Loci Investigations

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1. Restraining Order
Based on an XKCD.com cartoon, you have to construct the locus of points no closer than a fixed distance *and* no further than another distance, from a point.

2. Sniper at the Gates
You need to use knowledge of the locus of points within a given distance to determine optimal coverage of a courtyard by clever positioning of gunmen.

3. Border Patrol
You need to minimise the number of CCTV cameras needed to ensure all travel across a courtyard is recorded. First from left to right, then top to bottom, then in either direction. Pythagoras may come in handy for the optimal solution.

4. Castle Construction
This requires an understanding of the locus of points a fixed distance from a rectangle, extending to a compound shape, and calculation of circle area.

5. Pentagon Perimeter
How far would a security guard at the Pentagon need to walk to patrol the whole building if he were a given distance from the wall?

6. Radiation Leak
Application of equidistance loci will enable you to find the source of radiation which is known to be the same distance from three people. Can be used to enable students to investigate how any 3 non-linear points define a circle.

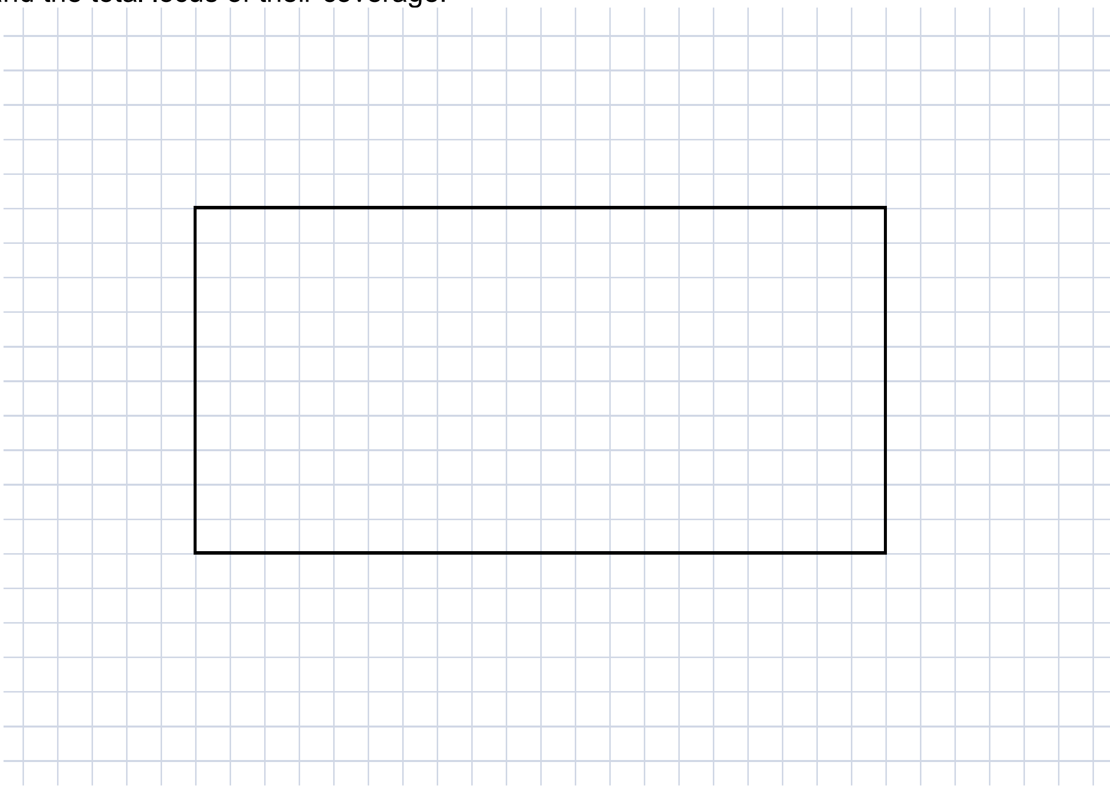
7. Around the Corner
First worksheet introduces the idea of the locus of a sheep tied by a rope to part of a barn wall. The radius of its circle of motion reduces whenever the rope has to go around a corner. How does the choice of attachment point affect the area of land available for the sheep to graze?
Second worksheet extends this idea by looking at a general rectangle and examining the maximum and minimum possible grazing areas.

