AQA

ASSESSMENT and QUALIFICATIONS ALLIANCE

Not Formulae and Statistical Tables for GCE Mathematics and GCE Statistics

The Not-Formula Book

Mechanics 1

Everything you need to remember that the formula book won't tell you

The Not-Formula Book for M1

Everything you need to know for Mechanics 1 that *won't* be in the formula book

Examination Board: AQA

Brief

This document is intended as an aid for revision. Although it includes some examples and explanation, it is primarily not for learning content, but for becoming familiar with the requirements of the course as regards formulae and results. It cannot replace the use of a text book, and nothing produces competence and familiarity with mathematical techniques like practice. This document was produced as an addition to classroom teaching and textbook questions, to provide a summary of key points and, in particular, any formulae or results you are expected to know and use in this module.

Contents

Chapter 1 – Mathematical modelling in mechanics

This chapter is primarily concerned with introducing the topic and defining key terms and concepts. This means the content tends to be examined only insofar as it is necessary for the proper interpretation of problems throughout the course. Sometimes questions will also be asked which require a wider-perspective analysis of a problem (eg, if air resistance were taken into account, the final velocity would be lower).

The **mathematical modelling cycle**:

- Formulate the problem
- Obtain a mathematical solution
- Validate and interpret the solution
- Prepare a report
- If necessary, refine the formulation of the problem and repeat

A **particle** has no size but does have mass.

A **rigid body** has size but does not change shape (deform) when forces are applied to it.

In order to solve real-life problems, a number of factors may be neglected to simplify the mathematical model. How significantly their inclusion would affect the solution should be taken into account, and if necessary a revised model should be formulated.

Although in reality an object cannot slide over another object with absolutely no friction, if the frictional force is very small, assuming there is no friction can simplify the problem without changing the validity of the solution very much.

Smooth means there is no friction

Rough means friction is present

Light means has no mass

Inelastic means does not stretch

Elastic means does stretch

Note: The more refined a problem becomes, the more factors must be taken into account in order to reduce the margin of error between the theoretical solution and the observable result.

Chapter 2 – Kinematics in one dimension

The **gradient** of a **displacement-time** graph gives the **velocity** at any given point.

Where the line is horizontal, the object is not moving, and where the gradient is negative the object is moving backwards.

The **gradient** of a **velocity-time graph** gives the **acceleration** at any given point.

Where the line is horizontal, there is no acceleration which implies a constant speed (which may be zero but doesn't have to be). A positive gradient implies the speed is increasing and a negative acceleration implies it is decreasing (but may still be positive, eg when braking in a car).

The **area** under a **velocity-time graph** gives the **displacement**.

For a graph made of straight line segments this is straightforward to calculate using the area of a triangle and area of a rectangle formulae.
Note: taking area below the axis as negative will give a total equal to the displacement, but taking the area above and below as positive will calculate total distance travelled in any direction.

The Kinematics Equations (often referred to as the SUVAT equations) apply **only** to situations where the **acceleration is constant**.

$$v = u + at \qquad s = \frac{u+v}{2}t$$

$$v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$$

Where: $s = Displacement\ (m)$ $\quad u = Initial\ Velocity\ (ms^{-1})$

$v = Final\ Velocity\ (ms^{-1})$ $\quad a = Acceleration\ (ms^{-2})$ $\quad t = Time\ (s)$

Usually you will know three of the variables and therefore, by choosing the correct equation, can deduce a fourth (and, if necessary, the fifth).

Note: Ensure you convert units if they are incompatible. For instance, it's fine to use miles per hour provided displacement is given in miles, time in hours and acceleration in miles per hour per hour.

Acceleration due to gravity is called g and is taken to be $g = 9.8ms^{-2}$.

