Summary Method C4 (Jun '09)

1.

a)

Recall that when $f(x)$ is divided by $(ax + b)$, the solution to $(ax + b) = 0$ is the number to substitute into $f(x)$ to find the remainder.

b)

Add the terms on the right together by making a common denominator, and multiply out, equating coefficients, to find values for a, b and c. Alternatively, use long division to divide the numerator by the denominator. Note that the value of c is the remainder calculated in part a).

2.

a)

Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$ then divide to find an expression for $\frac{dy}{dx}$. Remember to simplify your answer.

b)

Substitute $t = 1$ into your expression for $\frac{dy}{dx}$ to find the gradient, then use $m_1 = -\frac{1}{m_2}$ for the perpendicular (gradient of the normal). Substitute $t = 1$ into the original equations for x and y to find the coordinates of the point, and use $y - y_1 = m(x - x_1)$ to form a straight line equation.

c)

Find x^2 and $2xy$ in terms of t using the equations given at the start, combine and simplify to give an answer in the required form.

3.

a)

Use the binomial expansion formula (in the formula book), being careful with the negative sign. Note: this expansion is a very common one which you may even choose to learn by heart.

b)

i.

Use common denominators to add the fractions on the right, then either substitute useful values for x to eliminate A or B, or equate coefficients of the numerators to find the values of A and B.

ii.

Write the fraction in the partial fractions form from part b)i, then use the binomial expansion from part a) and form a similar expansion for $(2 - 3x)^{-1}$, combine and simplify.

c)

Use the validity inequality for the binomial expansion separately for each part, then apply the strictest one (ie, both must be satisfied).

4.

a)

i.

When $t = 0$, $k^t = 1$, so A must be the initial value of the car.

ii.

Substitute the information given into the formula (36 months and £7000) and solve for k.

b)

Substitute into the formula and solve for n, using logarithms to deal with the exponential component.

5.

Differentiate this equation implicitly and substitute the values given for x and y. Solve for $\frac{dy}{dx}$. Note: you will need to use the product rule for $3xy$.

6.

a)

i.

Use the $\cos^2 x$ form of the double angle formulae for $\cos 2x$, rearrange and simplify.

ii.

Solve the quadratic to find solutions for $\cos x$ using any valid method (although usually in such questions the quadratic will factorise nicely).

b)

i.

Write $\sin(\theta + \alpha)$ in terms of $\sin\theta$, $\sin\alpha$, $\cos\theta$ and $\cos\alpha$, compare to the expression given and equate coefficients of $\sin\theta$ and $\cos\theta$. Recall that $R^2 = a^2 + b^2$ and be careful that your solution for α is valid for both equations produced by equating coefficients.

ii.

Write the equation in the form $R\sin(\theta + \alpha) = 4$ and solve for θ. Remember to adjust the solution range accordingly if you intend to find solutions for $\theta + \alpha$ before converting into solutions for θ, and make sure you give all solutions in the range.

c)

i.

Write $\tan\beta$ in terms of $\cos\beta$ by using $\tan x = \frac{\sin x}{\cos x}$ and $\sin^2 x + \cos^2 x = 1$, then rearrange to find $\cos\beta$.

ii.

Use the double-angle formulae for $\sin 2x$ to write in terms of $\sin\beta$ and $\cos\beta$, then eliminate $\sin\beta$ using $\tan x = \frac{\sin x}{\cos x}$. Alternatively use $\sin^2 x + \cos^2 x = 1$ to write in terms of $\cos\beta$ exclusively. Finally, simplify and give your answer in the required form.

7.

a)

Use 3-D Pythagoras on the coordinates, or write as column vectors and find $|\overrightarrow{AB}|$ where $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$.

b)

Put the vector $\overrightarrow{OB}$ equal to the general point of the line l_1 and demonstrate that the three resulting equations are consistent (ie they yield a single value for λ).

c)

Find the point of intersection C by making the general points of l_1 and l_2 equal to one another and solving the resulting system of three simultaneous equations in μ and λ. Substitute one of these values back into the appropriate general point to find the coordinates of C. To show that A, B and C form an isosceles triangle it is sufficient to show that any two of $\overrightarrow{AB}$, $\overrightarrow{AC}$ or $\overrightarrow{BC}$ are equal in length. Alternatively the angles between the three vectors can be found, but this would require more work.

8.

a)

Rearrange to separate the variables onto the appropriate sides, and integrate. Use results from the formula book if necessary to deal with $\cos 2t$. Substitute in the values given and rearrange to leave in the required form.

b)

i.

Substitute $t = 13$ into your solution and solve for x.

ii.

Substitute $x = 11$ into your solution and solve for t. Note that this will involve solving a trigonometrical equation, and to find the time when the cradle is *first* at this height you will need to choose the appropriate solution from the trigonometric graph.