Carbon Dating

Carbon is the 15th most common element in the Earth's crust, the 4th most common element in the universe and the 2nd most common element in the human body. Carbon-14, or C-14, is an unstable isotope, which means it is the same chemically, but has 2 extra neutrons compared to the more common C-12, and, more importantly, is radioactive and decays predictably over time back into a more stable form of carbon.

Ca-14 is constantly being generated through bombardment of the atmosphere by cosmic rays, meaning the Earth's atmosphere contains roughly the same proportions of c-14 at any given time in history, give or take. Once an organism dies, since they are no longer interacting with the atmosphere, any carbon-14 in their system at the time of death can only be reduced through radioactive decay. By measuring how much C-14 is in the sample today, we can extrapolate backwards to predict how long it has been decaying for and therefore how old the sample is.

1. C-14 decays at a rate proportional to its mass. Given that a human body during life contains $16ng$ (nanograms) of C-14, and after 1000 years there will be $14.2ng$ still undecayed. Use this information to find the constant k in the general exponential decay formula below:

$$M = M_0 e^{-kt}$$

Where M is the mass, t is the time, M_0 is the initial mass and k is a positive constant.

2. The half-life of a radioactive isotope is the time a given sample of the isotope takes to decay to half of its original mass. Find the half-life of C-14. Hence estimate when Ötzi the Iceman was alive, given that approximately half of the C-14 in his system had decayed by the time he was discovered recently, preserved by a glacier.

Carbon Dating SOLUTIONS

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$$M_0 = 16 \quad and \quad 14.2 = 16e^{-1000k} \quad \Rightarrow \quad e^{-1000k} = 0.8875$$

$$\Rightarrow \quad -1000k = \ln 0.8875 \quad \Rightarrow \quad \boldsymbol{k = \frac{-\ln 0.8875}{1000}} \approx 0.000119347$$

Note: since $\ln p < 0$ for $0 < p < 1$, k is a positive constant despite the $-ve$ sign.

$$\boldsymbol{M = 16e^{\frac{\ln 0.8875}{1000}t}}$$

2. The half-life of a radioactive isotope is the time a given sample of the isotope takes to decay to half of its original mass. Find the half-life of C-14. Hence estimate when Ötzi the Iceman was alive, given that approximately half of the C-14 in his system had decayed by the time he was discovered recently, preserved by a glacier.

$$0.5M_0 = M_0 e^{\frac{\ln 0.8875}{1000}t} \quad \Rightarrow \quad 0.5 = e^{\frac{\ln 0.8875}{1000}t} \quad \Rightarrow \quad \ln 0.5 = \frac{\ln 0.8875}{1000}t$$

$$\Rightarrow \quad \boldsymbol{t = \frac{1000 \ln 0.5}{\ln 0.8875}} = 5800 \; years \; to \; 2 \; s.f.$$

$$Approx \; age \; of \; Iceman: \quad 2000 - 5800 = -3800 \quad \Rightarrow \quad \boldsymbol{3800BC}$$

Note: due to fluctuations in atmospheric levels of C-14, the method outlined above is only the first stage of carbon dating, and calibration by taking into account known C-14 levels throughout history is used to improve estimates. Ötzi is actually estimated to date from around 3300BC. And the half-life of C-14 is actually 5730 years $\pm$ 40 years. And I honestly don't know how much C-14 they found in Mr Ötzi. There was only 13.75kg of the poor chap left anyway by the time he was found in 1991, and we don't really know anything about him other than the fact that he was about 45 years old, weighed 50kg, grew up in a village down the mountain, moved north when he grew up, travelled a lot at high altitude, was a coppersmith, killed two men with one arrow and carried a bleeding companion over one shoulder within a few days of being killed himself by an arrow to the shoulder (despite the best efforts of his mates to save him). And that he had been ill three times during the last 6 months of his life, most recently 2 weeks before his accident, had shared a picnic of red deer and herb bread in a mid-altitude conifer forest 8 hours before dying and snacked on a bit of ibex later that day. He had some pretty cool tattoos as well. Oh, and he may well have been impotent, which might explain the wild outing with the lads that got him into trouble. Apart from that, he is a total enigma.