All objects experience the same acceleration when in freefall (neglecting air resistance).
Note: The 'standard' value of g is given as $9.80665ms^{-2}$, but since the earth is not completely spherical this varies around the planet, ranging from 9.78 to 9.82. In Physics a value of $9.81ms^{-2}$ may be used, but since 9.8 is sufficient for most practical purposes, this is the value required in M1.

Chapter 3 – Kinematics in two dimensions

An **object's position** can be described in **vector notation**: $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} = \begin{bmatrix} x \\ y \end{bmatrix}$ where $\boldsymbol{i}$ and $\boldsymbol{j}$ represent unit vectors, usually in the horizontal and vertical direction respectively (although sometimes in the East and North directions).

Eg:
A ship's position is given as $12\boldsymbol{i} - 4\boldsymbol{j}$ where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors of length $1\ mile$ East and North respectively.

This means the ship is 12 **miles east and 4 miles south of the origin**.

A particle's **changing position** can be described by a **vector** given in terms of the time, t.

The position at any given time can be found by substituting in values of t into the expressions.

Eg:
Find the position described by this vector at times 0, 1 and 2 seconds: $\boldsymbol{r} = (3t + 2)\boldsymbol{i} + (2t^2)\boldsymbol{j}$

$t = 0 \quad \Rightarrow \quad \boldsymbol{r} = 2\boldsymbol{i}$
$t = 1 \quad \Rightarrow \quad \boldsymbol{r} = 5\boldsymbol{i} + 2\boldsymbol{j}$
$t = 2 \quad \Rightarrow \quad \boldsymbol{r} = 8\boldsymbol{i} + 8\boldsymbol{j}$

The position of an object given as a **distance** away from the origin at a particular **angle** can be converted into **vector form** by **resolving** horizontally and vertically.

A point at a distance d from the origin at an angle θ above the positive $\boldsymbol{i}$ direction becomes:

$$\boldsymbol{r} = d\cos\theta\,\boldsymbol{i} + d\sin\theta\,\boldsymbol{j} = \begin{bmatrix} d\cos\theta \\ d\sin\theta \end{bmatrix}$$

Eg:

$$10\ metres\ away\ at\ an\ angle\ of\ 30°:$$

$$\boldsymbol{r} = \begin{bmatrix} 10\cos 30 \\ 10\sin 30 \end{bmatrix} = \begin{bmatrix} \boldsymbol{8.66\ldots} \\ \boldsymbol{5} \end{bmatrix}$$

To **add vectors**, add the $\boldsymbol{i}$ components and add the $\boldsymbol{j}$ components. Subtraction is similar.

$$\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a + c \\ b + d \end{bmatrix} \qquad \begin{bmatrix} a \\ b \end{bmatrix} - \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a - c \\ b - d \end{bmatrix}$$

To **multiply a vector by a scalar**, multiply each component by the scalar.

$$k\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} ka \\ kb \end{bmatrix}$$

Crucially, because motion can be considered separately in perpendicular directions, the **constant acceleration equations** (SUVAT) can be applied equally well to **2 or more dimensions**.

$$s = ut + \frac{1}{2}at^2 \quad \textit{becomes} \quad \boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2 + \boldsymbol{r_0}$$

$$s = \frac{u+v}{2}t \quad \textit{becomes} \quad \boldsymbol{r} = \frac{1}{2}(\boldsymbol{u}+\boldsymbol{v})t + \boldsymbol{r_0}$$

Note: $\boldsymbol{r_0}$ is simply the initial displacement. Since the standard SUVAT equations effectively calculate a *change* in displacement, adding this terms allows us to calculate the actual displacement given the initial displacement.

Note: $v = u + at$ can be treated in the same way, but since a vector cannot be squared in the conventional sense, $v^2 = u^2 + 2as$ cannot be applied directly (you will need to work around using a combination of the other equations).

