Finding Pythagorean Triples

Pythagoras’ Theorem:
\[ a^2 + b^2 = c^2 \]

A set of three integers which satisfy the theorem. A triangle with these side lengths would be right-angled. Eg: 3, 4, 5
\[ a^2 + b^2 = c^2 \]
\[ 3^2 + 4^2 = 5^2 \]

To generate a Pythagorean triple:
Using any whole number more than 1 for \( m \), substitute into these formulae:

\[ a = m^2 - 1 \quad b = 2m \quad c = m^2 + 1 \]
Where \( m \) is an integer (a whole number) greater than 1

Eg: \( m = 4 \) \[ \Rightarrow \]
\[ a = 4^2 - 1 = 15 \quad b = 2(4) = 8 \quad c = 4^2 + 1 = 17 \]
Pythagorean triple: 8, 15, 17. \[ \text{Check: } 8^2 + 15^2 = 64 + 225 = 289 = 17^2 \]

1. Use the formulae above to generate some of your own triples.

A primitive triple is one where the three numbers have no common factors (called ‘coprime’). Eg, 60, 80, 100 is not primitive – it is \( 20 \times \) the 3, 4, 5 triple.

2. What values of \( m \) produce primitive triples?

The formulae above will generate an infinite number of primitive triples, but not all possible triples. To do this it is necessary to extend it a little:

\[ a = m^2 - n^2 \quad b = 2mn \quad c = m^2 + n^2 \]
where \( m \) and \( n \) are both positive integers

Eg: \( m = 5 \quad n = 2: \)
\[ a = 5^2 - 2^2 = 21 \quad b = 2(5)(2) = 20 \quad c = 5^2 + 2^2 = 29 \]

3. Investigate different values for \( m \) and \( n \). Can you identify the conditions for generating primitive Pythagorean triples?
Finding Pythagorean Triples - SOLUTIONS

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Pythagorean triple:
A set of three integers which satisfy the theorem.
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Eg: 3, 4, 5

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\[ a = m^2 - 1 \quad b = 2m \quad c = m^2 + 1 \]
Where \( m \) is an integer (a whole number) greater than 1

1. Use the formulae above to generate some of your own triples.

<table>
<thead>
<tr>
<th>( m )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a, b, c )</td>
<td>3, 4, 5</td>
<td>8, 6, 10</td>
<td>15, 8, 17</td>
<td>24, 10, 26</td>
<td>35, 12, 37</td>
<td>48, 14, 50</td>
</tr>
</tbody>
</table>

2. What values of \( m \) generate primitive triples?

Even values of \( m \) generate primitive triples. \( m^2 + 1 \) and \( m^2 - 1 \) would be odd, and \( 2m \) would be even. Since \( m^2 - 1 \) and \( m^2 + 1 \) are exactly 2 apart, the only way they can share factors is if the factor is 2, but if they are both odd, all three numbers must be coprime.

\[ a = m^2 - n^2 \quad b = 2mn \quad c = m^2 + n^2 \]
where \( m \) and \( n \) are both positive integers

3. Investigate different values for \( m \) and \( n \). Can you identify the conditions for generating primitive Pythagorean triples?

<table>
<thead>
<tr>
<th>( n )</th>
<th>( m )</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
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<td>15, 8, 17</td>
<td>24, 10, 26</td>
<td>35, 12, 37</td>
<td>48, 14, 50</td>
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<td>45, 28, 53</td>
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<td>6</td>
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</table>

Primitives occur when both: exactly one of \( m \) and \( n \) are even and when they are coprime