Summary Method M2 (Jan '13)

1.

a)

Use the formula $KE = \frac{1}{2}mv^2$.

b)

Use the formula $GPE = mgh$, where h is the increase in height.

c)

i.

Use conservation of energy. The gain in gravitational potential, found in part b), is equal to the drop in kinetic energy from the value found in part a).

ii.

Use the kinetic energy formula with the value found in part c)i., and work backwards to find v.

2.

a)

Use the fact that $\boldsymbol{a} = \frac{dv}{dt}$. Differentiate the expression for $\boldsymbol{v}$ with respect to time, treating i and j components separately. The result should be a vector expression for $\boldsymbol{a}$.

b)

i.

Use $\boldsymbol{F} = m\boldsymbol{a}$ to find an expression for the vector quantity $\boldsymbol{F}$, given the acceleration found in part a).

ii.

Substitute the value $t = 3$ into your expression for $\boldsymbol{F}$ from part b)i., then use Pythagoras to find the magnitude of this vector.

c)

Use the fact that $\boldsymbol{v} = \frac{dr}{dt}$. Integrate the expression for $\boldsymbol{v}$ with respect to time to find an expression for the displacement, $\boldsymbol{r}$. Use the conditions given to find the value of the constant of integration (note that this should also be in the form of a vector, with i and j components).

3.

Resolve forces up the slope, taking into account the motive force of the van, the resistance force and the down-slope component of the weight. Since the van is travelling at a constant speed, all forces will be in equilibrium. Use the fact that $P = F_m v$ where F_m is the motive force of the engine to find the power output, P. Finally, remember to convert your answer to kilowatts.

4.

a)

Recall that the centre of mass of a uniform lamina must lie on any lines of symmetry of the lamina.

b)

Add up the moments of each part of the shape, and set this equal to the total moment of the shape (using the total mass or area, and the unknown distance to the centre of mass, $\bar{x}$. Finally, solve for $\bar{x}$.

c)

Recall that when an object is freely suspended from a point, the centre of mass will lie vertically below the point of suspension. A line drawn from H to the centre of mass is therefore vertical, and the angle to HG can be calculated using right-angled trigonometry. Finally, the angle between HG and the horizontal can be found.

5.

a)

Use the formulae $F = ma$ and $a = \frac{dv}{dt}$ to construct a differential equation linking $\frac{dv}{dt}$ to v. Solve by separating variables and integrating, using the initial conditions given to find the value of the constant of integration. Rearrange to make v the subject.

b)

Substitute the value $v = 0$ into the result from part a), and solve for t.

6.

a)

Resolve forces acting on the particle P vertically, using the fact that the particle is in equilibrium vertically to equate and hence find θ.

b)

Resolve forces radially (that is, towards the centre of the circle, B, not towards the end of the string at A) and use the formula $F = \frac{mv^2}{r}$ to find the speed v for a centripetal force F.

c)

Use the formula $v = r\omega$ to find the angular speed using the speed v calculated in part b). Use the result $T = \frac{2\pi}{\omega}$ to find the period of circular motion (that is, the time taken to complete a revolution). Note that this result can be derived relatively easily from the definition of angular speed.

7.

a)

Use conservation of energy to determine the kinetic energy, and therefore the speed, of the ball at A. Use the fact that the particle has gained gravitational kinetic energy at the expense of kinetic energy, and the formula mgh where h is the increase in height. Note that the increase in height can be found by considering the right angled triangle made by the points O, A and the downward vertical from O. The hypotenuse of this triangle is the radius of the circle, and the angle at O is known.

b)

Resolve forces towards the centre of the circle, taking into account the weight which will have a component acting directly away from the centre. Use the centripetal force formula $F = \frac{mv^2}{r}$, using the value of v found in part a).

8.

a)

Use Hooke's law, $T = \frac{\lambda x}{l}$, and recall that the work done by a variable force $F(x)$ over a given distance is given by $Work\ Done = \int F(x)\,dx$. Integrate the expression for T between the limits 0 and e to derive the formula for elastic potential energy.

b)

i.

Use $T = \frac{\lambda x}{l}$ and resolve forces vertically. Recall that if the particle hangs in equilibrium, the upward force, tension, will exactly balance the downward force (weight).

ii.

Calculate the extension in the string by subtracting the natural length from the distance below O, then use the formula for elastic potential energy $EPE = \frac{\lambda e^2}{2l}$.

iii.

Use conservation of energy, taking into account kinetic, gravitational potential and elastic potential at each point. At A, note that the only energy (counting the height of A as 0), will be elastic potential energy, which was calculated in part b)ii. At the moment when the speed reaches $0.8ms^{-1}$, you can calculate kinetic energy then form an equation for the remaining energy; a combination of EPE and GPE, both of which will depend on the height above A. Solve to find this height.

9.

a)

Recall that the normal reaction force from a surface will always act at right angles to the surface. For a sphere, at right angles to the surface will be along a radial line, which is always towards the centre.

b)

There should be a normal reaction force at A, pointing towards O as established in part a). The weight of the rod will be acting at a distance $\frac{l}{2}$ from A, and vertically downwards. Finally, the reaction at the point C must be pointing at right angles to the rod AB because both the rod and the hemisphere are smooth. If there is no friction, there can be no component of force acting along the direction of the rod at this point. Note that this diagram is crucial to being able to tackle the final part of the question.

c)

First, use the geometry of the bowl to determine the length AC in terms of a and θ. Also use the triangles within the diagram to determine the angle between the normal reaction at A and the rod AB. Resolve forces horizontally, using the fact that the rod is in equilibrium. This will yield an equation involving the normal reaction at A (R_A), the normal reaction at C (R_C) and θ. Next, take moments about the midpoint of the rod. This will enable you to form a second equation linking R_A and R_C without involving mg. This equation will involve a and l. Finally, since we need to eliminate both R_A and R_C, rewrite both of the equations to give $\frac{R_A}{R_C} = \cdots$ and equate them. Rearrange the resulting equation in l, a and θ to give the result in the desired form. Note that it may be necessary to use trigonometric identities such as $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = cos^2\theta - \sin^2\theta$ and $\sin^2\theta + \cos^2\theta = 1$.