A-Level Maths
in a week

Core Maths - *Co-ordinate Geometry of Circles*

Generating and manipulating graph equations of circles.

Statistics - *Binomial Distribution*

Developing a key tool for calculating probability for a large number of trials.

Mechanics - *Kinematics*

Extending formulae for velocity and acceleration to develop powerful tools for describing motion.

Overview of A-level Courses

<table>
<tr><th>Straight Statistics</th><th>Maths & Statistics</th><th>Maths & Mechanics</th><th>Further Maths</th></tr>
<tr><td>Modules:
Statistics × 6</td><td>Modules:
Pure Maths × 4
Statistics × 2</td><td>Modules:
Pure Maths × 4
Mechanics × 2</td><td>Modules:
Pure Maths × 7
Statistics × 2
Mechanics × 2
Decision × 1</td></tr>
<tr><td rowspan="2">This course focuses on statistical techniques without requiring an especially high level of algebraic manipulation.
Ideal for students who want to pursue Statistics but are not especially confident with A/A* level material from the GCSE course. You still need a solid basis of mathematics.</td><td colspan="2">Two-thirds of the course is pure mathematics, during which you will build on the basis of higher level GCSE skills in algebra, geometry and trigonometry, as well as learning and developing key skills and techniques in calculus.</td><td rowspan="2">Over half of this course is pure mathematics, covering all the content of the A-level Maths courses in addition to 3 ‘Further Pure’ modules which introduce new ideas such as matrices and complex numbers.
Includes an introduction to Decision Mathematics which is concerned with networks, algorithms and sorting. While the content is twice as much as a single A-level, the course is taught in just 7 lessons a week, not 10.
This demanding course requires a high level of competence.</td></tr>
<tr><td>One third of the course will focus on the development of statistical ideas such as probability, data handling and hypotheses testing.
It builds on GCSE topics such as scatter-graphs and tree diagrams.
This subject fits in well with Economics.</td><td>One third of the course will focus on the development of mechanical ideas such as forces, energy and motion.
It builds on GCSE topics such as vectors and compound measures. This subject fits in well with Physics.</td></tr>
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Contents

