Summary Method M2 (Jun '12)

1.

a)

Use $KE = \frac{1}{2}mv^2$ to find the kinetic energy, using the mass and the speed of the jumper.

b)

The change in gravitational potential energy is given by $GPE = mgh$ where h is the change in height.

c) i.

Use the conservation of energy principle, calculating the total energy at point A and point B. Alternatively, since all energy in the system is either kinetic or gravitational potential, use your answer to part b) – the gravitational potential lost – to determine how much kinetic energy has been gained, and add this to the original kinetic energy found in part a).

ii.

Use the kinetic energy formula to calculate v now you know the total kinetic energy.

2.

a) i.

Recall that $a = \frac{dv}{dt}$, so to find the acceleration it is only necessary to differentiate the expression for v with respect to t. Recall that e^{kt} differentiates to ke^{kt}.

ii.

Substitute $t = 0.5$ into your expression for the acceleration from part a)i.

b)

Use $F = ma$ to find the force for the acceleration calculated in part a)ii.

c)

Recall that $v = \frac{dx}{dt}$, so to find the displacement it is necessary to integrate v with respect to t. Note that you will need the initial conditions to determine the value of the constant of integration. Do not just assume it will be 0 because it starts at the origin – this is not always the case.

3.

a) i.

Find the moment of each lamina from AB, and add to give the total moment of the system. Add the masses to give the total mass of the system, and use $m\bar{x}$ to form an equation for the total moment of the system. Solve for $\bar{x}$.

ii.

Do the same, but measuring from AD instead of AB, to find the distance of the centre of mass from AD.

b)

Draw a force diagram, and take moments about either A or B (A would be best, as you already have the distance of the centre of mass from AB and AD). This will give you a value for the tension in the string attached at B. Resolve forces vertically to find the tension in the string attached at A.

4.

a)

For circular motion to take place, it is sufficient to demonstrate that the distance from the centre is constant. Find the magnitude of $\boldsymbol{r}$, recalling the fact that $\sin^2 t + \cos^2 t = 1$.

b)

Since $\boldsymbol{v} = \frac{dr}{dt}$, it is only necessary to differentiate our expression for $\boldsymbol{r}$ (differentiating i and j components separately, to give a vector expression for $\boldsymbol{v}$).

c)

Differentiate velocity to find acceleration.

d)
Compare the expressions for $\boldsymbol{r}$ and $\boldsymbol{a}$. The value of k should be readily apparent.
e)
Note that k is negative, meaning the acceleration is always pointing in the opposite direction to displacement. Since displacement is measured from the origin, acceleration must be acting towards the origin (that is, towards the centre of the circle, as you would expect for circular motion).

5.
a)
Resolve forces vertically for the particle B, using the fact that it hangs in equilibrium, to find the tension in the string. Resolve forces radially for the particle A using this value for the tension, and use the result $F = m\omega^2 r$ to find the angular speed ω.
b)
Use the result $v = r\omega$ to find the speed.
c)
The time period T is given by $T = \frac{2\pi}{\omega}$ since ω gives the number of radians traversed in one second.

6.
a)
Maximum speed will be reached at the lowest point of motion. Use the fact that, at the greatest height reached, his speed will be 0, and use conservation of energy to find the kinetic energy, and hence the speed, at the lowest point.
b)
Resolve towards the centre at the lowest point, and use the fact that centripetal force is given by $F = \frac{mv^2}{r}$.

7.
a)
Resolve forces in the direction of motion to find an expression for the resultant force acting on the stone. Use $F = ma$ to find an expression for acceleration in terms of velocity.
b)
Solve the differential equation by separating the variables, integrating and using the initial conditions given to find the constant of integration.

8.
a)
Use conservation of energy to find the total energy of the block when it reaches A. Recall that $EPE = \frac{\lambda e^2}{2l}$ and $KE = \frac{1}{2}mv^2$. Note that all elastic potential energy will be converted to kinetic by the time the block reaches A (in fact, the string goes slack once the block is within $5m$ of A). When you have found the total energy at A, and noted that this is all kinetic energy, use this value to find the speed.
b) i.
The total energy at A will be equal to the total energy at B minus the work done against the frictional force. Recall that $Work\ Done = Force \times Distance$, and use $F_r = \mu R$ for friction during motion. Once the new value for the final energy (which, again, will all be kinetic) is found, the speed can be calculated.
ii.
Energy is lost to the wall through the collision, so consider the total energy of the system immediately after the collision – it will all be kinetic, and the speed will be half of that calculated in part b)i. Given that all kinetic energy is either transferred to elastic potential or lost to friction between A and B, form an equation from your expressions for energy and solve to find μ.