Gender Balance

In some countries, one gender may be valued above another.

Suppose in a certain population, every family continued to have children until they had a son, then stopped.

Would the population end up with more boys or more girls?

1. Find the total number of boys that would be born to a population of $1{,}000{,}000$ families:

2. How many of these $1{,}000{,}000$ families will go on to have a 2nd child?

3. How many of those which have a 2nd child will go on to have a 3rd?

4. Write down the first 4 terms of the series representing the number of girls who are a:
1st child, 2nd child, 3rd child, 4th child, …

5. Use the formula $S_n = \frac{a(1-r^n)}{1-r}$ to calculate the total number of girl babies who are among the first 10 children born to a family.

6. Use the formula $S_\infty = \frac{a}{1-r}$ to calculate the total number of girls that would be born to such a population.

7. Use your answers to decide whether the population in question will end up with more males than females.

Gender Balance SOLUTIONS

In some countries, one gender may be valued above another.

Suppose in a certain population, every family continued to have children until they had a son, then stopped.

Would the population end up with more boys or more girls?

1. Find the total number of boys that would be born to a population of 1,000,000 families:

$1,000,000$ (every family will eventually have a boy)

2. How many of these 1,000,000 families will go on to have a 2nd child?

$500,000$ (half the families will have a boy for their first child)

3. How many of those which have a 2nd child will go on to have a 3rd?

$250,000$ (half of the families who have a 2nd child will have boys, then stop)

4. Write down the first 4 terms of the series representing the number of girls who are a:

1st child, 2nd child, 3rd child, 4th child, …

$500,000 \quad 250,000 \quad 125,000 \quad 62,500 \quad \ldots$

5. Use the formula $S_n = \frac{a(1-r^n)}{1-r}$ to calculate the total number of girl babies who are among the first 10 children born to a family.

$$S_{10} = \frac{500,000\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}} = \frac{500,000\left(\frac{1023}{1024}\right)}{\frac{1}{2}} = 1,000,000\left(\frac{1023}{1024}\right) \approx 999023$$

6. Use the formula $S_\infty = \frac{a}{1-r}$ to calculate the total number of girls that would be born to such a population.

$$S_\infty = \frac{500,000}{1-\frac{1}{2}} = \frac{500,000}{\frac{1}{2}} = 1,000,000$$

7. Use your answers to decide whether the population in question will end up with more males than females.

There will be the same number of boys as girls.

Another way to approach this problem is to consider the proportions of 1st children, 2nd children, etc. Half the 1st children will be boys, half the 2nd children will be boys, etc.