Averages Challenge Questions

Rugby-Jockeys

The New Zealand All-Black squad of 34 men has a mean average mass of around 100kg.
The mean average weight of a horse-racing jockey is about 50kg.
The mean average weight of a New-Zealander is approximately 80kg
How many jockeys would need to be added to the All-Blacks squad to bring the average weight below that of the average Kiwi?

The IQ Club

Twelve members of an IQ club have a mean average IQ of 150. Controversially, four new members are admitted to the club even though their average IQ is only 120. In protest, some of the original members leave. Ironically, their leaving did not reduce the average IQ of the club any further.
a) What was the average IQ of the members who left?
b) If the total number of members who left was 4, and one of them had an IQ of 180, what is the minimum possible range of their IQ scores?

Golden Gates:

The average annual salary of Bob Gates, Brian Gates, Barry Gates and Bill Gates is calculated to be $1,000,000,000.
a) What type of average was used?
b) Bill Gates makes $3,900,000,000. Barry Gates is unemployed. Work out the median average.

Speed Cameras

a) My average speed during the first 4 miles of a 5-mile 'Average Speed Camera' stretch of road was 45 mph.
What is the greatest average speed I can do during the final mile for my overall average not to exceed 50mph?
b) The next time I drive down this stretch of road, I average 55mph during the first 4 miles.
What must I average during the final mile in order to remain within the 50mph overall average speed limit?

Averages Challenge Questions SOLUTIONS

Rugby-Jockeys

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$34 \times 100 + 50x < 80(34 + x) \implies 3400 + 50x < 2720 + 80x \implies 680 < 30x \implies 22.\dot{6} < x \implies \boldsymbol{x = 23}$

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a) $\frac{12\times150+4\times120}{16} = 142.5 \implies$ *members who left have average IQ of* $\boldsymbol{142.5}$

b) Minimum range would occur if the other 3 all had the same IQ. Therefore $3x + 180 = 4 \times 142.5 \implies x = 130$. This would give a minimum range of $180 - 130 = \mathbf{50}$.

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a) Since Bill Gates is a famous billionnaire and the others are unknown, it's safe to assume the huge average is a result of one value (Bill's salary) skewing the result. This only happens with the **mean average**.

b) Given that Bill makes $3.9 billion and Barry makes $0, that only leaves 2 people who, together, make the remaining $100 million. Since there are only two of them, their mean average ($50 million) must also be their median average, and since they are also the two middle earners their median average must be the median average of all four. Therefore the median average is **$50,000,000**.
(Turns out at least one of them is also a big shot, although not quite in the same league as Bill).

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a) Total time: $\frac{4}{45} + \frac{1}{v}$ and: Total distance: 5. Average speed: $50 = \frac{5}{\frac{4}{45}+\frac{1}{v}} \implies \frac{4}{45} + \frac{1}{v} = \frac{1}{10} \implies \frac{1}{v} = \frac{1}{10} - \frac{4}{45}$

Therefore: $\frac{1}{v} = \frac{9}{90} - \frac{8}{90} = \frac{1}{90} \implies \boldsymbol{v = 90\ mph}$

b) Total time: $\frac{4}{55} + \frac{1}{v}$ and: Total distance: 5. Average speed: $50 = \frac{5}{\frac{4}{55}+\frac{1}{v}} \implies \frac{4}{55} + \frac{1}{v} = \frac{1}{10} \implies \frac{1}{v} = \frac{1}{10} - \frac{4}{55}$

Therefore: $\frac{1}{v} = \frac{11}{110} - \frac{8}{110} = \frac{3}{110} \implies v = \frac{110}{3} = \boldsymbol{36.\dot{6}\ mph}$

Common error: Note: a common misconception is to interpret average speed with respect to distance. The method below is **wrong** because it doesn't take into account the fact that speed is calculated *with respect to time.*
a) $4 \times 45 + x = 5 \times 50 \implies x = 250 - 180 = \boldsymbol{70mph}$ b) $4 \times 55 + x = 5 \times 50 \implies x = 250 - 220 = \boldsymbol{30mph}$