Core Maths

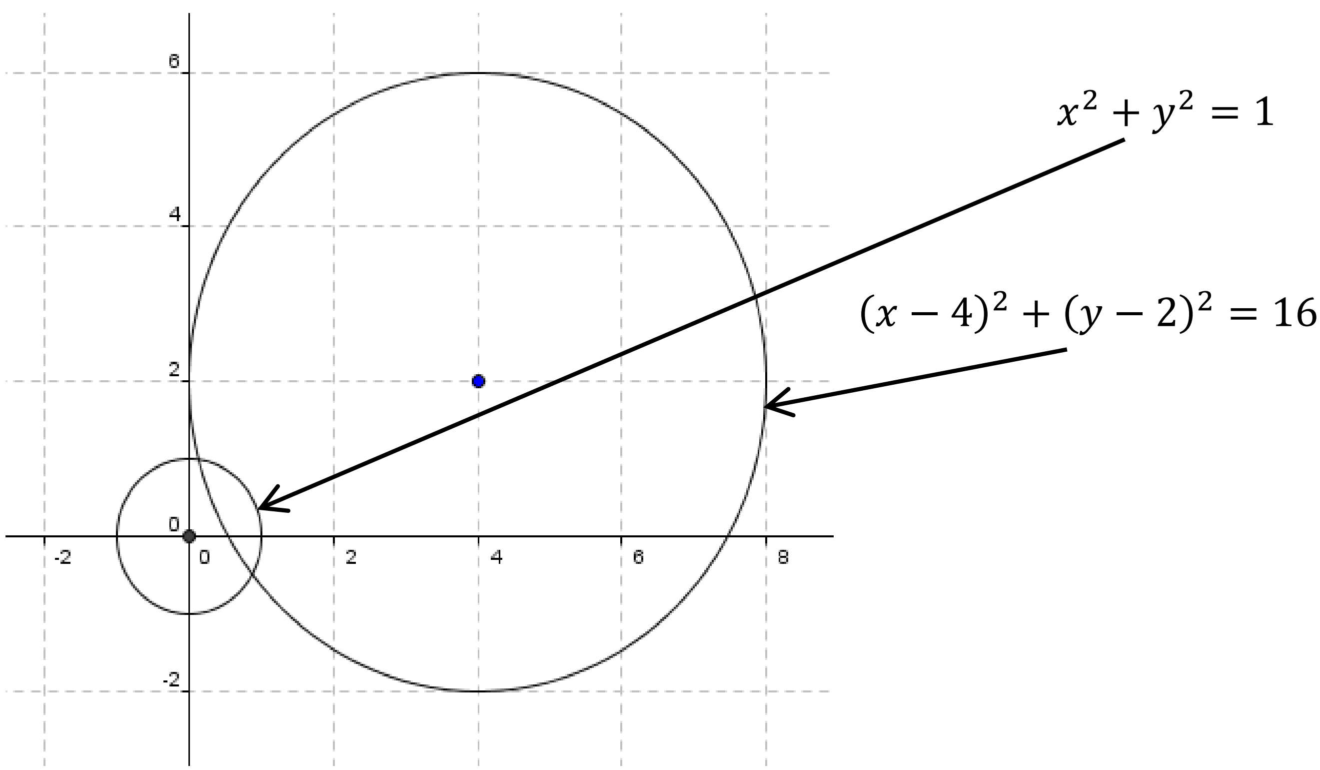
Co-ordinate Geometry of Circles

Prerequisites: You should already have some familiarity with:

- Straight line graphs ($y = mx + c$)
- Pythagoras' Theorem ($a^2 + b^2 = c^2$)
- Graph transformations (specifically translation)
- Quadratic expressions (Completing the Square, and $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

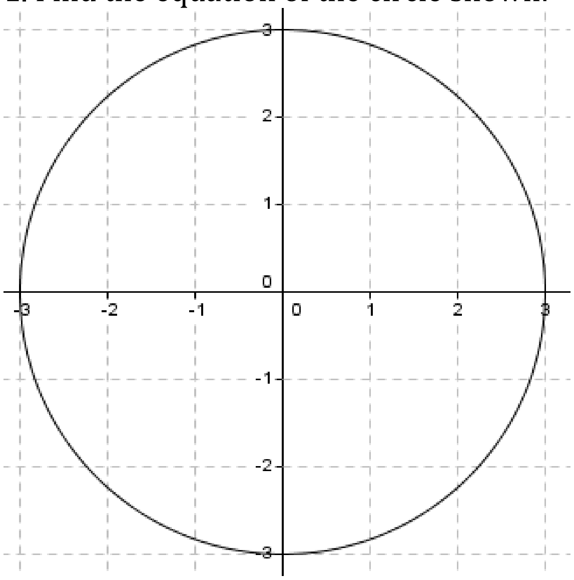
Key Points: You will learn about and use:

- A circle of radius 1 with centre (0,0) is given by: $x^2 + y^2 = 1$.
- A circle of radius R with centre (a,b) is given by: $(x-a)^2 + (y-b)^2 = R^2$