Eg:
A model boat initially at position $\begin{bmatrix} 2 \\ -5 \end{bmatrix} m$ travelling at a velocity of $\begin{bmatrix} 1 \\ 2 \end{bmatrix} ms^{-1}$ moves with constant acceleration $\begin{bmatrix} -0.5 \\ 1.5 \end{bmatrix} ms^{-2}$. Find an expression for the displacement after t seconds, and hence find the distance from the origin after 3 seconds.

$$\boldsymbol{r} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} t + \frac{1}{2}\begin{bmatrix} -0.5 \\ 1.5 \end{bmatrix} t^2 + \boldsymbol{r_0} = \begin{bmatrix} t - 0.25t^2 \\ 2t + 0.75t^2 \end{bmatrix} + \begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} t - 0.25t^2 + 2 \\ 2t + 0.75t^2 - 5 \end{bmatrix}$$

$$t = 3 \quad \Rightarrow \quad \boldsymbol{r} = \begin{bmatrix} 3 - 0.25(3^2) + 2 \\ 2(3) + 0.75(3^2) - 5 \end{bmatrix} = \begin{bmatrix} 2.75 \\ 7.75 \end{bmatrix}$$

$$\textit{Using Pythagoras, the } \boldsymbol{distance} \textit{ from the origin is:} \quad \sqrt{2.75^2 + 7.75^2} = \boldsymbol{8.22m\ to\ 2\ d.p.}$$

Note: A familiarity with bearings is important for certain questions. Recall that a bearing 'from A' means the angle is measured at the point A, that it is measured clockwise from North and written with 3 figures before the decimal point.

Note: Sometimes questions will require you to interpret phrases such as 'travelling due north' (the easterly component, $\boldsymbol{i}$, will be 0 and the northerly component, $\boldsymbol{j}$, will be positive). You should also be familiar with the Sine Rule and Cosine Rule in order to interpret vector triangle questions.

Chapter 4 – Forces

Force is a **vector** – it has both **magnitude** and **direction**. The units of force are **Newtons (N)**.

Common types of force (you should be familiar with all of these):

- **Weight:** $\boldsymbol{mg}$
  $m = mass\ (kg)\quad g = acceleration\ due\ to\ gravity\ (ms^{-2}) \text{: } 9.8\ on\ Earth, 1.6\ on\ the\ Moon$
  Note: This is *not* the same as mass (measured in kg), although the two are directly proportional. On the moon your weight (the force pulling you down) is six times smaller than on Earth, but your mass is unchanged (eg, your inertia is the same).
- **Normal Reaction/Contact Force:** $\boldsymbol{R}$ $(\boldsymbol{sometimes\ N})$
  $This\ is\ the\ force\ exerted\ by\ any\ surface\ a\ body\ rests\ on\ which\ balances\ any\ forces\ acting\ into\ the\ surface.\ It\ acts\ at\ right\ angles\ to\ the\ surface.$
  Note: A common mistake is to forget the force acts perpendicular to the surface, not always vertically upwards. Another mistake is to forget to include it altogether. Also note that the normal reaction often doesn't exactly balance the weight. If another force is also acting against the weight pulling down, for instance, the normal reaction will simply 'take up the slack', being just large enough to balance the forces into and out of the surface.
- **Tension** $\boldsymbol{T}$
  $A\ force\ exerted\ on\ an\ object\ via\ a\ string, rope, towbar\ or\ similar\ acts\ as\ tension.$
  Note: The tension is the same in both directions – the towbar exerts the same force on the car as it does on the trailer, just in opposite directions. This is true even when a string passes over a pulley – the tension may act in completely different directions at each end of the string, but the tension at each end will be the same.
- **Friction** $\boldsymbol{F_r}$ $(\boldsymbol{sometimes\ just\ F})$
  $The\ frictional\ force\ will\ always\ act\ against\ motion, sometimes\ stopping\ motion\ taking\ place\ altogether, sometimes\ (when\ its\ maximum\ value\ is\ not\ enough) simply\ reducing\ the\ overall\ resultant\ forwards\ force\ to\ reduce\ overall\ acceleration.$
  Note: Ensure friction is pointing the right way – it will always work against motion. The value of the frictional force is directly proportional to the normal reaction (hence why designing F1 cars to be pressed to the ground improves grip). The constant of proportionality depends on the surface, and is known as the *coefficient of friction*: μ.

