Quadratic Powers Problem

Find all real solutions to the equation:

\[(x^2 - 7x + 11)^{x^2-11x+30} = 1\]
Quadratic Powers Solution

Note that the right-hand-side is 1, and that there are a limited number of ways this can be accomplished:

1. **The power is zero.**
   Since $a^0 = 1$ for any $a$, solutions can be found by solving $x^2 - 11x + 30 = 0$:
   \[
   x^2 - 11x + 30 = 0 \implies (x - 5)(x - 6) = 0 \implies x = 5 \text{ or } x = 6
   \]

2. **The base is one.**
   Since $1^a = 1$ for any $a$, solutions can be found by solving $x^2 - 7x + 11 = 1$:
   \[
   \implies x^2 - 7x + 10 = 0 \implies (x - 2)(x - 5) = 0 \implies x = 2 \text{ or } x = 5
   \]

3. **The base is negative one and the power is even.**
   Since $(-1)^a = 1$ for even values of $a$, solutions to $x^2 - 7x + 11 = -1$ will be solutions provided they also satisfy $x^2 - 11x + 30 = 2n, n \in \mathbb{N}$ (that is, $x^2 - 11x + 30$ is even):
   \[
   \implies x^2 - 7x + 12 = 0 \implies (x - 3)(x - 4) = 0 \implies x = 3 \text{ or } x = 4
   \]
   Checking: $(3)^2 - 11(3) + 30 = 6 = 2(3)$ and $(4)^2 - 11(4) + 30 = 2 = 2(1)$

Both give even values, therefore the full list of solutions is:

\[
\begin{align*}
  x &= 2 & \iff & & 1^{12} = 1 \\
  x &= 3 & \iff & & (-1)^6 = 1 \\
  x &= 4 & \iff & & (-1)^2 = 1 \\
  x &= 5 & \iff & & 1^0 = 1 \\
  x &= 6 & \iff & & 4^0 = 1
\end{align*}
\]