Equations of Motion Investigation

The basics

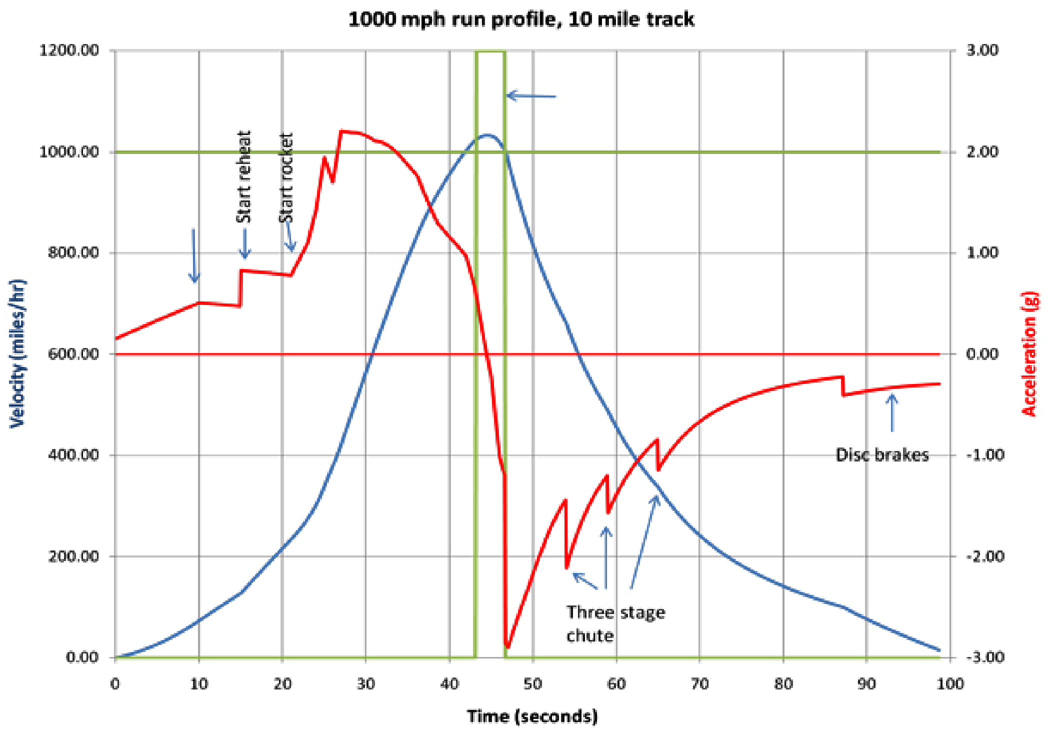
Definition: **Displacement** is the distance an object has travelled in a given direction. This is usually measured in *metres*, generally abbreviated to m.

Definition: **Velocity** is the speed of an object in a given direction. This is usually measured in *metres per second*, generally abbreviated to m/s or ms^{-1}. For comparison, $1mph \approx 0.5ms^{-1}$.

Definition: **Acceleration** is the rate at which the velocity of an object is changing. This is usually measured in *metres per second per second*, generally abbreviated to m/s^2 or ms^{-2}. For comparison, an acceleration of $11ms^{-2}$ corresponds to that of the Bugatti Veyron, going from $0mph$ to $60mph$ in 2.4 seconds.

Definition: A **velocity-time graph** measures velocity (y-axis) against time (x-axis). Since acceleration is the rate at which velocity changes, and velocity is the rate at which displacement changes, we can use the *gradient* of the graph and the *area* under the graph to describe acceleration and displacement.

Example of a compound graph, tracking the motion of Bloodhound SSC, contender for the land speed record.



Reading the left-hand vertical axis and the blue line gives a velocity-time graph.

Reading the right-hand vertical axis and the red line gives an acceleration-time graph.

The green lines indicate the location of the test track and the desired top speed ($1050mph$).

The investigation

The aim is to investigate connections between *displacement, initial velocity, final velocity, acceleration and time*.

