Summary Method C4 (Jun '08)

1.
a)
Substitute the appropriate value for x into $f(x)$ and find out what you get.
b)
i.
Substitute the value into the function.
ii.
Use the previous part to determine a factor, find the quadratic remaining and factorise it.
iii.
Use your previous factorisation, and factorise the numerator and denominator. Cancel any factors you can.

2.
a)
Find expressions for $\frac{dx}{dt}$ and $\frac{dy}{dt}$, then use these to find $\frac{dy}{dx}$ for the given value of t.
b)
Use your gradient to find the gradient of the normal, substitute the value of t into the original equations to find x and y values, and use the straight line equation formula.
c)
Find a way of eliminating t in the original equations in order to write an equation in just x and y.

3.
a)
Use the $\sin(A+B)$ formula from the formula book, and simplify.
b)
Use your relationship, rearranged, to write the integral in a simpler form, then integrate.

4.
a)
i.
Use the formula in the formula book to find the binomial expansion, remembering to use $(-x)$ instead of x.
ii.
Rearrange the bracket by taking 81 out (remember this produces an $81^{\frac{1}{4}}$ outside the bracket), then substitute the term in place of x into your previous expansion.
b)
Find an appropriate value for x which makes the bracket equal 80, then substitute this into the expansion.

5.

a)

i.

Use the identity $\sin^2\theta + \cos^2\theta \equiv 1$ to rearrange and solve for $\cos\alpha$ (using the positive solution since α is an acute angle.

OR

Construct a right-angled triangle with the appropriate side lengths, use Pythagoras to complete it and find the appropriate ratio of sides for $\cos\alpha$.

Note: Check your answer works for both, by using a calculator.

ii.

Use the $\cos(A - B)$ formula from the formula book.

iii.

Use a similar method to part i to determine $\sin\beta$, then substitute into your formula.

b)

i.

Use the $\tan(A + B)$ formula from the formula book, and rearrange to form a quadratic.

ii.

Substitute $22\frac{1}{2}°$ into your quadratic for x, and solve (given the solution they require, don't expect the quadratic to factorise).

6.

a)

Start with the final expression, and add the fractions to form a single fraction with the same denominator as the original expression but a numerator involving A and B. Equate the numerators and either compare coefficients of x and constants separately to form simultaneous equations or substitute strategic values in for x to eliminate either A or B to determine their values.

b)

Write the integral as the sum of two linear fractions (as you have done in part i), then use the integral result (in the formula book): $\int \frac{f'(x)}{f(x)} = \ln f(x)$.

c)

Separate the variables, so any expressions involving x are on the right-hand side with the dx and any expressions involving y are on the left with the dy. Then integrate both sides. Notice that the right-hand side integral will be the same as (or closely related to, depending on how you separate) in part b. Next, substitute the initial condition values given to find C and simplify.

7.
a)
To find the distance between two points, use 3-D Pythagoras. Find the vector between the two points, then find the magnitude of it by summing the squares of its elements and square rooting.
b)
The angle between two vectors is found using the dot product. The vectors to use are the vector $\overrightarrow{AB}$ and the direction vector of line l: $\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$. Use the dot product formula $a \cdot b = |a||b| \cos\theta$, and remember that the angle will be the angle between the two lines in their positive direction. To find the acute angle it would be necessary to subtract from 180° if your original angle were obtuse.
c)
First visualise the situation (draw a diagram). If the length AB is equal to the length AC then ABC forms an isosceles triangle, with the base being part of line l. Since C must lie on line l, write it as the general point of that line (for some value of λ). Then find the vector $\overrightarrow{AC}$ in terms of λ, and generate an expression for the magnitude of this vector. Set this equal to the magnitude of the vector $\overrightarrow{AB}$, and solve the resulting quadratic for λ. You should end up with two possible solutions, one of which represents the point B (also on line l) and therefore the other must be the point C.

8.
a)
i.
Recall that 'rate' means the derivative with respect to time, and the fact that it is decreasing means it will be negative (a negative sign will be required since the constant of proportionality is defined to be positive).
ii.
Using the initial conditions given, substitute directly into your differential equation to find k.
b)
i.
Substitute the values given into this new equation to find A.
ii.
Set $P = 1900$ and solve for t. This will give the minimum time to reach 1900 exactly, then you need to convert to years and write down the year *during which* the number exceeds 1900. So at the start of this year it had not exceeded 1900, but at the start of the following year it had.