Summary Method M2 (Jun '13)

1.

a)

Use the fact that velocity is the rate of change of displacement with respect to time. That is, $v = \frac{ds}{dt}$.

b)

Differentiate again to find an expression for acceleration from your expression for velocity, then apply $F = ma$ substituting in the value for t and for m.

2.

a)

Use $KE = \frac{1}{2}mv^2$ to calculate kinetic energy.

b)

Use conservation of energy, calculating the final kinetic energy by equating initial and final energy. Initially the energy is a combination of kinetic and potential (taking the starting point as $h = 8$), and the final energy is all kinetic.

c)

Rearrange $KE = \frac{1}{2}mv^2$ to find speed given the kinetic energy calculated in part b).

3.

a)

Integrate acceleration to find an expression for velocity, remembering to include a constant of integration in each part of the vector. Determine the value of these constants using the specific conditions provided.

b)

To determine initial speed, first substitute $t = 0$ into your expression to find initial velocity, then calculate the magnitude of this using Pythagoras to convert to speed.

4.

a)

i.

The most efficient method would be taking moments about the point P, then the point Q. This eliminates one of the two unknown tensions allowing you to calculate the other. Alternatively, resolve vertically for a link between the two, then choose any point to take moments about for a second equation linking the two and solve simultaneously.

ii.

Uniform means the mass is considered to be evenly distributed across the length of the plank, so the mass can be modelled as acting exactly in the centre. Another way of putting this is to say that the centre of mass can be considered to be exactly in the middle.

b)

Use the new force (mg), and relabel the tensions using the fact that they are equal. Taking moments about B would yield an equation in T, and resolving vertically allows you to find the value of m, given T.

5.

Resolve radially (towards the centre) and use the fact that the only force acting in this direction is friction, which can be linked to the normal reaction using $F_r = \mu R$ since the phone is on the point of slipping outwards if we need the least possible value of r (that is, limiting equilibrium). This radial force must be equal to $\frac{mv^2}{r}$ since it is the centripetal force causing circular motion. Rearrange and solve for r.

6.

a)

Use $P = F_m v$ to link the power output of the car to its motive force, resolve in the direction of motion to determine the resultant force (taking into account the resistive force acting against motion), and use $F = ma$ to link to acceleration. Recall that $a = \frac{dv}{dt}$, and use this to formulate the differential equation.

b)

Solve the differential equation using the separation of variables method. Recall that $\int \frac{f'(x)}{f(x)} = \ln f(x) + C$. Use the initial statement of the question to determine a value for C.

7.

Use $P = F_m v$ to link power output to motive force, resolve in the direction of motion to determine the resultant force and use $F = ma$ to find acceleration.

8.

a)

Use conservation of energy to form an equation in u, using the results $GPE = mgh$ and $KE = \frac{1}{2}mv^2$. Rearrange and simplify.

b)

Use energy considerations to determine the speed of the particle at S. Note that you should expect this to be between $2u$ and $5u$ since this is the range of speeds experienced by the particle. Finally, resolve forces radially and set equal to $\frac{mv^2}{r}$ to determine the magnitude of R.

9.

a)

i.

Resolve forces for particle A, taking into account friction and tension in the string. To calculate tension, use Hooke's law: $T = \frac{\lambda x}{l}$. Determine that there is a positive resultant in the direction of the hole even at maximum friction.

ii.

Resolve forces for particle B, taking into account the weight acting down and tension acting up. Determine that there is a positive resultant upwards.

b)

Use the formula for elastic potential energy $EPE = \frac{\lambda e^2}{2l}$.

c)

The system initially has total energy equal to the elastic potential energy in the string. Energy is lost from the system through work done against friction. This can be found using $Work\ Done = Force \times Distance$. By the time particle B is at rest for the first time, the total energy of the system will be a combination of gravitational potential for B (the GPE for particle A doesn't change so can be ignored) and elastic potential in the string. Use $Initial\ Energy - Work\ Done\ Against\ Friction = Final\ Energy$.
Next, form an algebraic expression for the extension in the string after A has moved $0.46m$ closer to the hole and B has moved upwards h metres. Finally combine these to give a quadratic equation in h which can be solved, and the solutions examined to determine which one fits the requirements of the situation.