Oil Drum Optimisation

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For a radius of x and a height of h: $V = 2x^2h = 0.2 \quad \Rightarrow \quad h = 0.1x^{-2}$

The curved surface has area: $2\pi xh = 2\pi x(0.1x^{-2}) = 0.2\pi x^{-1}$

The circular ends have area: $2\pi x^2$

The total cost is: $C = 14(0.2\pi x^{-1}) + 20(2\pi x^2) = 2.8\pi x^{-1} + 40\pi x^2$

Minimum cost at: $\frac{dC}{dx} = 0 \quad \Rightarrow \quad -2.8\pi x^{-2} + 80\pi x = 0 \quad \Rightarrow \quad -2.8 + 80x^3 = 0$

$$\Rightarrow \quad x^3 = \frac{2.8}{80} \quad \Rightarrow \quad \boldsymbol{x = 0.327m \text{ to } 3 \text{ s.f.}}$$

Cost for this radius: $C = \frac{2.8\pi}{0.327} + 40\pi 0.327^2 = \boldsymbol{£40.34}$