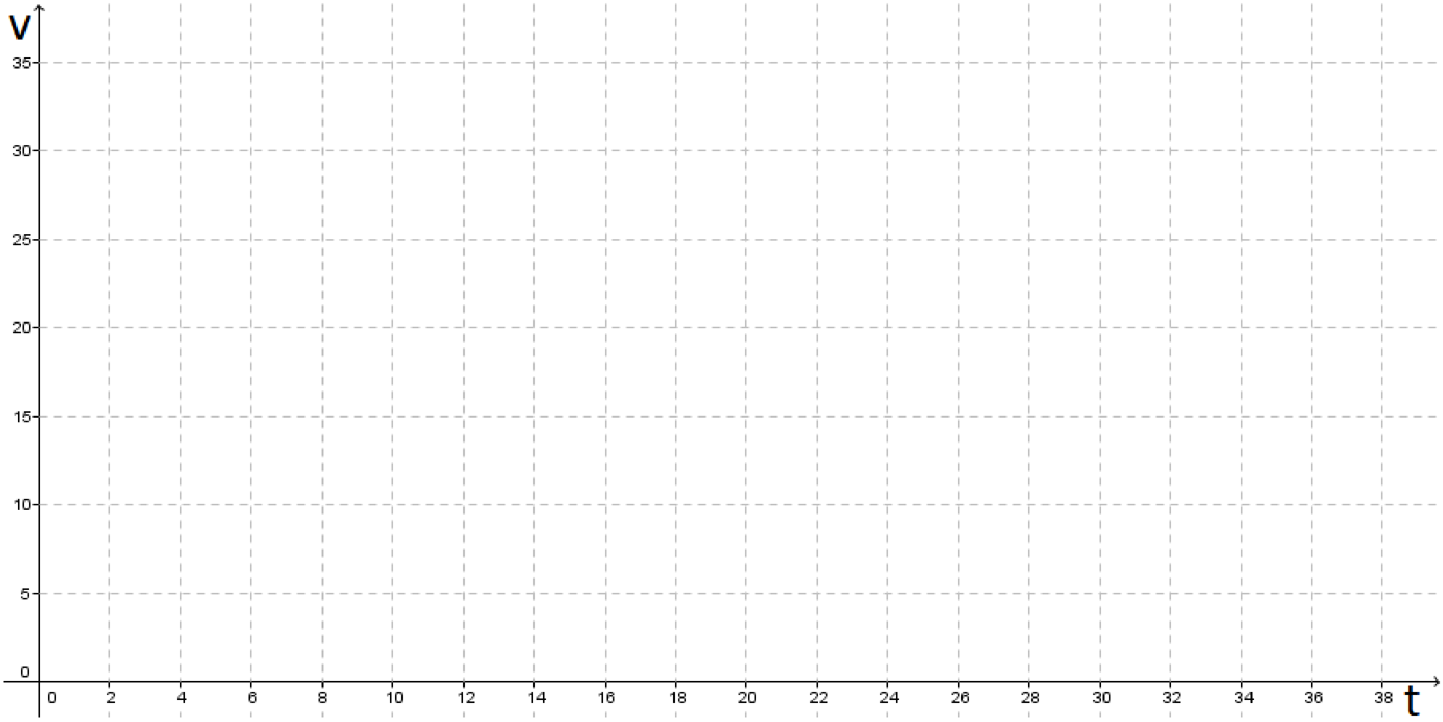
Getting started

For this investigation, you will be using a **velocity-time graph**. Follow the instructions over the page to generate the graph, then think about what the gradient and the area tell you about the situation.

Equations of Motion Investigation
Instructions

1. Using the information in the following paragraph, plot a velocity-time graph on the grid.

A car accelerates uniformly from rest to a speed of $30ms^{-1}$ *during the first* 10 *seconds.*
It then maintains this speed for 20 *seconds.*
At this point, the car performs an emergency stop, decelerating at a rate of $6ms^{-2}$*.*



Hint: Mark on the key *points* before drawing in the lines. Acceleration gives the *change in speed* over time.

2. Answer the following questions using the graph you have drawn:
   a) What was the acceleration during the first 10 seconds of motion?

   b) What distance was covered while travelling at a constant speed?

   c) What was the *average speed* during the emergency stop?

