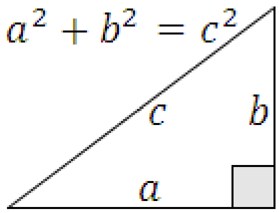
Finding Pythagorean Triples

Pythagoras' Theorem:



Pythagorean triple:

A set of three integers which satisfy the theorem.

A triangle with these side lengths would be right-angled.

Eg: 3, 4, 5

$$\boldsymbol{a^2 + b^2 = c^2} \qquad \qquad \mathbf{3^2 + 4^2 = 5^2}$$

To generate a Pythagorean triple:

Using any whole number more than 1 for m, substitute into these formulae:

$$\boldsymbol{a = m^2 - 1 \quad b = 2m \quad c = m^2 + 1}$$

Where m is an integer (a whole number) greater than 1

Eg: $m = 4 \quad \Longrightarrow \quad a = 4^2 - 1 = \mathbf{15} \quad b = 2(4) = \mathbf{8} \quad c = 4^2 + 1 = \mathbf{17}$

Pythagorean triple: $\mathbf{8, 15, 17}$. *Check:* $8^2 + 15^2 = 64 + 225 = 289 = 17^2$

1. Use the formulae above to generate some of your own triples.

A **primitive** triple is one where the three numbers have no common factors (called 'coprime'). Eg, $60, 80, 100$ is not primitive – it is $20 \times$ the $3, 4, 5$ triple.

2. What values of m produce primitive *triples?*

The formulae above will generate an infinite number of primitive triples, but not all possible triples. To do this it is necessary to extend it a little:

$$\boldsymbol{a = m^2 - n^2 \quad b = 2mn \quad c = m^2 + n^2}$$

where m and n are both positive integers

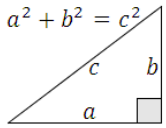
Eg: $m = 5 \quad n = 2$:

$$a = 5^2 - 2^2 = \mathbf{21} \quad b = 2(5)(2) = \mathbf{20} \quad c = 5^2 + 2^2 = \mathbf{29}$$

3. Investigate different values for m and n. Can you identify the conditions for generating primitive *Pythagorean triples?*

Finding Pythagorean Triples - SOLUTIONS

Pythagoras' Theorem:



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A set of three integers which satisfy the theorem.

A triangle with these side lengths would be right-angled.

Eg: 3, 4, 5

$\boldsymbol{a^2 + b^2 = c^2}$ $\qquad$ $\mathbf{3^2 + 4^2 = 5^2}$

$$\boldsymbol{a = m^2 - 1 \quad b = 2m \quad c = m^2 + 1}$$

Where $\boldsymbol{m}$ is an integer (a whole number) greater than 1

1. Use the formulae above to generate some of your own triples.

$m =$	2	3	4	5	6	7
$a, b, c =$	$3, 4, 5$	$8, 6, 10$	$15, 8, 17$	$24, 10, 26$	$35, 12, 37$	$48, 14, 50$

2. What values of m generate primitive *triples?*

Even values of m generate primitive triples. $m^2 + 1$ and $m^2 - 1$ would be odd, and $2m$ would be even. Since $m^2 - 1$ and $m^2 + 1$ are exactly 2 apart, the only way they can share factors is if the factor is 2, but if they are both odd, all three numbers must be coprime.

$$\boldsymbol{a = m^2 - n^2 \quad b = 2mn \quad c = m^2 + n^2}$$

where $\boldsymbol{m}$ and $\boldsymbol{n}$ are both positive integers

3. Investigate different values for m and n. Can you identify the conditions for generating primitive *Pythagorean triples?*

n ↓ **m** →	**2**	**3**	**4**	**5**	**6**	**7**
1	**3,4,5**	*8,6,10*	**15,8,17**	*24,10,26*	**35,12,37**	*48,14,50*
2		**5,12,13**	*12,16,20*	**21,20,29**	32,24,40	**45,28,53**
3			**7,24,25**	*16,30,34*	27,36,45	*40,42,58*
4				**9,40,41**	*20,48,52*	**33,56,65**
5					**11,60,61**	*24,70,74*
6						**13,84,85**
7						

Primitives occur when both: **exactly one of $\boldsymbol{m}$ and $\boldsymbol{n}$ are even** *and* **when they are coprime**