frac{3}{4}$	M1		Using (c)(i) to reach $\cos 2x = ¾$ **or** finding at least 3 solutions of $\cos\theta = 3/4$ **and** dividing them by 2.
	$2x = 0.722(7\ldots)$, $2\pi - 0.722(7\ldots)$, $2\pi + 0.722(7\ldots)$, $4\pi - 0.722(7\ldots)$,	m1		Valid method to find all four 'positions' of solutions.
	$x = 0.361$, 2.78, 3.50, 5.92	A1	3	CAO Must be these four 3sf values but ignore any values outside the interval $0 < x < 2\pi$.
	Total		**12**	