Deriving SUVAT Equations

Stated assumptions:

\[ \text{average speed} = \frac{\text{distance}}{\text{time}} \]  
\[ \text{acceleration} = \frac{\text{change in speed}}{\text{time}} \]  

Defining variables:

\[ s = \text{displacement} \ (m) \]
\[ u = \text{initial velocity} \ (ms^{-1}) \]
\[ v = \text{final velocity} \ (ms^{-1}) \]
\[ a = \text{acceleration} \ (ms^{-2}) \]
\[ t = \text{time} \ (s) \]

Deriving \( v = u + at \):

Writing (2) using the variables from (3):

\[ a = \frac{v - u}{t} \]

Rearranging:

\[ v = u + at \]  

Deriving \( s = \frac{u + v}{2}t \):

Writing (1) using the variables from (3):

\[ \frac{u + v}{2} = \frac{s}{t} \]

Rearranging:

\[ s = \frac{u + v}{2}t \]

Deriving \( s = ut + \frac{1}{2}at^2 \):

Substituting an expression for \( v \) from (4) into (5):

\[ s = \frac{u + (u + at)}{2}t \]

Rearranging:

\[ s = \frac{2u + at}{2}t \]

\[ s = u + \frac{1}{2}at^2 \]

Note: \( s = vt - \frac{1}{2}at^2 \) can be derived by substituting for \( u \) instead of \( v \).

Deriving \( v^2 = u^2 + 2as \):

Substituting an expression for \( t \) from (4) into (5):

\[ s = \frac{u + v}{2} \left( \frac{v - u}{a} \right) = \frac{(v + u)(v - u)}{2a} \]

\[ s = \frac{v^2 - u^2}{2a} \]

\[ v^2 = u^2 + 2as \]