A force with a certain **magnitude** in a given **direction** can be **resolved** into two perpendicular components in exactly the same way as a position or velocity vector:

A force of magnitude F acting at an angle θ to the horizontal can be written as: $\begin{bmatrix} F\cos\theta \\ F\sin\theta \end{bmatrix}$

A force of 20 Newtons acts at an angle of 30 degrees to the horizontal. What are the horizontal and vertical components of this force?

$$Horizontal\ component\text{:}\quad 20\cos 30 = \boldsymbol{17.3N}\qquad Vertical\ component\text{:}\quad 20\sin 30 = \boldsymbol{10N}$$

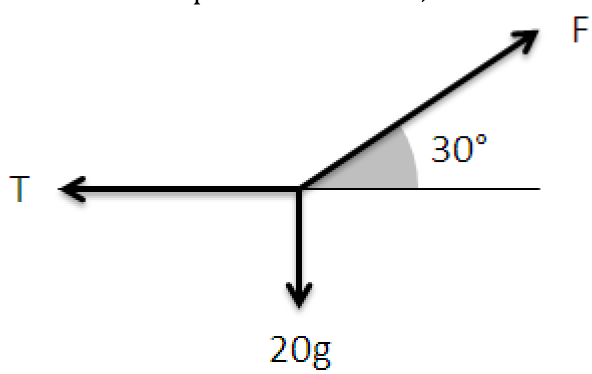
Forces can be combined in two ways: **Vector Addition** where each force is represented by a vector, these vectors are placed sequentially and the line between the start and the end represents the overall resultant, or **Resolving Perpendicularly** where forces are resolved into two perpendicular components – commonly vertical and horizontal, but may be parallel with and perpendicular to a slope – and the resultants in each direction are added together.

If the resultant of all forces acting on a particle is zero, the forces are said to be in **equilibrium**.

Note: This means if an object is at rest (or travelling at a constant speed), it will remain at rest (or at the same speed). An object may be in equilibrium vertically but not horizontally, eg a car accelerating along a horizontal road.

When forces are known to be in **equilibrium**, **unknown forces or angles** may be found by setting upwards forces **equal** to downwards forces, or rightwards and leftwards forces **equal**.

Eg:
Given that the particle is at rest, find F and T.



$$\textit{Resolving vertically}:$$

$$F \sin 30 = 20g \quad \Longrightarrow \quad 0.5F = 196$$

$$\boldsymbol{F = 392N}$$

$$\textit{Resolving horizontally}:$$

$$F \cos 30 = T \quad \Longrightarrow \quad T = 392 \cos 30$$

$$\boldsymbol{T = 339.5N \ to \ 1 \ d.p.}$$

The **frictional force** is proportional to the **normal reaction**. This inequality is always true:

$$F_r \leq \mu R$$

Where a particle is in **limiting equilibrium** or in **motion**, the following – stricter – formula applies:

$$F_r = \mu R$$

Eg:
A horizontal force of magnitude 20N acts on a particle of mass 5kg resting on a horizontal surface. Find the minimum possible value of the coefficient of friction for the particle to remain at rest.

$$\textit{Resolving vertically}: \quad R = 5g = 49 \qquad \textit{Resolving horizontally}: \quad F_r = 20$$

$$F_r \leq \mu R \quad \Longrightarrow \quad 20 \leq 49\mu \quad \Longrightarrow \quad \mu \geq \frac{20}{49} \quad \textit{Therefore } \boldsymbol{\mu_{min}} = \boldsymbol{\frac{20}{49}}$$

Note: While μ is often less than 1, it can take any non-negative value. In fact, the value of μ necessary to keep a particle at rest on a slope inclined at an angle of θ to the horizontal is $\tan\theta$.