Restraining Order

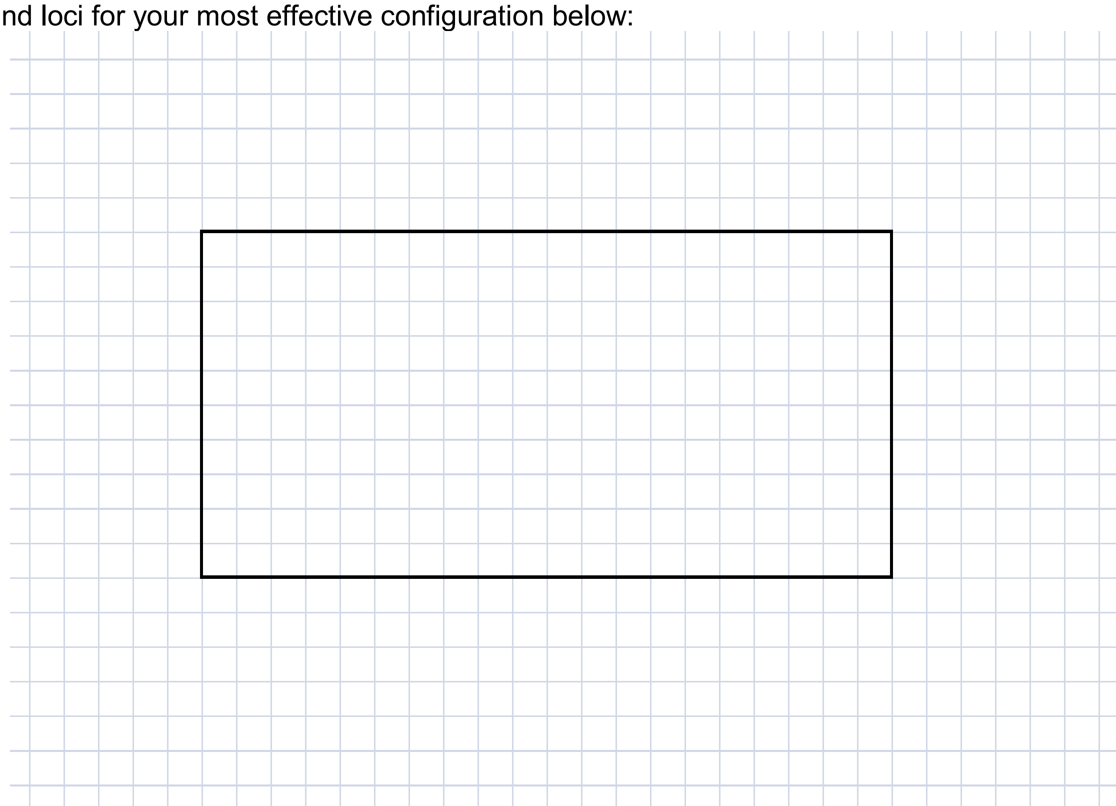
Construct the locus described in the space below, using an appropriate scale, and calculate the area within which he is free to move.

Sniper at the Gates

Four snipers are posted at the four corners of a 2km by 1km rectangular compound. If each sniper has an effective range of 500m, how many more would be needed to guard the entire perimeter fence? Draw on their positions, and the total locus of their coverage:



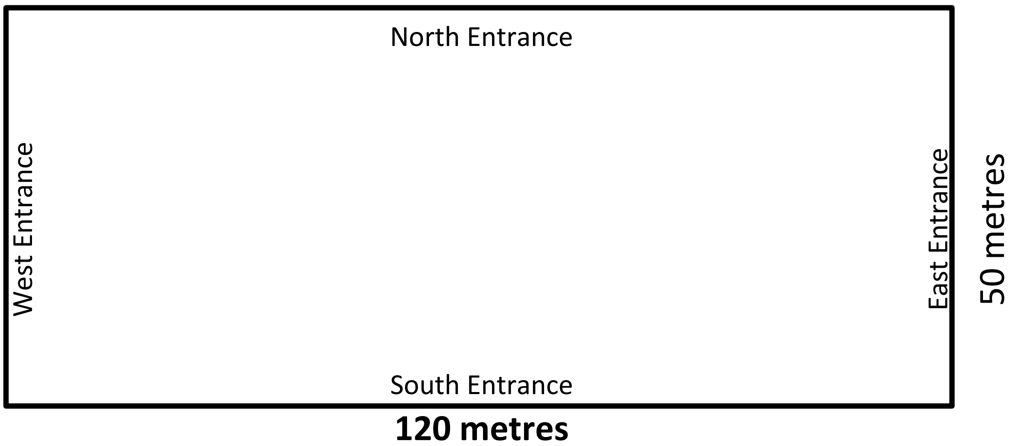
How many more would be needed to cover the entire area of the compound? Mark on their positions above.
Is it possible to cover the area with fewer snipers? You may change the position of any of the snipers.
On squared paper, investigate how much of the compound you could cover with just 5 snipers. Mark on the positions and loci for your most effective configuration below:



