Manufacturers of oil drums would like to minimise the cost of production by determining the optimal dimensions for a cylindrical drum.

The curved surface is made of 16-gauge steel which costs £14 per square metre.

The circular ends are made slightly differently, and costs £20 per square metre.

Given that the drum must have a volume of 200 litres (that is, 0.2$m^3$), find the optimal radius for the drum and the overall cost of materials.

For a radius of $x$ and a height of $h$: $V = 2x^2h = 0.2 \Rightarrow h = 0.1x^{-2}$

The curved surface has area: $2\pi x h = 2\pi x (0.1x^{-2}) = 0.2\pi x^{-1}$

The circular ends have area: $2\pi x^2$

The total cost is: $C = 14(0.2\pi x^{-1}) + 20(2\pi x^2) = 2.8\pi x^{-1} + 40\pi x^2$

Minimum cost at: $\frac{dC}{dx} = 0 \Rightarrow -2.8\pi x^{-2} + 80\pi x = 0 \Rightarrow -2.8 + 80x^3 = 0$

$\Rightarrow x^3 = \frac{2.8}{80} \Rightarrow x = 0.327m$ to 3 s.f.

Cost for this radius: $C = \frac{2.8\pi}{0.327} + 40\pi 0.327^2 = £40.34$