Chapter 5 – Newton's laws of motion

Newton's First Law states that a body will remain at rest or continue to travel in a straight line at a constant velocity unless it is compelled to change by the action of a **resultant force**.

Note: An object may be acted on by considerable forces, but unless there is some imbalance between them (an overall resultant force) there will be no resulting acceleration. The implications of this law are simply that resolving forces can be used to solve problems where the particle is in motion provided it is travelling at a constant speed in a given direction.

Newton's Second Law states that, for a **resultant force** F, mass m and acceleration a:

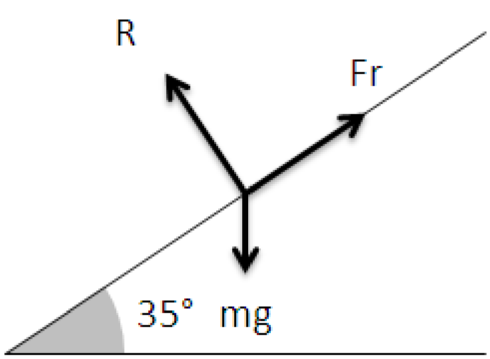
$$F = ma$$

Note: It is important to remember that this is the resultant force acting on the particle, which may be a combination of a number of different forces. Also note that m represents $mass$, not $weight$ (which is itself a force – the force of gravity at $F = mg$ where g is acceleration due to gravity). The implications of this law are that resolving forces to find a resultant force can inform us about the motion of the particle even if it is not in equilibrium. Frequently SUVAT equations are incorporated into these questions.

Eg:
A particle is released from rest on a rough slope inclined at an angle of 35° to the horizontal. If it takes 4 seconds to slide 10 metres, calculate the coefficient of friction.

$$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad 10 = \frac{1}{2}a \times 4^2 \quad \Rightarrow \quad a = 1.25ms^{-2}$$

$$Resultant\ downslope\ force: \quad F = ma \quad \Rightarrow \quad F = 1.25m$$



$$Resolving\ down\ the\ slope:$$
$$1.25m = mg\sin 35 - F_r \quad \Rightarrow \quad F_r = mg\sin 35 - 1.25m$$

$$Resolving\ perpendicular\ to\ the\ slope:$$
$$R = mg\cos 35$$

$$F_r = \mu R \quad \Rightarrow \quad m(g\sin 35 - 1.25) = \mu mg\cos 35$$

$$\Rightarrow \quad \mu = \frac{g\sin 35 - 1.25}{g\cos 35} = \mathbf{0.544\ to\ 3\ s.f.}$$

Newton's Third Law states that for every **action** there is an **equal** but **opposite reaction**.

Note: This is the basis for the Normal Reaction (contact) force. When you push a wall, the wall pushes back. Provided there is no resultant movement, the forces must be in equilibrium.

Chapter 6 – Connected particles

When two particles are connected by a light string, the **tension** in the string will have the **same magnitude** throughout (although it may well act in different directions).

Note: This is why when you pull on a rope over a pulley, the tension pulling against you is equal to the tension pulling against the weight on the other end of the rope.

While the string connecting two particles remains taut, the particles will experience the **same acceleration**.