Border Patrol

A rectangular courtyard needs surveillance to track passersby.
360° cameras can be installed at certain points, with a range of **10 metres** in all directions. However, these are expensive, so we want to find the minimum number of cameras required to monitor travel across the courtyard.

Find the minimum number of cameras required to ensure *all* travel between the East and West entrances of the courtyard is recorded:



Now find the minimum number required to record any travel between the North and South entrances.

Finally, what is the smallest number required that would record *both* travel between East and West and travel between North and South?

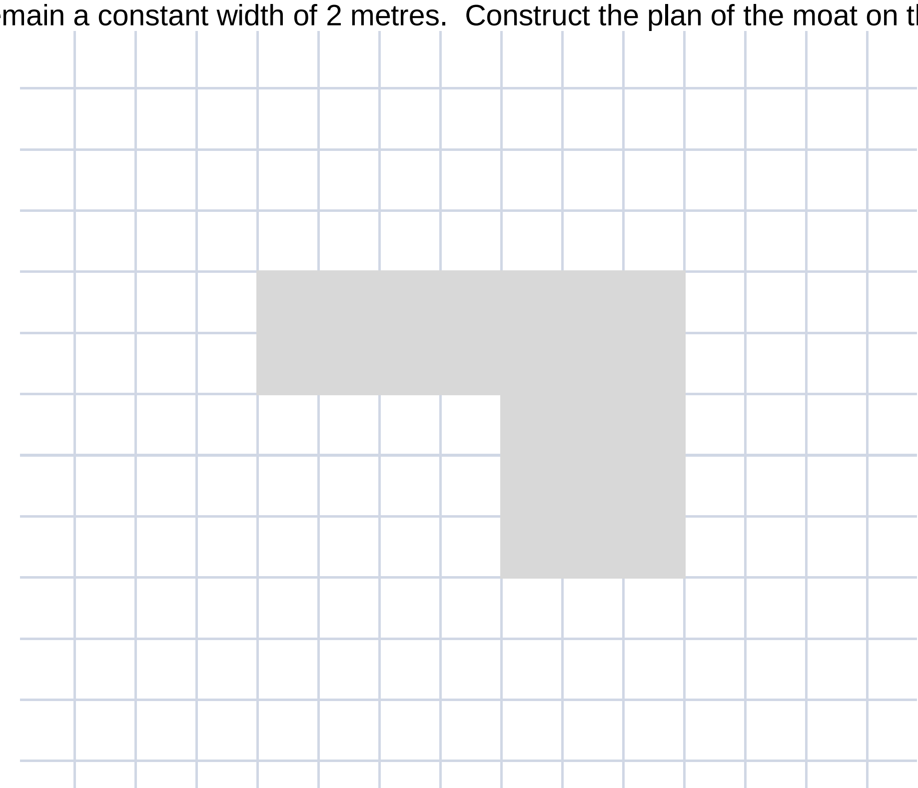
Hint:
Make sure the camera coverage forms a complete barrier, so that there are no possible paths from one side to the other without crossing the barrier.

For part 3, try to work out if there is a single line of cameras that would separate North from South as well as separating East from West.

