Deriving SUVAT Equations

Stated assumptions:

$$average\ speed = \frac{distance}{time} \qquad (1)$$

$$acceleration = \frac{change\ in\ speed}{time} \qquad (2)$$

Defining variables:

$$\begin{aligned} s &= displacement\ (m) \\ u &= initial\ velocity\ (ms^{-1}) \\ v &= final\ velocity\ (ms^{-1}) \\ a &= acceleration\ (ms^{-2}) \\ t &= time\ (s) \end{aligned} \qquad (3)$$

Deriving $v = u + at$: Writing (2) using the variables from (3):

$$a = \frac{v-u}{t}$$

Rearranging:

$$\boldsymbol{v = u + at} \qquad (4)$$

Deriving $s = \frac{u+v}{2}t$: Writing (1) using the variables from (3):

$$\frac{u+v}{2} = \frac{s}{t}$$

Rearranging:

$$\boldsymbol{s = \frac{u+v}{2}t} \qquad (5)$$

Deriving $s = ut + \frac{1}{2}at^2$: Substituting an expression for v from (4) into (5):

$$s = \frac{u + (u + at)}{2}t$$

Rearranging:

$$s = \frac{2u + at}{2}t$$

$$\boldsymbol{s = u + \frac{1}{2}at^2}$$

Note: $s = vt - \frac{1}{2}at^2$ can be derived by substituting for u instead of v.

Deriving $v^2 = u^2 + 2as$: Substituting an expression for t from (4) into (5):

$$s = \frac{u+v}{2}\left(\frac{v-u}{a}\right) = \frac{(v+u)(v-u)}{2a}$$

$$s = \frac{v^2 - u^2}{2a}$$

$$\boldsymbol{v^2 = u^2 + 2as}$$