Core Maths – Coordinate Geometry of Circles – Questions

1. Find the equation of the circle shown:

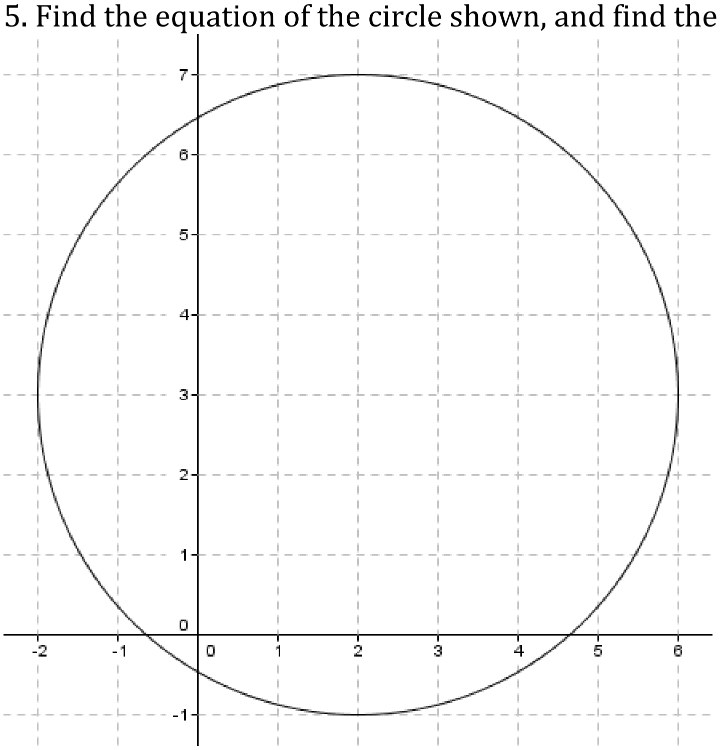


2. Find the equation of a circle centred at the origin with radius $\sqrt{5}$.

3. Write down the radius of the circle with equation $x^2 + y^2 = 36$.

4. Find the centre and radius of the circle with equation $(x - 2)^2 + (y + 1)^2 = 8$

5. Find the equation of the circle shown, and find the equation of a diameter line passing through $(0,0)$:



Statistics

Binomial Distribution

Prerequisites: You should already have some familiarity with:

- Probability: $P(A \text{ and } B) = P(A) \times P(B)$, $P(A \text{ or } B) = P(A) + P(B)$
- Tree diagrams.

Key points: You will learn about and use:

- Binomial Theorem: $P(X = r) = {}_nC_r p^r (1-p)^{n-r}$ where $r =$ number of successes, $p =$ probability of success and $n =$ number of trials.
- Factorial: $n! = n(n-1)(n-2) \dots (3)(2)(1)$, eg $4! = 4 \times 3 \times 2 \times 1 = 24$.
- Pascal's Triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Statistics – Binomial Distribution – Notes

The chance of getting a six 3 times, followed by not a six the next 2 times is:

$$\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}=\left(\frac{1}{6}\right)^3\times\left(\frac{5}{6}\right)^2=\frac{25}{7776}\approx 0.322\%$$

The chance of getting a six any 3 times in 5 rolls is greater than this, since there are lots of different orders in which this could happen:
11100, 11010, 11001, 10110, 10101, 10011, 01110, 01101, 01011 or 00111.

Since there are 10 different ways of getting 3 sixes in 5 rolls, the probability of getting 3 sixes in 5 rolls, in any order, is $10\times\left(\frac{1}{6}\right)^3\times\left(\frac{5}{6}\right)^2=\frac{250}{7776}\approx 3.22\%$

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For larger numbers, the ${}_nC_r$ function on your calculator will give you this.

Eg: The number of ways of choosing 7 items from 32 is: ${}_{32}C_7=3365856$.

To calculate, enter:

[3] [2] [$SHIFT$] [÷] [7] [=]

A situation can be modelled using the binomial distribution if it has:

- A fixed number of trials (n)
- Two possible outcomes (described as success or failure)
- The probability of success is the same for every trial (p)
- Trials are independent.

The chance of r successes in n trials with probability of success p is:

$${}_nC_r\times p^r\times(1-p)^{n-r}$$

Eg: The chance of getting a 4 on a six-sided dice five times in twelve throws is given by:

$${}_{12}C_5\times\left(\frac{1}{6}\right)^5\times\left(\frac{5}{6}\right)^7=792\times\frac{1}{7776}\times\frac{78125}{279936}=\frac{61875000}{2176782336}\approx 2.84\%$$

Statistics – Binomial Distribution – Questions

1. How many ways can the letters of the word ‘thousand’ be rearranged?

2. A team of 5 is to be chosen at random from each class of students in a school. How many different teams are possible for a class of 30 students?

3. A 5-card poker hand is dealt from a deck of 52 unique playing cards. How many possibilities are there?

4. What is the probability that a fair coin tossed 20 times shows heads exactly 10 times?

5. a) What is the probability that a fair six-sided die shows a 3 exactly once in 6 throws?

b) What is the probability that it shows a 3 exactly twice?

c) What is the probability that it never shows a 3?

d) What is the probability that it shows a 3 on at least half of the six throws?