Castle Construction

A man wants to lay a path of constant width around his summer house (shown below). If it costs £10 per square metre to lay the path, how much would it cost if the path was two metres wide? What if it were three metres wide? (*one square represents one metre – use the square lengths to set your compass, not a ruler!)*

He now plans to dig a moat around his house (shown below). It will begin exactly 1 metre from the wall all the way around, and remain a constant width of 2 metres. Construct the plan of the moat on the diagram:



If the moat will be an average depth of 2 metres, how much water will be needed, in litres? ($1m^3$ = 1,000 litres = 1 tonne)

Pentagon Perimeter

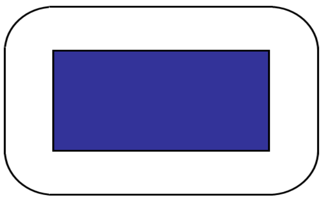
The defense base of the United States is The Pentagon.
It is a huge building in the shape of a regular pentagon where each side is 280 metres long.

If you were a security guard whose job was to walk all the way around the building, how far would you need to walk to get all the way round?

If the Pentagon security forces need someone to patrol around the building *exactly 70 metres away from the wall* at all times, draw the locus of this path around the Pentagon:

Finally, what is the total distance this outer security guard would have to walk to complete the circuit?

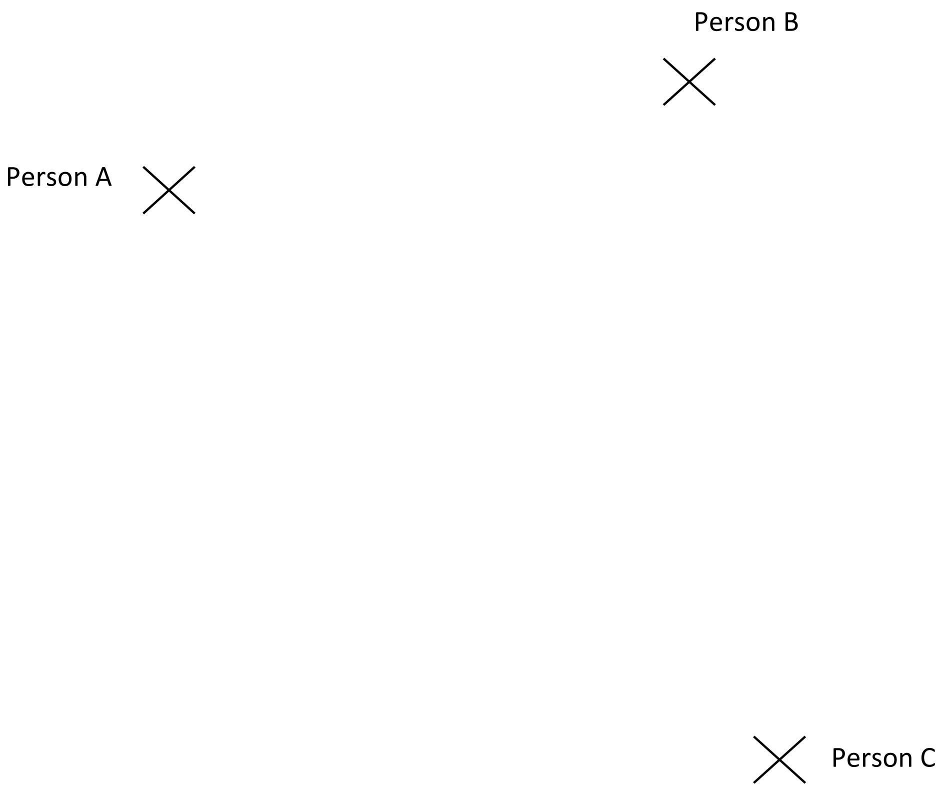
Hint:
Use the information about the length of each side, and think carefully about what happens to the path at each corner. This example shows a path a fixed distance away from the edge of a rectangular building:



Think carefully about how much of a circle the arcs at each corner make altogether.