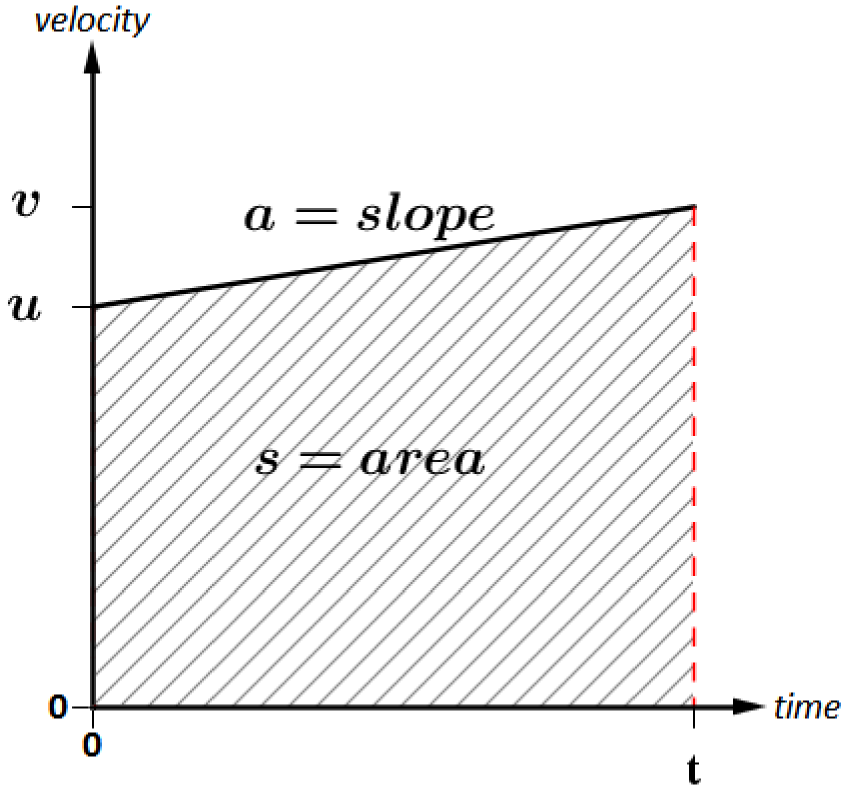
   d) Using your answer to c, work out the distance travelled in the last 5 seconds.

3. How does the *area under the graph* relate to the *distance travelled*? Use your observation to calculate the total distance travelled by the car during the entire journey.

4. How does the *gradient of the graph* relate to the *acceleration* of the car?

Involving Algebra

Consider the following velocity-time graph:



The interval shown describes a general scenario with constant acceleration:

The **displacement** is $\boldsymbol{s}\ m$

The **initial velocity** is $\boldsymbol{u}\ ms^{-1}$

The **final velocity** is $\boldsymbol{v}\ ms^{-1}$

The **acceleration** is $\boldsymbol{a}\ ms^{-2}$

The **time** is $\boldsymbol{t}\ s$

5. Use the following formulae for gradient and the area of a trapezium to generate your own formulae linking the five variables s, u, v, a and t:

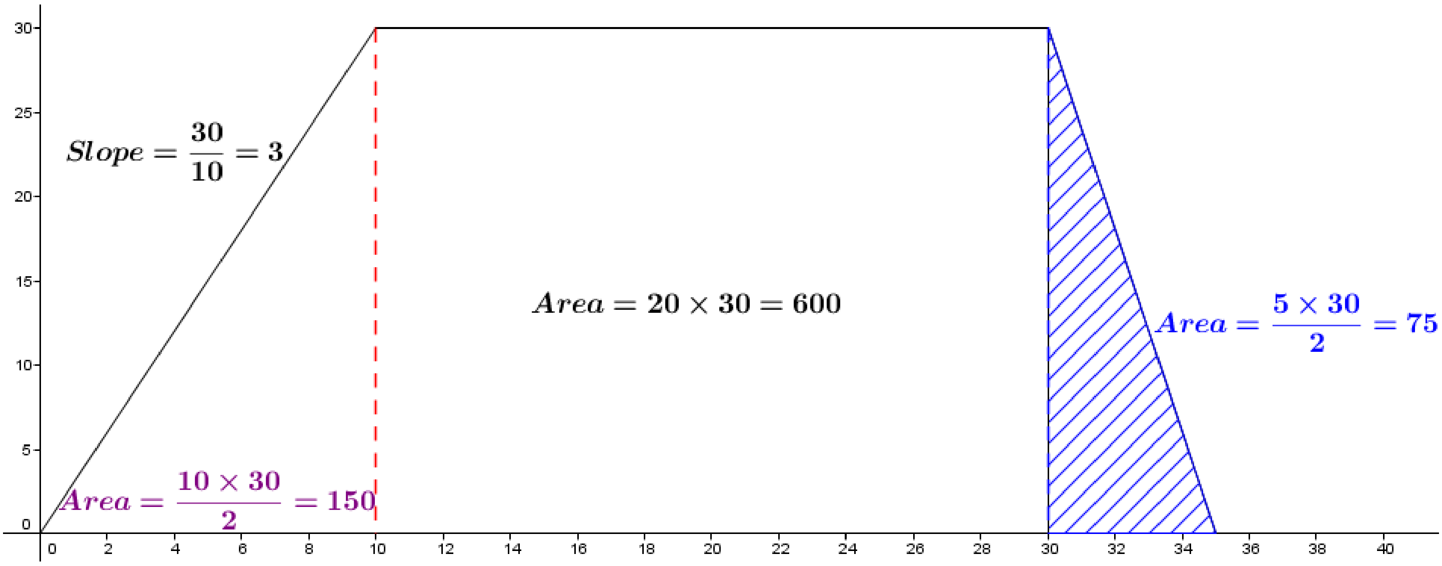
Gradient of a line:	**Area of a trapezium:**
$\frac{y-step}{x-step} \quad or \quad \frac{y_2-y_1}{x_2-x_1}$ where (x_1, y_1) and (x_2, y_2) are two points on the line	$Average\ Width \times Height \quad or \quad \frac{a+b}{2}h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between them