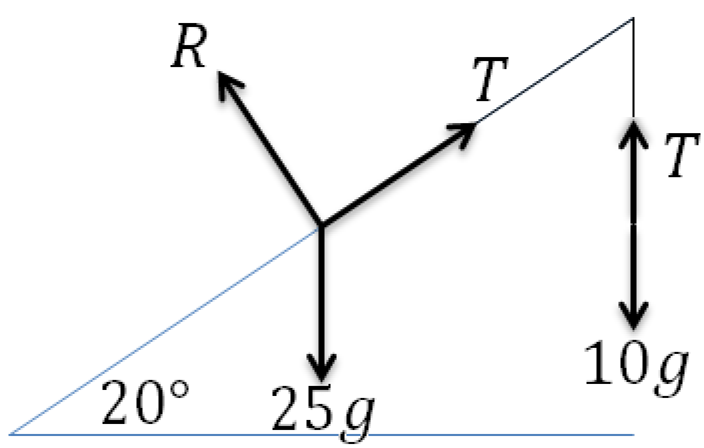
Note: We assume for the purposes of these problems that the string is light (has negligible mass) and is inextensible (does not stretch or deform at all). Elasticity in strings is dealt with in more detail in a subsequent module.

Connected particle problems are best dealt with by drawing a diagram and considering **each particle** and forces acting on it **separately**.

Note: The fact that the acceleration and the tension is the same for each particle means we can often construct a pair of simultaneous equations by examining the forces on each particle, and solve to find both a and T.

For connected particles on an inclined plane, forces **perpendicular** to the direction of motion are in **equilibrium**, and forces in the **direction of motion** can be dealt with using $\boldsymbol{F = ma}$.

Eg:
A 25kg weight is initially at rest on a slope inclined at 20° to the horizontal, attached to a light inextensible rope which passes over a pulley and supports a 10kg weight hanging freely from the other end. The system is released from rest. Calculate the tension in the string and the resulting acceleration of the 25kg weight.



$$Resolving\ vertically\ (10kg):$$
$$10g - T = 10a$$

$$Resolving\ parallel\ to\ slope\ (25kg):$$
$$T - 25g\sin 20 = 25a$$

$$Solving\ simultaneously:$$
$$10g - 25g\sin 20 = 35a$$
$$\Rightarrow \quad \boldsymbol{a = 0.406ms^{-2}\ to\ 3\ s.f.}$$

$$T = 10g - 10a = \boldsymbol{94.0N\ to\ 3\ s.f.}$$

Note: If friction were taken into account it would be necessary to resolve perpendicular to the slope to find R, use $F_r = \mu R$ to find F_r and then resolve parallel to the slope to find a and T.

Chapter 7 – Projectiles

The techniques used in chapter 3 (2-D kinematics) can be applied to projectile motion. A particle set in motion with a certain initial velocity and has no force other than gravity acting on it (eg no air resistance or lift) will follow a parabolic trajectory.

As a particle moves under the influence of **gravity**, it will have a **constant acceleration** of magnitude $g\ ms^{-1}$. Using horizontal and vertical unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ this can be written as;

$$a = -g\boldsymbol{j} = \begin{bmatrix} 0 \\ -g \end{bmatrix}$$

By **resolving** horizontally and vertically, an initial velocity of $V\ ms^{-1}$ at an angle of θ to the horizontal can be written in **vector form** as:

$$\boldsymbol{u} = V\cos\theta\,\boldsymbol{i} + V\sin\theta\,\boldsymbol{j} = \begin{bmatrix} V\cos\theta \\ V\sin\theta \end{bmatrix}$$

The familiar SUVAT equations, with **variables** given as **vectors**, allow us to find the **velocity** and **position** (both as vectors) at time t ($\boldsymbol{r}$ gives position and $\boldsymbol{r_0}$ gives initial displacement):

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t \quad and \quad \boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2 + \boldsymbol{r_0}$$

Note: It is often helpful to solve projectile problems by treating horizontal and vertical components separately. Horizontally the acceleration is $0\ ms^{-2}$, and the speed will therefore always be the same as the horizontal component of the initial velocity. Vertically the acceleration is – $g\ ms^{-2}$, and the SUVAT equations can be used to calculate vertical displacement at a given time.

Time is usually the key variable in projectile problems. Often the given information can be applied to either horizontal or vertical motion to find a value for t, then this value of t can be used in the other direction to solve the problem.