Radiation Leak

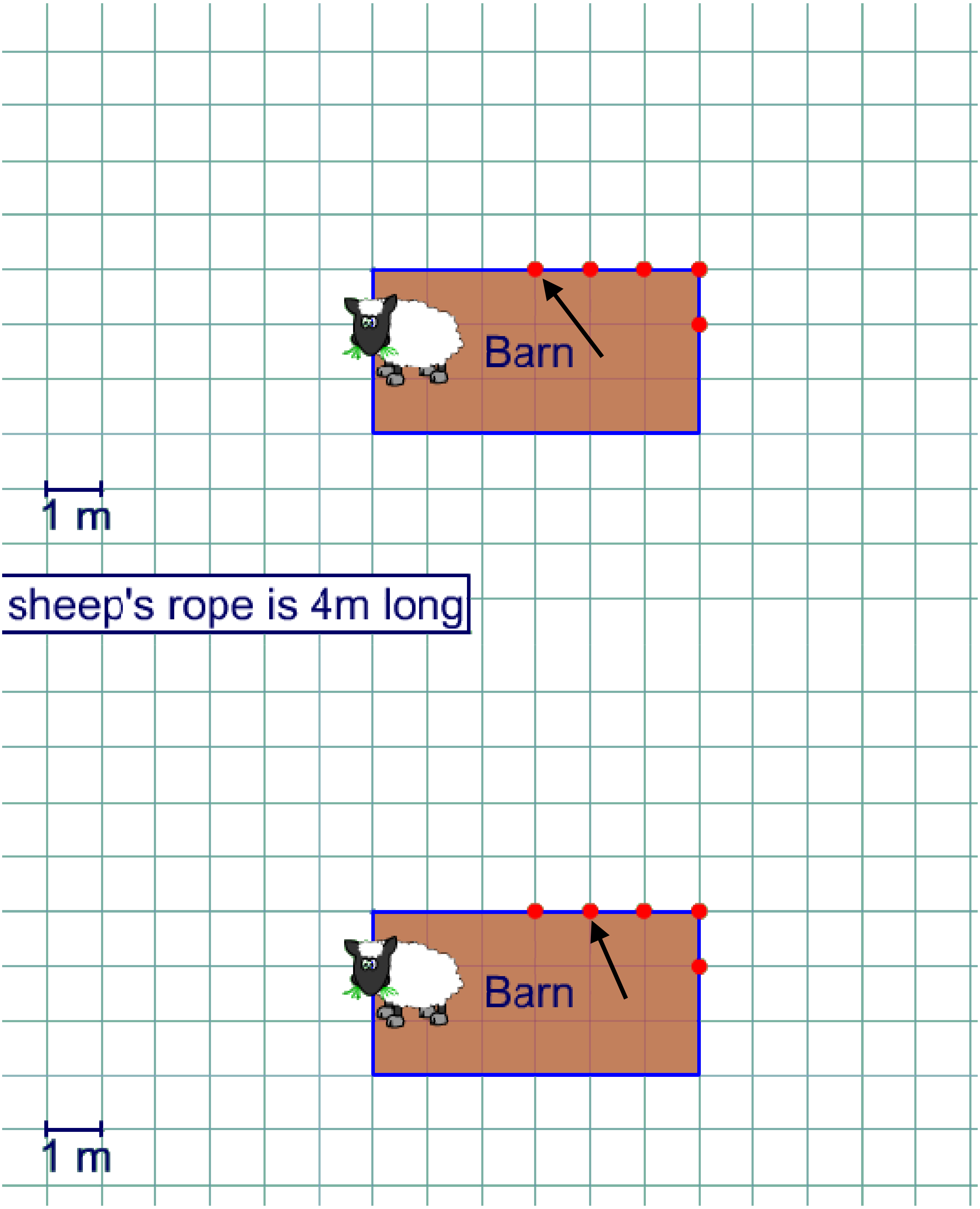
Three people are trying to locate a source of radioactivity.
Their instruments are giving all of them the same level of radiation in their current position, meaning the radiation source must be somewhere between them, exactly the same distance away from each person.
Find the location of the radioactive material, and their distance from it.