What's next?

The equations you have generated belong to a family of five formulae known variously as the Kinematics Equations (a fancy way of saying equations of motion), the Constant Acceleration Equations (useful, since they only hold true when acceleration doesn't change) and the SUVAT Equations (named after the variables – only don't ask me why we use s for distance; sometimes we use r or x, but hardly ever d. No-one knows why…). These five formulae are used to describe all sorts of motion, including free-fall (since, close to Earth's surface, the acceleration experienced due to gravity is effectively constant). Once you refine your mathematical models to take into account, say, air resistance, or have to deal with situations where acceleration changes, you will need a method for investigating the gradient of, and the area under, *curves* as well as straight lines, and that's where Calculus comes in…

Equations of Motion Investigation SOLUTIONS

1.



2. Answer the following questions using the graph you have drawn:

a) What was the acceleration during the first 10 seconds of motion?

$$Acceleration = \frac{Change\ in\ speed}{Time} = \frac{30}{10} = \mathbf{3ms^{-2}}$$

b) What distance was covered while travelling at a constant speed?

$$Speed = \frac{Distance}{Time} \implies Distance = Speed \times Time = 30 \times 20 = \mathbf{600m}$$

c) What was the *average speed* during the emergency stop?

$$Speed\ changes\ at\ a\ steady\ rate\ from\ 30ms^{-1}\ to\ 0ms^{-1} \implies \frac{30+0}{2} = \mathbf{15ms^{-1}}$$

d) Using your answer to c, work out the distance travelled in the last 5 seconds.

$$Total\ Distance = Average\ Speed \times Time = 15 \times 5 = \mathbf{75m}$$

3. How does the *area under the graph* relate to the *distance travelled*? Use your observation to calculate the total distance travelled by the car during the entire journey.

The area under the graph represents the distance travelled. If the speed were $0ms^{-1}$, there would be no distance covered. For a constant speed, the formula $distance = speed \times time$ is equivalent to the area of the rectangle, and for a constant acceleration, by using the average speed we are effectively finding the area of a triangle (or, in some cases, a trapezium).

4. How does the *gradient of the graph* relate to the *acceleration* of the car?

The gradient of the graph represents the acceleration. Since acceleration is simply the increase in speed per second, the gradient, which is change in y divided by change in x, is identical to this, since for a velocity-time graph it means "change in speed divided by change in time".

5. Use the formulae for gradient and the area of a trapezium to generate your own formulae linking the five variables s, u, v, a and t:

$$Gradient = \frac{y-step}{x-step} \implies a = \frac{v-u}{t} \quad or \quad \boldsymbol{v = u + at}$$

$$Area = Average\ width \times height \implies \boldsymbol{s = \frac{u+v}{2}t}$$

Note that 'width' refers to the length of the two parallel lines, and 'height' to the perpendicular distance between them, so when we apply this to the graph, 'width' is the height at either side while 'height' is the distance along the bottom.

The other three formulae (each one links four of the variables, ignoring one) can be found by substituting either $v = u + at$, $u = v - at$ or $t = \frac{v-u}{a}$ into $s = \frac{u+v}{2}t$.