6. A game pays out if you can throw five dice and get the same number on each. The chance of this happening is $\frac{1}{1296}$. Find the probability that, after 1000 people play the game, no more than 2 people win.

Mechanics

Kinematics

Prerequisites: You should already have some familiarity with:

- Velocity: $v = \frac{d}{t}$.
- Acceleration: $a = \frac{v}{t}$.
- Vectors (specifically the concept of a direction in addition to magnitude).

Key points: You will learn about and use:

$\boldsymbol{s}$	Displacement	m
$\boldsymbol{u}$	Initial velocity	m/s
$\boldsymbol{v}$	Final velocity	m/s
$\boldsymbol{a}$	Acceleration	m/s^2
$\boldsymbol{t}$	Time	s

Equation	Quantities involved				
$v = u + at$		**u**	**v**	**a**	**t**
$s = vt - \frac{1}{2}at^2$	**s**		**v**	**a**	**t**
$s = ut + \frac{1}{2}at^2$	**s**	**u**		**a**	**t**
$s = \frac{u+v}{2}t$	**s**	**u**	**v**		**t**
$v^2 = u^2 + 2as$	**s**	**u**	**v**	**a**	

Mechanics – Kinematics – Questions

1. A car accelerates at a rate of $5m/s^2$ from an initial speed of $12m/s$ for 8 seconds. Find the final speed.

2. A 2007 Aston Martin V8 Vantage accelerates from 0 to $60mph$ $(26.8m/s)$ in 4.8 seconds. Find the distance the car would travel during this time, assuming acceleration is at a constant rate.

3. Felix Baumgartner performed a freefall jump from a height of $40km$ (to reduce to practically nothing the effect of air resistance). His constant acceleration due to gravity was $9.8m/s^2$.
a) Find the distance he fell during the first 10 seconds of motion.

b) Find the distance travelled during the next 10 seconds. (Hint: find the total displacement for 20 seconds)

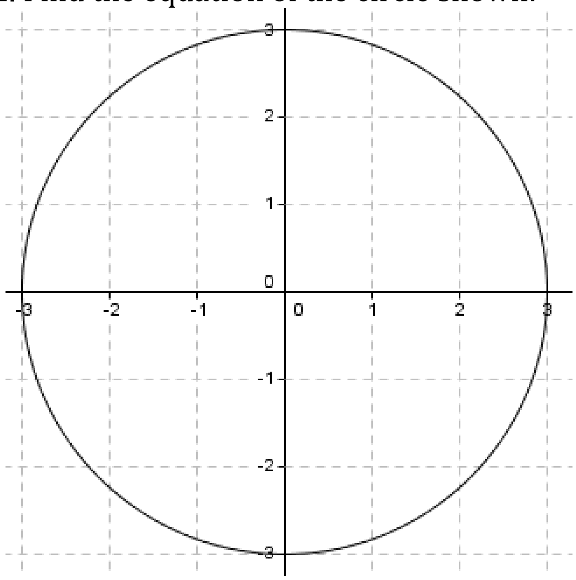
4. Top Gear raced a Porsche 911 against a VW Beetle. The Porsche drove along a mile-long track $(1600m)$ while the Beetle was dropped from a helicopter a mile above the finish line.
a) The Porsche accelerates from rest at a constant rate of $4.5m/s^2$ for the first 10 seconds, then continues at a constant speed. Find the top speed, the distance covered in the first 10 seconds and hence the total time taken to reach the finish.

b) Ignoring air resistance, calculate the time taken for the Beetle to reach the ground. Take the acceleration due to gravity as $9.8m/s^2$.

c) Explain why your answer to b) would, in reality, be an overestimate.