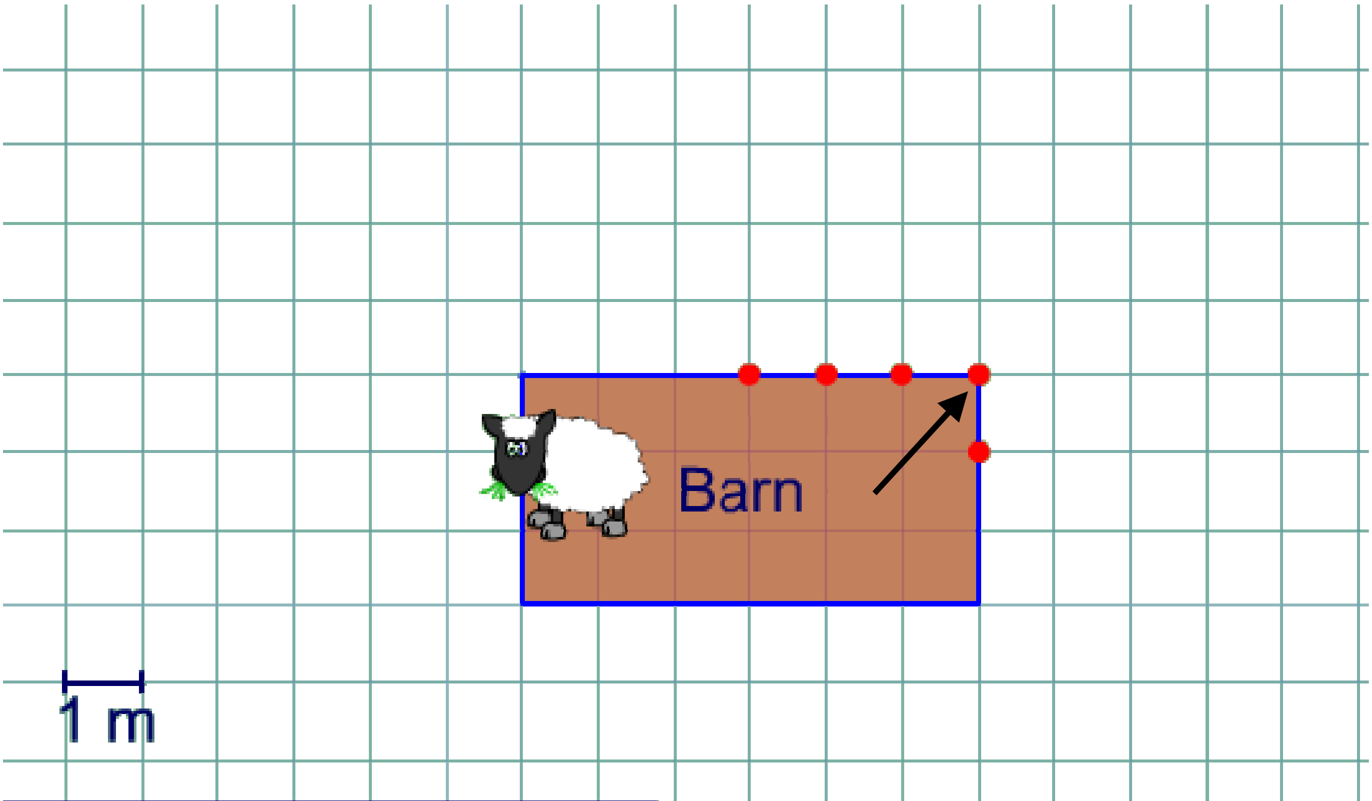


Hint:
*These three points are all on a circle whose centre we are trying to find. The centre of the circle is **equidistant** from A and B, and **equidistant** from B and C (and from A and C, but two pairs of points should be enough).*

