Currency

When buying foreign currency, you never get exactly the same as the exchange rate. Even if no commission is charged, the price is different for buying and for selling.

Airport Bureau De Change are notorious for charging a high rate to sell you currency, and paying a low rate to buy it back.

A typical example of non-airport foreign exchange rates is shown below:

We Sell:	We Buy:
\$1 for £0.64	\$1 for £0.56

Eg:
If you want to buy \$100 of holiday money, it will cost you £64.
If you have \$100 to exchange, you can get £56 for it.

Now imagine an indecisive traveller who can't make up his mind whether to go to America or not. He takes out £1000, exchanges it all for dollars and then thinks better of it and promptly exchanges his dollars back into pounds. How much of his original £1000 does he have remaining?

If our changeable tourist had repeatedly changed his mind, exchanging his money back and forth twelve times at these exchange rates, how much would he have remaining?

If our fickle globetrotter had been less savvy and forgot to exchange his money until he arrived at the airport, the rates would have been considerably less favourable:

We Sell:	We Buy:
\$1 for £0.80	\$1 for £0.50

Our capricious wayfarer still exchanges all his pounds for dollars and back again, over and over again. How many times altogether can he perform this double-exchange before he has less than £10 remaining?

Currency Solutions

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$$£1000 \div £0.64 = \$1562.50 \quad \$1562.50 \times £0.56 = £\mathbf{875}$$

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$$\textit{Geometric Series with}: \quad a = 875 \quad r = 0.875 \quad n = 12$$

$$U_n = ar^{n-1} \quad \Rightarrow \quad U_n = 875 \times 0.875^{11} \approx £\mathbf{201.42}$$

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$$\textit{Geometric series with}: \quad a = 625 \quad r = 0.625$$

$$U_n = ar^{n-1} \quad \Rightarrow \quad 10 = 625 \times 0.625^{n-1} \quad \Rightarrow \quad 0.016 = 0.625^{n-1}$$

$$\textit{Taking logs}:$$

$$\ln 0.016 = \ln 0.625^{n-1} \quad \Rightarrow \quad \ln 0.016 = (n-1)\ln 0.625 \quad \Rightarrow \quad n = \frac{\ln 0.016}{\ln 0.625} + 1 \approx 9.798$$

$$\textit{Since } 10 \textit{ is the smallest integer that satisfies the condition, he must exchange } \mathbf{10\ \textit{times}}.$$