Core Maths – Coordinate Geometry of Circles – SOLUTIONS

1. Find the equation of the circle shown:



The centre is at (0,0) and the radius is 3, so:

$$\boldsymbol{x^2 + y^2 = 9}$$

2. Find the equation of a circle centred at the origin with radius $\sqrt{5}$.

$$\boldsymbol{x^2 + y^2 = 5}$$

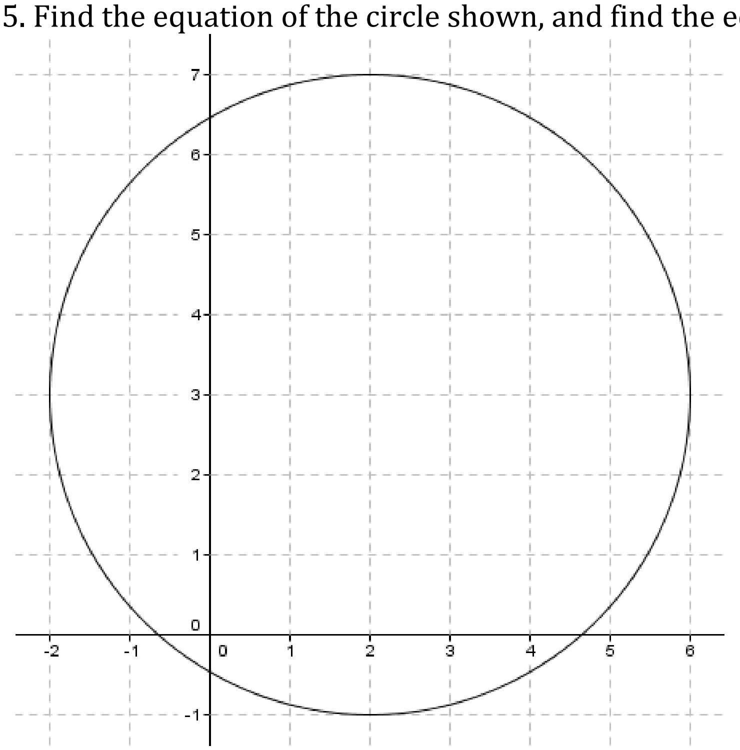
3. Write down the radius of the circle with equation $x^2 + y^2 = 36$.

$$Radius = \mathbf{6}$$

4. Find the centre and radius of the circle with equation $(x-2)^2 + (y+1)^2 = 8$

$$Centre: (\mathbf{2}, \mathbf{-1}) \quad Radius = \sqrt{8} = \mathbf{2\sqrt{2}} \approx 2.83$$

5. Find the equation of the circle shown, and find the equation of a diameter line passing through (0,0):



The centre is at (2,3) and the radius is 4, so:

$$\boldsymbol{(x-2)^2 + (y-3)^2 = 16}$$

The diameter passing through (0,0) must also pass through the centre, (2,3).

Writing the line in the form $y = mx + c$, since the y-intercept is 0, the line is:

$$y = mx$$

Since the point (2,3) is on the line:

$$3 = m(2) \quad \Rightarrow \quad m = 1.5$$

So the line equation is:

$$\boldsymbol{y = 1.5x}$$

Statistics – Binomial Distribution – SOLUTIONS

1. How many ways can the letters of the word 'thousand' be rearranged?

$$8! = 8 \times 7 \times 6 \times 5 \times 43 \times 2 \times 1 = \mathbf{40320}$$

2. A team of 5 is to be chosen at random from each class of students in a school. How many different teams are possible for a class of 30 students?

$${}_{30}C_5 = \mathbf{142506}$$

3. A 5-card poker hand is dealt from a deck of 52 unique playing cards. How many possibilities are there?

$${}_{52}C_5 = \mathbf{2598960}$$

4. What is the probability that a fair coin tossed 20 times shows heads exactly 10 times?

$${}_{20}C_{10} \times \left(\frac{1}{2}\right)^{10} \times \left(\frac{1}{2}\right)^{10} = \frac{184756}{1048576} = \mathbf{17.6}\%$$

5. a) What is the probability that a fair six-sided die shows a 3 exactly once in 6 throws?

$${}_{6}C_1 \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^5 = \mathbf{40.2}\%$$

b) What is the probability that it shows a 3 exactly twice?

$${}_{6}C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^4 = \mathbf{20.1}\%$$

c) What is the probability that it never shows a 3?