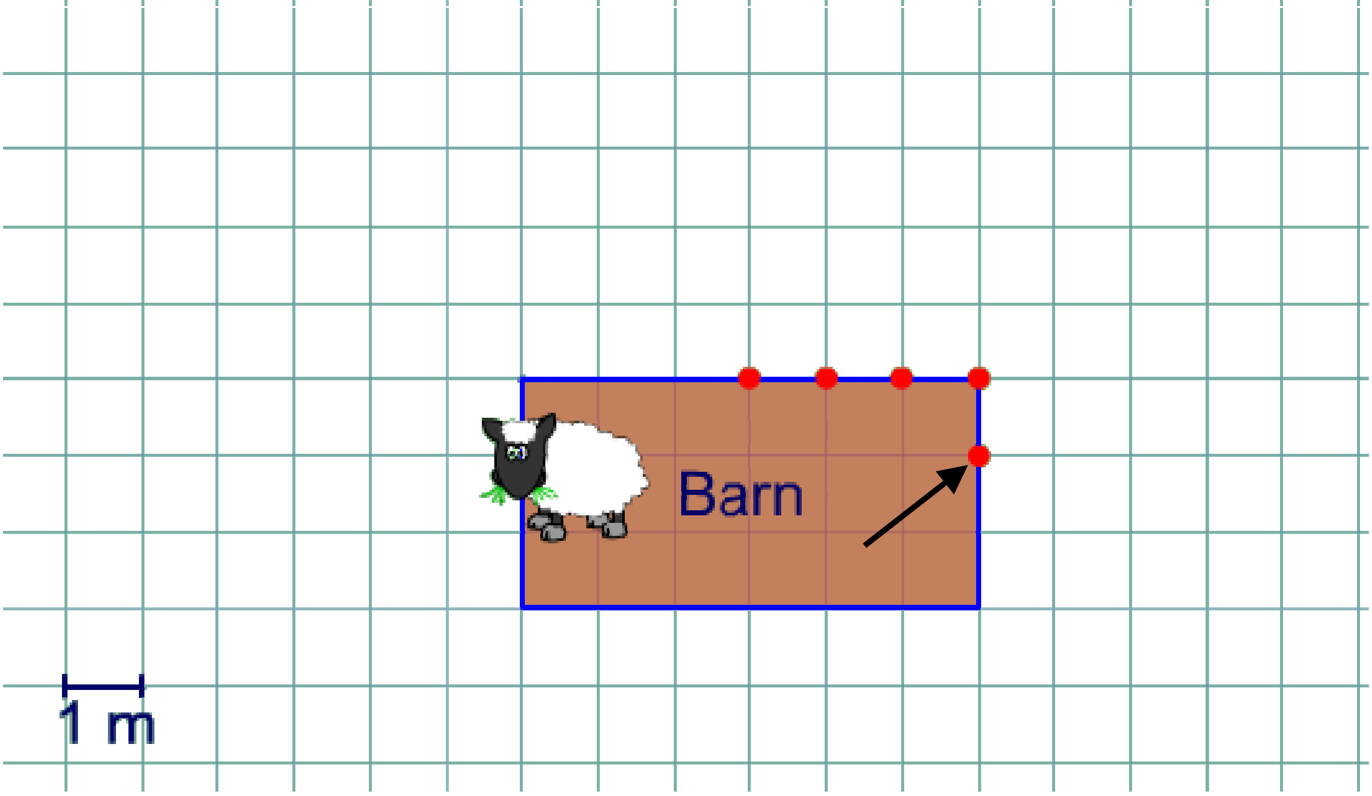
Use a perpendicular bisector to plot the locus of points equidistant from two points.

Loci – Around The Corner

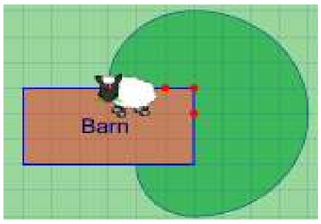




sheep's rope is 4m long



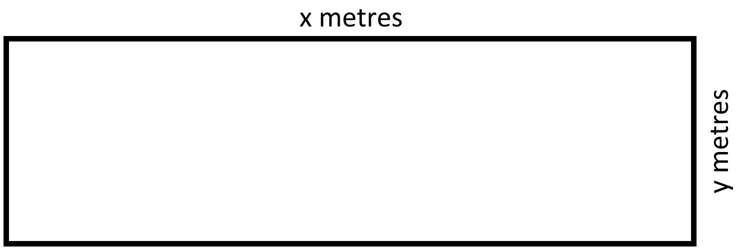
Around The Corner



If a sheep is tied by a fixed length of rope to the side of a barn, the area it is free to graze depends on exactly where it is tied.

What is the best place to tie the sheep, and can you explain why?

If a sheep is tied to a rectangular building by a long enough piece of rope to enable it to reach any point along the outside wall, what is the maximum area the sheep will be able to graze?



Assuming that the length y metres is the smaller of the two, what is the minimum area the sheep would be able to graze?

Hint:
First think about where the sheep would need to be tied for maximum or for minimum coverage. What affects this? Is it better to have a few large sections of circles, or lots of small sections?

Then use algebra to find an answer. What do you notice about your answer for the maximum coverage?