How to: Find the HCF or LCM of numbers using prime factors

Key ideas:

- Any integer greater than 1 can be written as a **product of primes**. Eg: $28 = 2^2 \times 7$.
- The Highest Common Factor (HCF), also known as the Greatest Common Divisor (GCD), of two (or more) numbers, is the **largest number which is a factor of each**.
- The Lowest Common Multiple (LCM) of two (or more) numbers, is the **smallest number which is a multiple of each**.

Highest Common Factor	Lowest Common Multiple
The basic method: + It's easy to understand + It's fairly quick for small numbers - You have to find **all** the factors of a number - It is very slow for large numbers 1. List all the factors of each number. 2. Find the largest number which is in each list.	**The basic method:** + It's easy to understand + It's fairly quick for small numbers - You may have to find a lot of multiples of a number - It is very slow for large numbers 1. Start listing multiples of each number. 2. Find the smallest number which is in each list.
The primes method: *+ You don't need all the factors of the numbers* *+ It's really quick for numbers in prime factor form, even if the numbers are massive* *- You have to be able to prime factorise a number* *- If you need a number in non-prime-factor form at the end, you'll have to do some multiplying.* 1. Break down the numbers into their prime factors 2. Look at how many of each prime occur in each number, and include the **lowest number of each**. This is the greatest number you can include that is a part of (a factor of) each of your numbers.	**The primes method:** *+ You don't need to list any multiples of the numbers* *+ It's really quick for numbers in prime factor form, even if the numbers are massive* *- You have to be able to prime factorise a number* *- If you need a number in non-prime-factor form at the end, you'll have to do some multiplying.* 1. Break down the numbers into their prime factors 2. Look at how many of each prime occur in each number, and include the **highest number of each**. This is the lowest number you can include that contains all of (is a multiple of) each of your numbers.

Example: *Find the HCF and LCM of* 240 *and* 450.

HCF

Basic method:

1×240	1×450
2×120	2×225
3×80	3×150
4×60	5×90
5×48	6×75
6×40	9×50
$8 \times \mathbf{30}$	10×45
10×24	15×30
12×20	18×25
15×16	

The largest number in both lists: $HCF = \mathbf{30}$

LCM

Basic method:

$\mathbf{240}$	...	$\mathbf{450}$
480	2160	900
720	2400	1350
960	2640	1800
1200	2880	2250
1440	3120	2700
1680	3360	3150
1920	$\mathbf{3600}$	$\mathbf{3600}$
...		

The smallest number in both lists: $LCM = \mathbf{3600}$

Primes method:

$$240 = \mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times \mathbf{5}$$

$$450 = \mathbf{2} \times \mathbf{3} \times \mathbf{3} \times \mathbf{5} \times \mathbf{5}$$

The **largest** collection that **both lists contain**:

$$2 \times 3 \times 5 = HCF = \mathbf{30}$$

The **smallest** collection that **contains both lists**:

$$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = \mathbf{3600}$$

Further example (very large numbers)

Find the HCF and LCM of $10!$*,* 850×19 *and* 15^{12}*.* Note that $10! = 10 \times 9 \times \ldots \times 2 \times 1$.

The three numbers, written in prime factor form, are:
$10! = (2 \times 5) \times (3^2) \times (2^3) \times 7 \times (2 \times 3) \times 5 \times (2^2) \times 3 \times 2 = \mathbf{2^8 \times 3^4 \times 5^2 \times 7}$
$850 \times 19 = 85 \times 10 \times 19 = (5 \times 17) \times (2 \times 5) \times 19 = \mathbf{2 \times 5^2 \times 17 \times 19}$
$15^{12} = (3 \times 5)^{12} = \mathbf{3^{12} \times 5^{12}}$

HCF: The greatest number of each featured prime contained in any of the three is:

$$2^0 \times 3^0 \times 5^2 \times 7^0 \times 17^0 \times 19^0 = HCF = \mathbf{5^2}$$

LCM: The lowest number of each featured prime that would contain any of the three is:

$$LCM = \mathbf{2^8 \times 3^{12} \times 5^{12} \times 7 \times 17 \times 19}$$

How it works

HCF:
Essentially, the highest common factor will be built from as many primes as possible *which could be sourced from either of your numbers*.
Therefore the powers of primes given in the prime factorisation should be read as a *maximum*.
When choosing which primes to include, you can **only take primes which appear in both lists**. If one list has twelve 7's, but the other has only five, you can only take five.

Every prime in the HCF must be in each number, but you should include as many as possible.
(2^5 *is the largest number which is a factor of both* 2^8 *and* 2^5*).*

LCM:
The lowest common multiple will be built from as few primes as possible *which must include everything in either of your numbers*.
Therefore the powers of primes given in the prime factorisation should be read as a *minimum*.
When choosing primes to include, you **must take all the primes which appear in either list**. If one list has twelve 7's, but the other has only five, you must take twelve.

Every prime in each number must be in the LCM, but they can be shared.
(2^8 *is the lowest multiple of both* 2^8 *and* 2^5*)*

In algebraic form:
Two numbers, A and B, whose prime factorisations are:

$$A = 2^a \times 3^b \times 5^c \times 7^d \times \ldots \qquad B = 2^w \times 3^x \times 5^y \times 7^z \times \ldots$$

Have their highest common factor given by:

$$HCF(A, B) = 2^{\min(a,w)} \times 3^{\min(b,x)} \times 5^{\min(c,y)} \times 7^{\min(d,z)} \times \ldots$$

And lowest common multiple given by:

$$LCM(A, B) = 2^{\max(a,w)} \times 3^{\max(b,x)} \times 5^{\max(c,y)} \times 7^{\max(d,z)} \times \ldots$$