Summary Method C4 (Jun '12)

1.
a)
i.
Add the two fractions on the right and use the fact that the numerators will then be identical to find A and B.
ii.
Use your partial fractions to rewrite the integral in a more manageable form. Recall that $\int \frac{f'(x)}{f(x)} = \ln f(x)$.
b)
i.
Use inspection or multiply out the RHS and equate coefficients.
ii.
Use the form given in part b)i. to rewrite the integral, rearrange and evaluate.

2.
a)
Use the $\sin(A - B)$ formula to rewrite $R\sin(x - \alpha)$, then equate coefficients of $\sin x$ and $\cos x$. Recall that $R = \sqrt{a^2 + b^2}$.
b)
Rewrite using the form given in a), rearrange and solve for $x - \alpha$ (changing the solution range accordingly), then convert to find all solutions for x. Ensure you round your answers as required.

3.
a)
Use the binomial expansion formula in the formula book if needed.
b)
i.
Take out 4 as a factor (giving $4^{-\frac{1}{2}}$ at the front) to change the number in the bracket to a 1, then expand using the binomial expansion formula.
ii.
Recall that the expansion $(1 + A)^n$ is only valid (when n is negative or fractional) when $|A| < 1$. Rearrange to give your inequality in terms of x.
c)
Rewrite as brackets, note the connection to the expansions already calculated in parts a) and b), and use your expansions to find this one.

4.
a)
i.
Substitute the values given into the formula and round your answer as required.
ii.
Substitute the values into the formula, rearrange (using logarithms where necessary) and solve for n. Note that N is the number of complete years, so the value of n which solves the equation will need to be rounded up to find N.
b)
Substitute each set of values into the formula and put the expressions for V equal to one another. As before, round up to find the number of complete years, T.

5.

a)

i.

Find $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ and use the chain rule to combine them for $\frac{dy}{dx}$.

ii.

The gradient of the normal will be $\frac{1}{\frac{dy}{dx}}$, or $\frac{dx}{dy}$. Substitute in the value of θ and evaluate.

b)

Find expressions for x^2 and y^2 in terms of θ, and demonstrate that the LHS is identical to the RHS.

6.

Differentiate this equation implicitly, making sure to properly apply the product rule where appropriate. Set $\frac{dy}{dx} = 0$ to form an equation in x and y. Solve simultaneously with the original equation to find all coordinates of stationary points.

7.

a)

If two lines intersect, their general points are equal for some value of μ or λ. Put the general points equal and solve the resulting simultaneous equations to find μ, λ and q. Substitute your value of μ or λ back into one of the general points to find the coordinates of P.

b)

If two lines are perpendicular, the dot product of their direction vectors is 0.

c)

i.

Substitute $\lambda = 1$ into the general point of l_1 to find the point A. AP^2 means the square of the distance between A and P (since they are points, not vectors, AP represents the length of the line, not the vector $\overrightarrow{AP}$). Find the vector $\overrightarrow{AP}$ and calculate $\left|\overrightarrow{AP}\right|^2$.

ii.

A diagram would be a good idea here. A is on l_1, B on l_2 and P on both. The point B could be either 'above' the point of intersection (P), or 'below'. Either way, the length of BP must be equal to the length of AP, which was the subject of part c)i. Use a general point on l_2 to represent B, and calculate the length $\left|\overrightarrow{BP}\right|$. Setting $\left|\overrightarrow{AP}\right|^2 = \left|\overrightarrow{BP}\right|^2$ is the easiest way of finding the two solutions (they will be two different values of μ which can then be substituted into the general point of l_2 to find the two possible positions of B).

8.

a)

Interpret the information in the form of a differential equation. Recall that 'the rate at which the depth of the water in the tank increases' means $\frac{dh}{dt}$.

b)

i.

Use separation of variables to write with in the form $f(x)dx = f(t)\ dt$. Integrate (a substitution might be useful here) and solve, using the initial conditions given to find the value of the constant of integration.

ii.

Substitute $x = 2$ into your solution equation and solve for t, making sure you round your answer as required.