Eg:
A bullet is fired at a speed of $100ms^{-1}$ at an angle of 5° above the horizontal, from $1.5m$ above the ground. It hits a target $1m$ above the ground. How far away is the target?

$$Vertically: \quad s = -0.5 \quad u = 100\sin 5 \quad v = \cdots \quad a = -9.8 \quad t = ?$$
$$-0.5 = 100\sin 5\,t - 4.9t^2 \quad \Rightarrow \quad t = -0.056\ldots\ or\ 1.834\ldots \quad \Rightarrow \quad t = 1.834\ldots$$

$$Horizontally: \quad u = v = 100\cos 5 \quad t = 1.834\ldots \quad s = ?$$
$$100\cos 5 = \frac{s}{1.834} \quad \Rightarrow \quad \boldsymbol{s = 183m\ to\ 3\ s.f.}$$

Note: Maximum range is a result of a trade-off between horizontal velocity (how quickly it covers the distance) and vertical velocity (how much time it has before it hits the ground). Therefore – provided initial and final height are equal – it is greatest when the angle to the horizontal is 45°.

Chapter 8 – Momentum

Momentum allows us to predict the outcome of collisions, since a difference in the masses of two objects will produce a proportional difference in their velocities.

The **momentum** of an object is defined as $\boldsymbol{mv}$ where m is the **mass** and v is the **velocity**.

The law of **conservation of momentum**, which applies to any collision where no external forces are acting, states that:

$$Initial\ Momentum = Final\ Momentum$$

Or, for two colliding particles A and B: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

Note: This is the basis for all collision questions. The total initial momentum of the system is calculated, and then the total final momentum, and the two are put equal to one another.

Eg:
A fully loaded AK-47 gun weighs 4.78kg, and fires bullets weighing 8g at a velocity of $715ms^{-1}$. Calculate the velocity of the recoil.

$$Initial\ Momentum = 4.78 \times 0 + 0.008 \times 0 = 0$$

$$Final\ Momentum = 4.78v + 0.008 \times 715 = 4.78v + 5.72$$

$$M_I = M_F \quad \Longrightarrow \quad 0 = 4.78v + 5.72 \quad \Longrightarrow \quad v = -\frac{5.72}{4.78} = -1.20ms^{-1}\ to\ 3\ s.f.$$

$$Recoil\ speed = \mathbf{1.20}\boldsymbol{ms^{-1}\ to}\ \mathbf{3}\ \boldsymbol{s.f.}$$

The principle of **conservation of momentum** can be written in **vector** form:

$$m_A \boldsymbol{u_A} + m_B \boldsymbol{u_B} = m_A \boldsymbol{v_A} + m_B \boldsymbol{v_B}$$

Eg:
In a game of billiards, a cue ball of mass 170g travelling at a velocity of $\begin{bmatrix}3\\5\end{bmatrix}$ hits a stationary ball of mass 160g and subsequently moves at a velocity of $\begin{bmatrix}-2\\4\end{bmatrix}$. Calculate the velocity of the second ball.

$$0.17\begin{bmatrix}3\\5\end{bmatrix} = 0.17\begin{bmatrix}-2\\4\end{bmatrix} + 0.16\begin{bmatrix}a\\b\end{bmatrix} \quad \Longrightarrow \quad \begin{bmatrix}0.51\\0.85\end{bmatrix} = \begin{bmatrix}-0.34 + 0.16a\\0.68 + 0.16b\end{bmatrix}$$

$$\Longrightarrow \quad 0.51 = -0.34 + 0.16a \quad \Longrightarrow \quad a = 5.3125 \quad and\ 0.85 = 0.68 + 0.16b \quad \Longrightarrow \quad b = 5.27$$

$$\Longrightarrow \quad velocity\ of\ ball = \begin{bmatrix}\mathbf{5.3125}\\\mathbf{5.27}\end{bmatrix}\boldsymbol{ms^{-1}}$$