$${}_{6}C_0 \times \left(\frac{1}{6}\right)^0 \times \left(\frac{5}{6}\right)^6 = \mathbf{33.5}\%$$

d) What is the probability that it shows a 3 on at least half of the six throws?

$$100\% - 40.2\% - 20.1\% - 33.5\% = \mathbf{6.23}\%$$

6. A game pays out if you can throw five dice and get the same number on each. The chance of this happening is $\frac{1}{1296}$. Find the probability that, after 1000 people play the game, no more than 2 people win.

$${}_{1000}C_0 \times \left(\frac{1}{1296}\right)^0 \times \left(\frac{1295}{1296}\right)^{1000} + {}_{1000}C_1 \times \left(\frac{1}{1296}\right)^1 \times \left(\frac{1295}{1296}\right)^{999} + {}_{1000}C_2 \times \left(\frac{1}{1296}\right)^2 \times \left(\frac{1295}{1296}\right)^{998}$$

$$= 46.2\% + 35.7\% + 13.8\% = \mathbf{95.7}\%$$

Mechanics – Kinematics – SOLUTIONS

1. A car accelerates at a rate of $5m/s^2$ from an initial speed of $12m/s$ for 8 seconds. Find the final speed.

$$v = u + at = 12 + 5 \times 8 = \boldsymbol{52m/s}$$

2. A 2007 Aston Martin V8 Vantage accelerates from 0 to $60mph$ $(26.8m/s)$ in 4.8 seconds. Find the distance the car would travel during this time, assuming acceleration is at a constant rate.

$$s = \frac{u+v}{2}t = \frac{0+26.8}{2} \times 4.8 = \boldsymbol{64.32m}$$

3. Felix Baumgartner performed a freefall jump from a height of $40km$ (to reduce to practically nothing the effect of air resistance). His constant acceleration due to gravity was $9.8m/s^2$.
a) Find the distance he fell during the first 10 seconds of motion.

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 9.8 \times 10^2 = \boldsymbol{490m}$$

b) Find the distance travelled during the next 10 seconds. (Hint: find the total displacement for 20 seconds)

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 9.8 \times 20^2 = 1960m \quad \Rightarrow \quad 2^{nd}\ 10\ seconds: \quad 1960 - 490 = \boldsymbol{1470m}$$

4. Top Gear raced a Porsche 911 against a VW Beetle. The Porsche drove along a mile-long track $(1600m)$ while the Beetle was dropped from a helicopter a mile above the finish line.
a) The Porsche accelerates from rest at a constant rate of $4.5m/s^2$ for the first 10 seconds, then continues at a constant speed. Find the top speed, the distance covered in the first 10 seconds and hence the total time taken to reach the finish.

$$v = u + at = 0 + 4.5 \times 10 = \boldsymbol{45m/s}$$

$$s = \frac{u+v}{2} \times t = \frac{0+45}{2} \times 10 = \boldsymbol{225m} \quad or \quad s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 4.5 \times 10^2 = \boldsymbol{225m}$$

Distance to be covered at a constant speed of $45m/s$: $1600 - 225 = 1375m$
Time taken (at constant speed): $speed = \frac{distance}{time} \quad \Rightarrow \quad 45 = \frac{1375}{t} \quad \Rightarrow \quad t = \frac{1375}{45} = \boldsymbol{30.6s\ to\ 3\ s.f.}$

b) Ignoring air resistance, calculate the time taken for the Beetle to reach the ground. Take the acceleration due to gravity as $9.8m/s^2$.

$$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad 1600 = 0 + \frac{1}{2} \times 9.8 \times t^2 \quad \Rightarrow \quad \frac{1600}{4.9} = t^2 \quad \Rightarrow \quad \boldsymbol{t = 18.1s\ to\ 3\ s.f.}$$

c) Explain why your answer to b) would, in reality, be an overestimate.

In reality, air resistance would have a large effect on a large object like a car falling at high speed. If the object only fell a short distance, it wouldn't get fast enough for air resistance to make much difference, but falling a mile means the car is limited by terminal velocity (around 100mph) so it can't just keep getting faster.